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Taxation and Wage Subsidies in a Search-Equilibrium Labor Market

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Labor Market

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Abstract

This paper studies the equilibrium outcomes of levying various types of taxes and subsidies on labor in an equilibrium labor search market. It is shown that despite the existence of labor market frictions, there is tax equivalence between an income tax and a payroll tax, exactly as in a competitive labor market. In addition, it is shown that a proportional subsidy has a non-linear, regressive impact on gross and net wages in equilibrium and hence it increases wage inequality. At the same time, a wage subsidy that guarantees a minimum net income has a dampening effect on wages above the guaranteed minimum since it reduces the degree of competition in the market. Thus, the results indicate that given labor market imperfections, wage subsidies may have some undesirable features.

מיסוי וסבסוד שכר בשוק עבודה לא-משוכלל

עמית פרידמן

תקציר

המאמר בוחן את ההשפעות בשיווי-משקל של הטלת מס ומתן סובסידיה לשכר, וכן את השלכות של הפעלת תוכנית להבטחת הכנסה, בשוק עבודה לא משוכלל בו לעובדים יש אינפורמציה חלקית בלבד על הצעות עבודה. התוצאה הראשונה היא שמס מעסיקים ומס הכנסה המוטל על העובדים הם שקולים, בדומה למצב בשוק עבודה משוכלל, ולכן העובדים והמעסיקים אדישים לסוג המס. תוצאה זו איננה ברורה מאליה, שכן בתהליך של קביעת השכר בשוק עבודה לא-משוכלל יש למעסיקים יתרון על פני העובדים. תוצאה נוספת היא כי לתוכניות המיועדות להעלות את שכרם הפנוי של עובדים המשתכרים שכר נמוך עלולות להיות מספר השלכות לא רצויות: אף שהסובסידיה פרופורציונלית לשכר, יש לה השפעה לא-ליניארית על השכר לפני מס, ולמעשה היא גרסיבית בשיווי-משקל, כלומר, מגדילה את אי-השוויון בהתפלגות השכר. שנית, תוכנית להבטחת הכנסה, המבטיחה לעובדים השלמת שכר עד לגובה מסוים, פוגעת ברמת התחרותיות בשוק העבודה. כתוצאה, השכר במשרות בהן השכר היה גבוה מהשכר המינימלי שמבטיחה התוכנית להבטחת הכנסה, מופחת, והעלייה בתוחלת ההכנסה הפנויה של העובדים קטנה יחסית. תוצאה זו - שסובסידית שכר יכולה להוריד שכר במשרות מסוימות - היא תוצאה ייחודית, הנובעת מניתוח באמצעות מודל של שוק עבודה לא משוכלל שבו נוצרת התפלגות של השכר.

1 Introduction

This paper studies the equilibrium outcomes of levying various types of taxes, particularly wage subsidies, using an equilibrium labor search model. The desire to analyze general equilibrium outcomes of wage subsidies, such as earned income tax credit programs and guaranteed minimum income programs, to assess the general equilibrium effects of taxation and to compare the effects of income and payroll taxes provides the motivation for this paper.

Wage subsidies are a commonly used instrument for raising the net income of low-paid workers. For example, earned income tax credit programs have been adopted in the US, the UK, Canada, France and the Netherlands, while guaranteed minimum income programs have been adopted in the US, Germany, Brazil, Portugal and Israel. The fact that wage subsidies are so commonly used suggests that whether it reflects social preferences or pure political opportunism, raising the net income of low-paid workers and thus reducing income inequality is a widely-shared goal among policy makers.

Generally speaking, equilibrium labor search theory is a natural candidate for analyzing labor market policies since it is a structural framework that can be used for policy analysis but is not subject to the Lucas critique, as argued by Eckstein and van den Berg (2007). It is particularly appropriate for analyzing the effects of taxation since it generates an endogenous wage distribution and can thus be used to analyze the division of the tax burden between workers and firms. In particular, policies that target the lower end of the wage distribution, such as a

guaranteed income program, can be accurately assessed since the theory is based on the concept of an endogenous reservation wage, i.e. the lowest wage workers are willing to work for.

Although equilibrium search theory has been used extensively for policy analysis, it has seldom been used to assess the impact of taxation. As claimed by Alan Manning:

Search theory has proved its value in many parts of labor economics... it has been widely used to analyze the impact of unemployment benefits... but surprisingly little work has been done on the impact of the tax system.

(Manning, 2001)

The exceptions include studies by Wright and Loberg (1987), Ljungqvist and Sargent (1995a, 1995b) and Manning (2001). All these studies, however, treat the pre-tax, gross wage offer distribution as exogenous and therefore do not take into account the impact of taxation on the distribution of gross earnings, even though equilibrium labor search models imply that such an impact does exist, as acknowledged by Manning.¹ Taxation has been analyzed using bargaining models, which are part of a different literature on labor market theories that incorporate information frictions. Examples are Pissarides (1983), Pissarides (1998) and Mortensen and Pissarides (2003). However, these models yield a single wage in equilibrium, and therefore are of limited value in analyzing policies that affect the wage distribution.

¹According to Manning's interpretation, the assumption of an unchanged wage offer distribution is appropriate for cases in which taxation is sectoral and thus does not affect the economy's aggregate wage offer distribution.

This study is based on a model in which the wage offer distribution is determined endogenously, thus taking into account the general equilibrium effect of taxation on gross wages. Use is made of a simplified version of an equilibrium labor search model due to Burdett and Mortensen (1998, BM hereafter) in order to account for labor market frictions. Although the model assumes that firms and workers are homogeneous, the qualitative results are general.²

First, the outcomes of levying a proportional payroll tax or subsidy are analyzed. This is followed by an analysis of a proportional income tax.³ In both cases, it is found that a proportional tax rate has a non-linear impact on the gross wage offer distribution and that a proportional tax is progressive in equilibrium. This result is due to the fact that the elasticity of labor supply in the model is decreasing in wage and that workers have a reservation wage. For negative tax rates, i.e. wage subsidies, this result is reversed, such that a proportional wage subsidy is regressive in equilibrium. The implication is that a proportional wage subsidy is expensive and does not substantially increase the net income of low-paid workers and in addition it increases wage inequality. In order to quantify this theoretical result, a general search model is solved and calibrated.

Next, the equilibrium outcomes of levying an income tax are com-

²The main features of the BM model are robust to the introduction of firm heterogeneity in which low-productivity firms offer lower wages than those offered by high-productivity firms.

³In most countries, both types of tax are levied. Payroll taxes are usually used to finance social security programs and health care insurance, while income tax revenues are not earmarked for any specific purpose. This difference is, however, somewhat artificial. Interestingly, while income tax rates are usually progressive, payroll tax rates in many countries are flat.

pared to those of levying a payroll tax. In both cases the tax burden is divided between workers and firms and the tax wedge narrows the gap between total labor cost and labor productivity. In competitive labor markets, income and payroll taxes are equivalent in the sense that the price and quantity of labor and the division of the tax burden are determined by the relative elasticities of supply and demand for labor and by the total tax wedge, not by the type of tax. It is shown that this *tax equivalence* also holds in an equilibrium labor search model. This result is neither trivial nor intuitive since the wage posting mechanism, which is the wage setting mechanism in equilibrium search theory, gives employers a kind of first-mover advantage which creates an asymmetry between workers and firms. Tax equivalence in matching and bargaining models has been shown to hold in previous studies, such as Cahuc and Zylberberg (2005).

The final section studies the outcomes of a guaranteed minimum income policy, which ensures that the net income of low-paid workers does not fall below some value w^s and therefore is a preferred instrument for policy makers who are interested in reducing poverty. It is shown that such a subsidy has some undesirable features. First, it is an expensive program since some employers react by reducing their wage offers down to the lowest possible wage. Second, it is shown that such a program usually creates a non-degenerate equilibrium in which some firms offer the minimum wage while others engage in competition by offering wages above w^s . This has a dampening effect on wages above the guaranteed minimum and therefore the rise in the expected wage is lower than the rise in wages up to the guaranteed minimum, which is in force throughout

the wage offer distribution, due to the general equilibrium effect of the program. If the guaranteed minimum w^s is set above a critical value, which is lower than some observed wages, all firms will offer the minimum wage in equilibrium as in Diamond (1971). I refer to this case as a *degenerate equilibrium*.

If firms could credibly coordinate their wage offers, they would offer all workers an identical low wage equal to the reservation wage. However, in this case, there is an incentive for an individual firm to offer an arbitrarily small wage increment which will lead to a discontinuous jump in its number of employees but only an infinitesimal fall in profits per employee. This argument explains why the wage offer distribution cannot be degenerate in equilibrium, i.e. it must be *nowhere dense*. Because firms cannot credibly coordinate among themselves to offer the reservation wage, firms choose different combinations of profit per employee and employment that yield the same profit. In contrast to the original equilibrium, the guaranteed minimum wage policy results in a mass of firms that offer an identical low wage equal to the reservation wage. This is because it eliminates the incentive to offer wages lower than the guaranteed minimum income since workers are now indifferent between wages up to that level.

This pool of firms, which offer the minimum wage, equally share the mass of workers who are employed at this wage, such that the profits of all firms rise, including firms that offered the minimum wage before the subsidy was introduced. Thus, the monopsonistic power of firms increases, while competition between firms weakens throughout the wage offer distribution.

The results of the theoretical analysis have some important policy implications for the optimal design of earned income tax credit programs. These programs may suffer from some inherent problems due to their adverse effect on competition and increased wage inequality. These effects can be partially offset by other policy instruments that increase the reservation wage, such as a minimum wage policy. Finally, a flat income tax or payroll tax decreases wage inequality and therefore tax rates do not have to increase in wage in order to reduce wage inequality.

2 The Model

The model is a simplified version of the BM equilibrium search model. Assume L infinitely-lived equally-productive workers and M firms.⁴ Workers are utility (wage) maximizers and firms are profit maximizers. The labor productivity of an employed worker is given by p and the production function is characterized by constant returns to scale.

The wage-setting mechanism is based on wage posting, i.e. firms make take-it-or-leave-it wage offers to workers and there is no bargaining. Wage offers are set to maximize steady-state profits and each firm must make identical wage offers to all of its workers.

Both employed and unemployed workers receive job offers at a rate λ (a simplifying assumption that is relaxed later on). The offers are drawn randomly from the equilibrium wage distribution offer F . Workers leave their jobs at an exogenous destruction rate δ and, while unemployed, receive a flow of benefits b . After accepting a job offer, workers receive a flow of wages w . Time is continuous and workers and firms do not

⁴In the following subsections, both are normalized to 1, unless specified otherwise.

discount the future.

Three types of taxes and labor subsidies are then introduced into the model: a proportional payroll tax or subsidy, which is levied on employers; a proportional income tax or subsidy, which is paid or received, respectively, by employees; and a guaranteed minimum income scheme, which ensures that a worker's net income does not fall below a certain threshold w^s .

The model is solved by applying the following BM equilibrium conditions:

1. Steady State: the flow of employed workers into unemployment equals the flow of unemployed workers into employment.
2. Worker behavior is optimal: utility maximization.
3. Firm behavior is optimal: profit maximization.

In equilibrium, the optimal behavior of workers is to accept job offers greater than or equal to the reservation wage while unemployed and to move to higher-paying jobs while employed. An endogenous non-degenerate wage offer distribution emerges in equilibrium and firms make identical profits with different combinations of profit-per-worker and employment. The reservation wage is set optimally in order to maximize workers' asset value. When taxation is introduced, the relevant wage that fulfils the second and third conditions is not identical. While the relevant wage for the worker's optimization problem is the net wage, the relevant wage from the firm's perspective is the gross wage or, more accurately, the total cost of labor. Thus, the effect of taxation on the pre-tax wage distribution is taken into account by the model.

In the following subsections, basic results are derived for the payroll

tax, income tax and guaranteed income subsidy.

2.1 A Payroll Tax

Assume that a proportional payroll tax τ is imposed on the employer. In this case, the total labor cost of employing a worker who is paid w is given by $(1 + \tau)w$. The solution of the model is derived by applying the three BM equilibrium conditions:

Worker behavior. The assumption that the job offer rate for unemployed and employed workers is identical leads directly to the solution for the optimal reservation wage R . Since the option value of search is zero, workers are indifferent between searching off- and on-the-job. Thus, a worker's optimal reservation is to accept any job offer with a flow of benefits greater than or equal to the flow of benefits received while unemployed. Thus:

$$R = b \tag{1}$$

Steady state. This condition determines equilibrium unemployment, employment per firm posting a wage offer w and the relation between offered and actual wages. In the steady state, unemployment is constant. This means that the flow of unemployed workers to employment equals the flow of employed workers to unemployment:

$$(1 - u)\delta = \lambda u \tag{2}$$

This condition determines the level of equilibrium unemployment:

$$u = \frac{\delta}{\delta + \lambda} \quad (3)$$

Let G be the cumulative distribution function of actual wages. The steady-state condition implies that, for every w , $G(w)$ is fixed over time; hence, the rate at which workers enter the group $G(w)$ equals the rate at which they leave it:

$$F(w)\lambda u = (1 - u)G(w)(\lambda(1 - F(w)) + \delta) \quad (4)$$

Thus, $G(w)$ is given by:

$$G(w) = \frac{\delta F(w)}{(\delta + \lambda(1 - F(w)))} \quad (5)$$

Let $l(w|F)$ represent the employment per firm posting a wage offer w , given a wage offer distribution F . The steady-state condition implies that the rate of recruitment to the firm equals the rate at which workers quit the firm. Thus:

$$l(w|F)(\delta + \lambda(1 - F(w))) = \lambda(u + (1 - u)G(w)) \quad (6)$$

By substituting for $G(w)$, $l(w|F)$ is now given by:

$$l(w|F) = \frac{\lambda\delta}{[\delta + \lambda(1 - F(w))]^2} \quad (7)$$

Equation (7) reflects the monopsonistic nature of the labor market. Unlike a competitive labor market, in which an individual firm faces an infinitely elastic labor supply curve at the market-clearing wage, in

this case, firms face an upward-sloping labor supply curve due to labor market frictions.

Firm behavior. In equilibrium, profit maximization implies that all firms make equal profits. This condition is a necessary one.⁵ The firm's profit function is given by:

$$\pi = (p - (1 + \tau) w) l(w|F) \quad (8)$$

where $(p - (1 + \tau) w)$ equals the after-tax profit per worker for a firm that pays w ⁶ and $l(w|F)$ equals the number of employees in the firm offering a wage w , given a wage offer distribution F . Equating the profit of the lowest-paying firm, which pays $w = R$, to that of a firm paying $w > R$, yields the following equilibrium condition:

$$(p - (1 + \tau) b) l(R|F) = (p - (1 + \tau) w) l(w|F) \quad (9)$$

Substituting for $l(w|F)$ completes the solution for the endogenous wage offer distribution function F in terms of the structural parameters of the search model (p, b, δ, λ) and τ , and the offered wage w :

$$F(w) = \frac{[\delta + \lambda]}{\lambda} \left[1 - \left[\frac{(p - (1 + \tau) w)}{(p - (1 + \tau) b)} \right]^{\frac{1}{2}} \right] \quad (10)$$

Note that the solution in the case of $\tau = 0$ is identical to the original solution in the BM model. Following are some of the equilibrium

⁵But not sufficient. For example, if all firms offer the reservation wage b , profits still equalize; however, in this case, firms have an incentive to offer an arbitrarily small wage increment above b .

⁶In order for a firm to have an incentive to produce, profits must be non-negative. Hence, the maximal tax rate is given by $\tau_{\max} \leq \frac{p-b}{b}$ and any higher tax rate will shut down production.

properties:

The wage offer distribution. Raising τ results in a wage offer distribution that is stochastically dominated by the wage offer distribution with a lower tax rate, for all wages above the reservation wage b . To see this, note that:

$$\begin{aligned} \frac{dF(w)}{d\tau} &= \frac{[\delta + \lambda]}{2\lambda} \left[\frac{(p - (1 + \tau)w)}{(p - (1 + \tau)b)} \right]^{-\frac{1}{2}} \left(\frac{w(p - (1 + \tau)b) - b(p - (1 + \tau)w)}{(p - (1 + \tau)b)^2} \right) \\ &= \frac{[\delta + \lambda]}{2\lambda} \left[\frac{(p - (1 + \tau)b)}{(p - (1 + \tau)w)} \right]^{\frac{1}{2}} \left(\frac{p(w - b)}{(p - (1 + \tau)b)^2} \right) > 0, \forall w > b \end{aligned} \quad (11)$$

The tax burden falls completely on firms that employ workers at the reservation wage since workers will not work for less than b . Specifically, the wage offer distribution, under positive tax rates, is stochastically dominated by the wage distribution without taxation since:

$$\frac{[\delta + \lambda]}{\lambda} \left[1 - \left[\frac{(p - (1 + \tau)w)}{(p - (1 + \tau)b)} \right]^{\frac{1}{2}} \right] > \frac{[\delta + \lambda]}{\lambda} \left[1 - \left[\frac{(p - w)}{(p - b)} \right]^{\frac{1}{2}} \right], \forall w > b \quad (12)$$

The maximum wage. This special case clearly illustrates the effect of taxation. Workers who are paid the maximal wage, which is solved for by substituting $F(w) = 1$ in equation (10) earn

$$w_{\max} = \frac{1}{(1 + \tau)} \left(p - \left(\frac{\delta}{[\delta + \lambda]} \right)^2 (p - (1 + \tau)b) \right) \quad (13)$$

which is decreasing in τ .

Note that all the results so far have been independent of the sign of

τ . In the case of $\tau < 0$ (i.e. a wage subsidy), $F(w|\tau < 0) < F(w|\tau = 0)$ and $(w_{\max}|\tau < 0) > (w_{\max}|\tau = 0)$.

The results thus far suggest that the tax burden affects outcomes throughout the wage offer distribution, with the exception of workers who are paid at the reservation wage and thus remain indifferent. This leads to the following proposition:

Proposition 1 *The workers' tax burden is increasing in wage.*

Proof. Let w^x be the wage of the x percentile worker. The worker's wage is given by $\frac{p}{(1+\tau)} - \left(\frac{\delta+\lambda[1-x]}{[\delta+\lambda]}\right)^2 \left(\frac{(p-(1+\tau)b)}{(1+\tau)}\right)$. The relative wage of the "same" worker after taxation $\frac{(w^x|\tau>0)}{(w^x|\tau=0)}$ is decreasing in x since $\frac{d}{dx} \frac{(w^x|\tau>0)}{(w^x|\tau=0)} < 0$. Thus, a proportional payroll tax compresses the wage offer distribution and therefore has a progressive impact on gross and net wages in equilibrium. In other words, it reduces wage inequality. ■

The intuition behind this result is as follows: Each firm makes the same profit in equilibrium and firm size is increasing in wage. Since workers who are paid the reservation wage are unaffected by taxation, the firms that employ them absorb the full tax burden. As we move to higher-paying and thus larger firms, the tax burden on the workers they employ must increase. Since profits are equal across firms, the tax burden on workers who are employed by larger firms is higher and therefore the total tax burden of a large firm equals that of a small one.

The results thus far have shown that a payroll tax or subsidy on earnings has a distributional effect and that the effect of a flat-rate wage subsidy on net wages is increasing in wage. At this point, it is interesting to compare this policy to a minimum wage policy which sets the mini-

mum wage at some level higher than b . The effect of a minimum wage on low-paid workers is stronger than a wage subsidy since the reservation wage is now equal to the minimum wage (assuming full compliance) and the effect is decreasing in wage, which is a general result of the BM model.

2.2 An Income Tax

Suppose that a proportional tax τ is imposed on the employee. The employee's net (i.e. after-tax) wage can be no lower than b ; otherwise, he would prefer to be unemployed.⁷ The employer is aware of this and therefore the lowest gross wage offer R^g is given by:

$$R^g = \frac{b}{(1 - \tau)} \quad (14)$$

From the employer's perspective, this is equivalent to raising b . This has a positive effect on offered wages throughout the gross wage offer distribution F^g since the condition of equal profits for all firms states that:

$$(p - R^g) l(R^g | F^g) = (p - w^g) l(w^g | F^g) \quad (15)$$

Substituting for employment per firm that pays w given the wage offer distribution F (see eq. (7)), we obtain:

⁷Feasible tax rates: In order for firms to have an incentive to produce, profits have to be non-negative. Hence, the maximal tax rate is given by $\tau_{\max} \leq \frac{p-b}{p}$. A higher tax rate will shut down production since the gross reservation wage will exceed labor productivity.

$$\left(p - \frac{b}{(1-\tau)}\right) \frac{\lambda\delta}{[\delta + \lambda]^2} = (p - w^g) \frac{\lambda\delta}{[\delta + \lambda(1 - F^g(w^g))]^2} \quad (16)$$

Thus, the gross wage offer distribution is explicitly given by:

$$F^g(w^g) = \frac{[\delta + \lambda]}{\lambda} \left[1 - \left[\frac{(p - w^g)}{\left(p - \frac{b}{(1-\tau)}\right)} \right]^{\frac{1}{2}} \right] \quad (17)$$

From the worker's perspective, it is the net wage offer distribution that is relevant. If the gross wage offer is w^g , the worker faces a net offer of $w^g(1 - \tau)$. The implied net wage offer distribution F^n therefore satisfies:

$$F^n(w^g(1 - \tau)) = F^g(w^g)$$

By substitution, the explicit net wage offer distribution is given by:

$$F^n(w^n) = \frac{[\delta + \lambda]}{\lambda} \left[1 - \left[\frac{\left(p - \frac{w^n}{(1-\tau)}\right)}{\left(p - \frac{b}{(1-\tau)}\right)} \right]^{\frac{1}{2}} \right] \quad (18)$$

As in the case of a payroll tax, it is easy to show that $F^n(w^n|\tau > 0) \geq F^n(w^n|\tau = 0)$, i.e. that a linear tax has a progressive effect in equilibrium, and in addition that $\frac{dF^g(w^g)}{d\tau} < 0$ and $\frac{dF^n(w^n)}{d\tau} > 0$, which imply that the tax burden is divided between workers and firms.

2.2.1 Comparison of Equilibrium Outcomes

The explicit solutions for the wage offer distributions in the cases of an income tax and a payroll tax enable a comparison of the labor market outcomes. The following proposition summarizes the main result of this

comparison:

Proposition 2 *A linear payroll tax τ is equivalent in equilibrium to a linear income tax $\frac{\tau}{1+\tau}$ for all values of τ (whether negative or positive).*

Proof. *By substituting τ into equation (10) and $\frac{\tau}{1+\tau}$ into equation (18), we obtain $F^n = F$. ■*

Thus, there is tax equivalence between income and payroll taxes, which implies that both employers and employees are indifferent between a payroll tax τ and an income tax $\tau/(1 + \tau)$. The intuition behind this result states that, as in competitive models, the tax burden is divided according to relative elasticities of supply and demand though in this case there is no single intersection of demand and supply curves in equilibrium. Furthermore, although information on other job opportunities is incomplete, there is no information asymmetry between workers and firms once matched. The tax equivalence holds because the equilibrium condition of equal profits across firms takes into account total labor cost. Accordingly, note that workers are better off in the case of a tax imposed on employers at the rate τ than in the case of the same tax being imposed on employees since $F^n(w^n) \geq F(w)$.

A note on profit taxation. Assume that a proportional profit tax τ_f is levied on firm's gross revenue. Net profit is then given by:

$$\pi = (1 - \tau_f)(p - w)l(w|F)$$

Such a tax does not change any equilibrium outcome apart from reducing firms' net profits. This result implies that wage subsidies can

be financed without additional changes in equilibrium and, in addition, that taxation of profits can be used to offset the negative marginal effect of a guaranteed net income program on labor share due to the dampening effect on wages above the guaranteed minimum (see section 3.6).

Indeed, such a tax is sufficient in order to achieve the outcomes of a competitive equilibrium. Thus, if τ_f approaches 1, it can be used to finance a guaranteed net income that approaches p . This result is comparable to that of the present model, according to which a minimum wage equal to p replicates the results of the competitive model. In fact, these outcomes indicate that the model is not rich enough in order to rigorously analyze these policies since it does not allow firms to react to policy changes. For example, relaxing the assumption of a fixed number of firms, and instead allowing the number of active firms to be determined endogenously through a fixed entrance cost equal to expected profits, would alter these results.

2.3 The Reservation Wage in the General Case

In previous sections, a simplifying assumption was made whereby job offers arrive at a rate λ , both for employed and unemployed workers. This assumption resulted in an immediate solution for the reservation wage, i.e. $R = b$, since the option value of labor search is set to zero and taxation did not have any effect on the net reservation wage. In this section, the assumption is relaxed for two reasons: First, in order to show that this assumption does not result in a loss of generality and second, in order to quantify the effect of taxation on the reservation wage.

Assume that the job offer arrival rate is given by λ^0 for the unemployed and by λ^1 for the employed. Usually, $\lambda^0 > \lambda^1$ (see Eckstein and van den Berg, 2007), such that the reservation wage is higher than b and is a function of the wage offer distribution. Therefore, the reservation wage changes when taxation is introduced.

Let x be the net wage. The general form of the reservation wage R is given by:⁸

$$R = b + [k^0 - k^1] \int_R^\infty \left[\frac{(1 - F(x))}{1 + k^1(1 - F(x))} \right] dx \quad (19)$$

where $k^0 = \frac{\lambda^0}{\delta}$, $k^1 = \frac{\lambda^1}{\delta}$.

Assume that a payroll tax τ is levied. The total cost to the firm of paying a wage w is then given by $(1 + \tau)w$ and therefore the firm's net profit is equal to:

$$\pi = (p - (1 + \tau)w) l(w|F) \quad (20)$$

The lowest wage offer in equilibrium is given by:

$$\underline{w} = R \quad (21)$$

The employment per firm offering the reservation wage $l(R|F)$ is given by (see BM):

$$l(R|F) = \frac{k^0}{(1 + k^0)(1 + k^1)}$$

The net profit of firms paying the reservation wage is obtained by

⁸See BM for the derivation of this solution.

substituting for $l(R|F)$, which yields:

$$\pi = (p - (1 + \tau) R) \left(\frac{k^0}{(1 + k^0)(1 + k^1)} \right) \quad (22)$$

The condition of equal profits implies that:

$$(p - (1 + \tau) R) \left(\frac{k^0}{(1 + k^0)(1 + k^1)} \right) = (p - (1 + \tau) w) l(w|F) \quad (23)$$

The wage offer distribution is given by:

$$F(w) = \left[\frac{1 + k^1}{k^1} \right] \left[1 - \left(\frac{p - (1 + \tau) w}{p - (1 + \tau) R} \right)^{\frac{1}{2}} \right] \quad (24)$$

which is equivalent to (10) for the case $\lambda_0 = \lambda_1 = \lambda$.

Substituting equation (24) into (19) yields a relation between R and the maximal wage \bar{w} :

$$\begin{aligned} R - b &= [k^0 - k^1] \int_R^\infty \left[\frac{1 - \left[\frac{1+k^1}{k^1} \right] \left[1 - \left(\frac{p-(1+\tau)x}{p-(1+\tau)R} \right)^{\frac{1}{2}} \right]}{1 + k^1 \left[1 - \left[\frac{1+k^1}{k^1} \right] \left[1 - \left(\frac{p-(1+\tau)x}{p-(1+\tau)R} \right)^{\frac{1}{2}} \right] \right]} \right] dx \quad (25) \\ &= \left[\frac{k^0 - k^1}{k^1} \right] \int_R^{\bar{w}} \left[1 - \left(\frac{1}{1 + k^1} \right) \left[\frac{p - (1 + \tau) x}{p - (1 + \tau) R} \right]^{-1/2} \right] dx \\ &= \left[\frac{k^0 - k^1}{k^1} \right] \left[x + \left(\frac{2(p - (1 + \tau) R)}{(1 + k^1)(1 + \tau)} \right) \left(\frac{p - (1 + \tau) x}{p - (1 + \tau) R} \right)^{1/2} \right]_R^{\bar{w}} \\ &= \left[\frac{k^0 - k^1}{k^1} \right] \left[\bar{w} - R + \left(\frac{2(p - (1 + \tau) R)}{(1 + k^1)(1 + \tau)} \right) \left(\left(\frac{p - (1 + \tau) \bar{w}}{p - (1 + \tau) R} \right)^{1/2} - 1 \right) \right] \end{aligned}$$

In addition, setting $F(\bar{w}) = 1$ in (24) yields a second equation in R

and the maximal wage \bar{w} :

$$\bar{w} = \frac{p}{(1 + \tau)} - \frac{(p - (1 + \tau) R)}{(1 + k^1)^2 (1 + \tau)} \quad (26)$$

Substituting the explicit solution for the maximum wage in the previous equation yields:

$$R = \frac{(1 + k^1)^2 b + \frac{1}{(1 + \tau)} (k^0 - k^1) k^1 p}{(1 + k^1)^2 + (k^0 - k^1) k^1} \quad (27)$$

Note that if $\lambda^0 = \lambda^1$, the solution is $R = b$, and that $\frac{dR}{d\tau} < 0$ if $\lambda^0 > \lambda^1$, i.e. in the case that the option value of search is positive. In addition, setting $\tau = 0$ yields the original BM solution.

The generality of the previous results. The results obtained under the simplifying assumption that $R = b$ also hold in the general case. To see this, note that the reservation wage is a weighted average of b and p and levying a **payroll tax** is equivalent to a decrease in labor productivity. Thus, the reservation wage can be written as:

$$R = \alpha b + (1 - \alpha) \frac{p}{(1 + \tau)}$$

where α is a function of the structural parameters of the model. By substitution, the general wage offer distribution can be expressed as:

$$\begin{aligned} F(w) &= \frac{[\delta + \lambda^1]}{\lambda^1} \left[1 - \left[\frac{(p - (1 + \tau) w)}{\left(p - (1 + \tau) \left(\alpha b + (1 - \alpha) \frac{p}{(1 + \tau)} \right) \right)} \right]^{\frac{1}{2}} \right] \\ &= \frac{[\delta + \lambda^1]}{\lambda^1} \left[1 - \left[\frac{(p - (1 + \tau) w)}{\alpha (p - (1 + \tau) b)} \right]^{\frac{1}{2}} \right] \end{aligned}$$

Thus, the result that $\frac{dF(w)}{d\tau} > 0$ and proposition 1 remain unchanged. In other words, a proportional payroll tax compresses the wage offer distribution and thus has a progressive impact on net wages in equilibrium. The results are analogous to those of a decrease in labor productivity, which are well-known.

Solving the general form of the reservation wage in the case of an **income tax** τ yields:

$$R = \frac{(1 + k^1)^2 b + (1 - \tau) (k^0 - k^1) k^1 p}{(1 + k^1)^2 + (k^0 - k^1) k^1}$$

where R is the net reservation wage. Thus, proposition 1 and proposition 2 continue to hold since a linear payroll tax τ and a linear income tax $\frac{\tau}{1+\tau}$ yield an identical reservation wage and an identical wage offer distribution.

2.3.1 Calibration

In order to illustrate the magnitudes of the qualitative outcomes, some initial calibration results are presented for the UK. Based on Ridder and van den Berg (2003), the following parameters were used: $\lambda^0 = 0.15$, $\lambda^1 = 0.12$, $\delta = 0.009$ and $b/p = 0.32$. According to the results, a 10-percent payroll subsidy increases the maximum wage by 11.1 percent in equilibrium, while the reservation wage increases by only 4.5 percent. A 20-percent payroll subsidy increases the reservation wage by 10 percent. A 10-percent payroll tax reduces the reservation wage by 3.7 percent. The results demonstrate that the increase in wages near the reservation wage is relatively small in magnitude and that gross and net wage inequality is increased. Similarly, a flat tax (payroll or income) compresses

the gross wage distribution, i.e. decreases gross and net wage inequality.

2.4 Guaranteed Minimum Income

This section analyzes the equilibrium outcomes of a wage subsidy, which guarantees every worker a minimal net income. In this case, low-paid workers apply for an income tax credit and the tax authorities pay these workers directly in order to guarantee a net income equal to w^s , where w^s is set by the policymaker.⁹

Assume that all workers receive job offers at an identical rate λ and that unemployed workers receive a benefit flow b . Assume that the policymaker sets w^s such that $b < w^s < w_{\max}$, where w_{\max} is the maximum observed wage prior to the implementation of the guaranteed minimum income policy. In addition, assume that firms do not pay below b , which can be interpreted as the threshold gross wage for receiving a wage subsidy. These assumptions result in a reservation wage $R = b$.

The following subsections describe the various equilibria in the case of a guaranteed minimum income program and examine their properties.

2.4.1 A Degenerate Equilibrium

A degenerate equilibrium is one in which all firms offer the minimal wage b , as in the case of no on-the-job search, which was studied by Diamond (1971). The conditions that produce this equilibrium are derived below.

The guaranteed minimum income program affects the incentives of both workers and firms. Thus, workers become indifferent between jobs that pay up to w^{s10} and as a result, firms do not have an incentive

⁹This program is similar to the Israeli Guaranteed Income program, which was in force until 2003.

¹⁰In most cases, guaranteed minimum income programs are conditioned on employ-

to offer wages between b and w^s . In other words, the existence of a guaranteed minimum income eliminates competition between firms in the range $b \leq w \leq w^s$.

Assume that there are L workers and M firms in the economy. In the degenerate equilibrium, $L(1 - u)$ workers are employed by M firms paying b . Firms cannot attain a higher level of profit by offering wages above w^s . The condition for a degenerate wage offer in equilibrium is as follows:

Proposition 3 *If the guaranteed minimum income w^s is higher than $p - (p - b) \frac{\delta}{\lambda + \delta}$, all firms offer the reservation wage b in equilibrium.*

Proof. *If all firms remain pooled together, firms share the employed workforce equally. Thus, each firm's profit is given by $\Pi^p = (p - b) \frac{L(1-u)}{M}$ where $p - b$ is profit per worker, $\frac{L(1-u)}{M}$ is employment per firm offering a wage b and $u = \frac{\delta}{\delta + \lambda}$ is the equilibrium unemployment rate. The profit to be gained by leaving the pool is computed as follows: Assume that a single firm leaves the pool by offering an arbitrarily small wage increment above w^s . The employment in this firm will then be given by $\frac{\lambda L}{\delta M}$ since all workers will accept its job offers and no worker will leave the firm on his own accord. Hence, the firm's profit will be given by $\Pi = (p - (w^s + \varepsilon)) \frac{\lambda L}{\delta M}$ and if $w^s > p - (p - b) \frac{\delta}{\lambda + \delta}$, then Π^p is always higher than Π . ■*

Note that the critical value for the degenerate equilibrium is lower than the maximum observed wage prior to the guaranteed minimum

ment in order to avoid moral hazard problems. Here, the extreme case is analyzed in which the program ensures a minimum net income w^s for all workers, whether employed or not.

income being introduced since $p - (p - b) \frac{\delta}{\lambda + \delta} < p - (p - b) \left(\frac{\delta}{\lambda + \delta}\right)^2 = w_{\max}$. Therefore, a degenerate wage offer can produce an equilibrium even when w^s is set below some of the observed wages in the economy. Thus, the whole wage distribution is affected and not just wages up to w^s .

2.4.2 A Non-Degenerate Equilibrium

In the non-degenerate equilibrium, some firms offer b while others are engaged in competition above the guaranteed minimum income w^s . Therefore, this equilibrium is a synthesis of Diamond's (1971) degenerate equilibrium, in which all firms offer the reservation wage, and the BM (1998) equilibrium, in which the wage offer distribution is nowhere dense.

The previous proposition implies that if the guaranteed minimum income w^s is set below $p - (p - b) \frac{\delta}{\lambda + \delta}$, there is an incentive for firms to leave the pool of firms offering b . This provides the intuition for the results in this section.

I show that in the case that w^s is lower than a certain threshold, there is a unique non-degenerate equilibrium in which a fraction M^s of firms offer b and a fraction $1 - M^s$ engage in competition in the range above w^s .¹¹ A fraction L_e^s of the employed workers are employed by firms that offer b while a fraction $1 - L_e^s$ are employed by higher-paying firms.

Following are the formal conditions under which such a non-degenerate equilibrium exists and a proof for its existence and uniqueness.

¹¹It is assumed that workers accept a job offer $w = w^s$ which is equivalent to the standard assumption in search theory that workers accept job offers at the reservation wage R , though they are indifferent between accepting such an offer and remaining unemployed.

1. **Steady state.** In this case, there are three groups of workers: unemployed, workers employed by low-paying firms and workers employed by high-paying firms. The conditions for a steady state are as follows:

A. Fixed unemployment:

$$\lambda u = (1 - u) \delta \quad (28)$$

This condition, which determines equilibrium unemployment, remains unchanged.

B. Fixed employment at b -paying firms. Substituting $G(b) = L_e^s$ and $F(b) = M^s$ in (5) yields:

$$L_e^s = \frac{\delta M^s}{(\delta + \lambda(1 - M^s))}$$

which provides an explicit relation between the fraction of low-paying firms M^s and that of low-paid workers L_e^s in the steady state. Rearrangement yields:

$$M^s = \frac{L_e^s (\delta + \lambda)}{(\lambda L_e^s + \delta)} \quad (29)$$

These two conditions ensure that the size of the remaining group, i.e. workers employed by high-paying firms, is also fixed over time.¹²

2. **Equal profits.** In equilibrium, firms are indifferent between staying in the pool and leaving it. In particular, they are indifferent between offering b and offering $w^s \leq w$, where w is within the support of F (the wage offer distribution in equilibrium). As in the original BM

¹²The explicit condition is $\lambda[(1 - u)L^s + u(1 - M^s)] = (1 - u)(1 - L^s)\delta$ which holds if the previous two conditions hold.

equilibrium, firms that offer wages above w^s face a trade-off between profit-per-worker and employment, such that competition rules out a single market wage and the wage offer distribution in equilibrium must be nowhere dense. The equal profits condition for b -paying and high-paying firms states that, for every $w \in F$:

$$(p - b) \frac{L_e^s}{M^s} = (p - w) l(w|F) \quad (30)$$

where $l(w|F)$ denotes the employment per firm posting a wage offer $w \geq w^s$, given the wage offer distribution F and is solved by equating the quits and the recruits of each firm. The quits $q(w)$ from a firm that pays $w \geq w^s$, are given by:

$$q(w) = l(w|F) (\delta + \lambda(1 - F(w))) \quad (31)$$

and the recruits $r(w)$ of a firm that offers w , are given by:

$$r(w) = \lambda(u + (1 - u)G(w)) \quad (32)$$

Therefore, $l(w|F)$ is given by:

$$l(w|F) = \frac{\lambda(u + (1 - u)G(w))}{(\delta + \lambda(1 - F(w)))} \quad (33)$$

Using the condition for equal profits yields:

$$(p - b) \frac{L_e^s}{M^s} = (p - w) \frac{\lambda(u + (1 - u)G(w))}{(\delta + \lambda(1 - F(w)))} \quad (34)$$

This condition states that all firms, whether or not they are in the

pool of low-paying firms, make equal profits in equilibrium, which makes it possible to solve explicitly for $F(w)$.

Equations (29) and (34) form a system of two equations in two variables: the share of firms in the pool and the share of workers who are employed in these firms. These constitute necessary conditions for equilibrium.

Proposition 4 *For policy w^s , such that $b < w^s < p - (p - b) \frac{\delta}{\lambda}$, there exists a unique non-degenerate equilibrium.*

Proof. See appendix 2. ■

Proposition 5 *For policy w^s , such that $p - (p - b) \frac{\delta}{\lambda} \leq w^s \leq p - (p - b) \frac{\delta}{\lambda + \delta}$, no equilibrium exists.*

Proof. See appendix 2. ■

The properties of the degenerate and non-degenerate equilibria. In both cases, all net wages in jobs that paid up to w^s previous to the wage subsidy are increased and the net wages in jobs that paid more than w^s previous to the wage subsidy are reduced. This outcome is due to the fact that in equilibrium profits are equalized across firms, such that higher-paying firms benefit from the wage subsidy indirectly in the form of wage reductions. Thus, the introduction of a guaranteed minimum income affects all wages in the economy in general equilibrium.

In competitive labor markets, a wage subsidy is divided between workers and firms. Thus, the above result, according to which the wage paid for some jobs is reduced, is not a standard one. However, recall

that workers in this model are identical and therefore all workers are better off since the mean net wage has been raised. Furthermore, in the real world, workers do not live forever and if the information structure is such that job spells are long (i.e. low λ and δ), then the short-run effect of such a wage reduction might be significant.

The dampening effect of a guaranteed minimum income on high-paid jobs is a result of the reduced incentives for workers to move to higher-paying jobs and for firms to attract them by offering higher wages (up to w^s). These changes reduce the level of competition throughout the job market. This effect cannot exist in either a competitive model or any other model with a single wage in equilibrium and therefore is a novel insight into policy analysis within an equilibrium search model.

3 Conclusions and Policy Implications

This paper has examined the effects of various types of taxes and subsidies on the outcomes in the labor market, assuming that there are search frictions in the labor market. The main motivation was to assess the impact of policies that attempt to raise the net income of low-paid workers, such as earned income tax credit programs, and to assess the general equilibrium effects of taxation.

In general, there was found to be tax equivalence between income and payroll taxes, exactly as in competitive labor markets. This result is neither trivial nor intuitive since the wage-posting mechanism provides employers with a kind of first-mover advantage that creates asymmetry between workers and firms.

A proportional linear tax is non-linear in equilibrium since the tax

burden increases in wage. This result implies that tax rates should increase more slowly than is usually the case in order to reduce net wage inequality and suggests that the problems associated with the discrete jumps in the marginal tax rate may be less severe than is usually thought.

The results indicate that policies which aim to raise the net income of low-paid workers may have some undesirable features. Thus, a proportional subsidy is an expensive way to raise the net wage in low-paid jobs since its effect on net wages at the lower end of the wage distribution is relatively small and moreover it increases wage inequality. Although only a proportional tax was analyzed, these results can be easily extended to the more general case of a linear tax.

The structure of the wage subsidies analyzed here does not fully correspond to real-life subsidies. It is usually the case that wage subsidies form a trapezoid on the linear budget constraint in the income-leisure plane. This is the result of the scheme's structure: a proportional subsidy in the "phase-in" stage, a fixed sum in the "plateau" and a "phase-out" stage in which the fixed amount of the subsidy is gradually reduced, as in the case of the Earned Income Tax Credit program in the US. A proportional subsidy corresponds to the phase-in stage.¹³ However, the qualitative results for proportional taxes, i.e. tax equivalence and the non-linear effect on net wages, also hold for other tax schedules, although the exact solutions differ. The result that wage subsidies might negatively affect higher wages, which was demonstrated in the case of a guaranteed minimum income scheme, also holds in the more general

¹³For example, the phase-in of the EITC provides a proportional subsidy of 40 percent up to an annual income of \$11,340 (couples with children; for 2006).

case no matter how smooth the "phase-out" stage is. This is because there is always a "marginal" worker who does not receive the subsidy and therefore his employer must reduce the wage it pays in order to equalize its profit to those of firms whose workers do receive the subsidy.

The guaranteed minimum income scheme suffers from several shortcomings, among them a dampening effect on wages above the guaranteed minimum due to reduced competition in the market. This is explained by the properties of the resulting equilibrium, in which a group of firms offer the same low wage, thus reducing the degree of competition throughout the market. In the case that the guaranteed minimum income level is set above some critical value, which is lower than some observed wages, a degenerate equilibrium results in which all firms offer the same low wage.

In equilibrium, although workers are better off, the increase in their average wage is lower than one would have expected. Standard economic wisdom, which is based on competitive labor markets, implies that in the "worst" case of a completely inelastic labor supply curve, employers receive the whole wage subsidy in equilibrium. The possibility that a wage subsidy might reduce the wage in some jobs is a unique outcome of this study.

The results of this study indicate that a minimum wage can complement an earned-income-tax-credit policy, in contradiction to the common view that these policies are substitutes in raising low wages. This conclusion contradicts Cahuc and Laroque (2007), who show that in a classic monopsonistic market, i.e. in which there is a single employer, it is possible to completely offset monopsonistic power by a combina-

tion of revenue taxation and wage subsidies, such that there is no room for a minimum wage. I conclude that when firms are heterogeneous, as may be in the case of incomplete information, the minimum wage is indispensable. A higher minimum wage can offset the dampening effect of wage subsidies on gross wages and, at the same time, wage subsidies can offset the negative effect of a minimum wage on employment. This prediction is consistent with a recent empirical study by Neumark and Wascher (2007) who show that during the last 10 years in the US, the minimum wage has increased wages of low-skilled workers by more than they have been reduced by the Earned-Income-Tax-Credit program and that the EITC in turn has had a positive and significant effect on employment.¹⁴

¹⁴Tables 7 and 8 show the average effect of these policies on the wages and employment of workers with a high school education or less while Table 6 shows the opposite effect on females with a high school education or less.

Appendix 1: Wage Offer Distributions under Different Policies

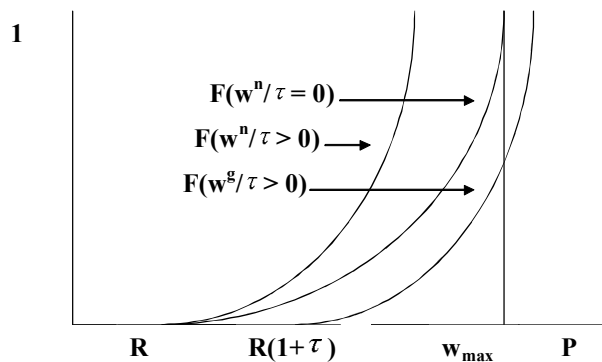


Figure 1: Gross and net wage offer distributions with a positive income tax

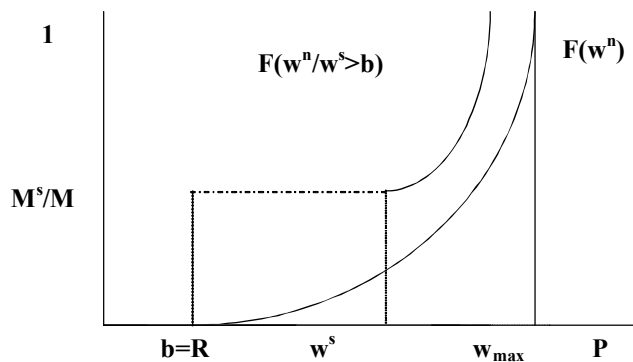


Figure 2: Wage offer distribution with a guaranteed minimum income policy that results in a non-degenerate equilibrium

Appendix 2: Existence and Uniqueness of the Non-Degenerate Equilibrium

Proof of proposition 4

The conditions for a non-degenerate equilibrium are given by the steady state condition (SS):

$$M^s = \frac{L_e^s (\delta + \lambda)}{(\lambda L^s + \delta)}$$

and the equal profits condition (EP):

$$(p - b) \frac{L_e^s}{M^s} = (p - w) \frac{\lambda(u + (1 - u)G(w))}{(\delta + \lambda(1 - F(w)))}$$

By substituting $w = w^s$, this condition yields:

$$(p - b) \frac{L_e^s}{M^s} = (p - w^s) \frac{\lambda(u + (1 - u)L_e^s)}{(\delta + \lambda(1 - M^s))}$$

since the lowest-paying firm that leaves the pool pays a wage that exceeds w^s by an arbitrarily small amount and $F(w^s) = M^s$ and $G(w^s) = L_e^s$.

The EP condition can now be re-written as:

$$M^s = \frac{\left[\frac{(p-b)(\delta+\lambda)}{(p-w^s)} L_e^s \right]}{\left[\lambda(u + (1 - u)(L_e^s)) + \lambda \frac{(p-b)}{(p-w^s)} L_e^s \right]}$$

The SS and EP conditions form curves in the L_e^s, M^s plane. In order to prove the existence of a non-degenerate equilibrium, it needs to be shown that these curves have at least one intersection point in the interval $L_e^s \in (0, 1), M^s \in (0, 1)$ since these variables represent shares. The proof for existence is based on the following four lemmas and on the

uniqueness of the solution, which is shown later on.

Lemma 6 *The EP and SS curves are increasing and concave in the L_e^s, M^s plane.*

Proof. A. The SS curve: $\frac{dM^s}{dL_e^s} = \frac{(\delta+\lambda)(\lambda L_e^s + \delta) - \lambda L_e^s(\delta+\lambda)}{(\lambda L_e^s + \delta)^2} = \frac{(\delta+\lambda)(\lambda L_e^s - \lambda L_e^s + \delta)}{(\lambda L_e^s + \delta)^2} = \frac{(\delta+\lambda)\delta}{(\lambda L_e^s + \delta)^2} > 0$ since δ and λ are positive. $\frac{d^2M^s}{dL_e^s} = -2 \left[\lambda \frac{(\delta+\lambda)\delta}{(\lambda L_e^s + \delta)^3} \right] < 0$.

B. The EP curve: $\frac{dM^s}{dL_e^s} = \frac{\left(\frac{p-b}{p-w^s}\right) \left[\lambda(u+(1-u)(L_e^s)) + \lambda L_e^s \frac{(p-b)}{(p-w^s)} - L_e^s \left(\lambda(1-u) + \lambda \frac{(p-b)}{(p-w^s)} \right) \right]}{\left(\lambda(u+(1-u)(L_e^s)) + \lambda \frac{(p-b)}{(p-w^s)} L_e^s \right)^2} = \frac{\left(\frac{p-b}{p-w^s}\right) \lambda u}{\left(\lambda(u+(1-u)(L_e^s)) + \lambda L_e^s \frac{(p-b)}{(p-w^s)} \right)^2} > 0$ since $p > w^s$ by assumption. $\frac{d^2M^s}{dL_e^s} = -2 \frac{\left(\frac{p-b}{p-w^s}\right) \lambda u \left(\lambda(1-u) + \lambda \frac{(p-b)}{(p-w^s)} \right)}{\left(\lambda(u+(1-u)(L_e^s)) + \lambda L_e^s \frac{(p-b)}{(p-w^s)} \right)^3} < 0$ since all the terms in brackets are positive. Thus, for both curves, $\frac{dM^s}{dL_e^s} > 0$, $\frac{d^2M^s}{dL_e^s} < 0$. ■

Lemma 7 *The EP and SS curves cross the origin and the SS curve crosses $(1, 1)$.*

Proof. By substitution. ■

Lemma 8 *The slope of the EP curve is greater than that of the SS curve at the origin.*

Proof. The slope of the SS curve at $L_e^s = 0$ is $\frac{(\delta+\lambda)}{\delta}$. The slope of the EP curve at $L_e^s = 0$ is $\left(\frac{(p-b)(\delta+\lambda)}{(p-w^s)} \right) (\lambda u)^{-1}$. The slope at the origin increases in w^s and therefore the minimal slope (for $w^s = b$) is $\frac{(\delta+\lambda)}{\lambda u} > \frac{(\delta+\lambda)}{\delta}$. ■

Lemma 9 *Substituting $L_e^s = 1$ in the equal profits equation yields $M^s < 1$ for the policy $w^s < p - (p-b) \frac{\delta}{\lambda}$.*

Proof. Substituting $L_e^s = 1$ in the EP condition yields $M^s = \frac{\lambda(p-b) + \delta(p-b)}{\lambda(p-b) + \lambda(p-w^s)}$.

Since $w^s > b$ by assumption, $M^s < 1$. ■

The above facts (i.e. that the curves are increasing and concave (lemma 6) and cross the origin (lemma 7), that the EP curve's slope at

the origin is greater than that of the SS curve (lemma 8) and that the M^s value at $L_e^s = 1$ on the EP curve is less than that on the SS curve (lemmas 8 and 9) ensure that the curves have at least one intersection point, such that $M^s, L_e^s \in (0, 1)$ (see Appendix 3 for examples).

In order to prove the uniqueness of the non-degenerate equilibrium, the explicit non-zero solution for (L_e^s, M^s) are presented:¹⁵

By substitution: $\frac{[\frac{(p-b)(\delta+\lambda)}{(p-w^s)} L_e^s]}{[\lambda(u+(1-u)(L_e^s))+\lambda\frac{(p-b)}{(p-w^s)} L_e^s]} = \frac{L_e^s(\delta+\lambda)}{(\lambda L_e^s+\delta)}$. The explicit solution for L_e^s is given by $L_e^s = \frac{\delta}{\lambda} \left(\frac{(\delta+\lambda)(p-b)}{\lambda(p-w^s)} - 1 \right)$ and the explicit solution for M^s is given by $M^s = \frac{\frac{\delta}{\lambda} \left(\frac{(\delta+\lambda)(p-b)}{\lambda(p-w^s)} - 1 \right) (\delta+\lambda)}{\delta \left(\frac{(\delta+\lambda)(p-b)}{\lambda(p-w^s)} - \frac{1}{\delta} \right)}$. As proven earlier, $M^s, L_e^s \in (0, 1)$.

Proof of proposition 5

Proposition 3 states that the critical value for a degenerate equilibrium is given by $w^s > p - (p - b) \frac{\delta}{\lambda + \delta}$. Thus, if w^s is set equal or below this value, there is an incentive to leave the pool by offering an arbitrarily small wage increment above w^s in order to attract all workers in the pool.

However, proposition 4 states that the highest w^s for the existence of a non-degenerate equilibrium satisfies $w^s < p - (p - b) \frac{\delta}{\lambda}$. Otherwise, the intersection point with the SS curve lies outside $[0, 1]$.

Thus, for the policy w^s , such that $p - (p - b) \frac{\delta}{\lambda} \leq w^s \leq p - (p - b) \frac{\delta}{\lambda + \delta}$, there is no stable equilibrium. However, since in most cases $\lambda \gg \delta$, this range is not large.

¹⁵The equal profits and steady-state conditions are necessary but not sufficient for equilibrium. Thus, $M^s = L^s = 0$, which represents the original BM equilibrium, cannot be an equilibrium here because firms do not have an incentive to offer wages between b and w^s .

Appendix 3: The Non-Degenerate Equilibrium - Simulation

Results

The simulation is based on values for the US, which were estimated by Ridder and van den Berg (2003). Specifically, the parameters used were $\lambda = 0.54$ (both on- and off-the-job, on average), $\delta = 0.03$, $p = 1$ and $b = 0.27$. Using these values, equilibrium unemployment is 5.3 percent.

For these values, the threshold w^s for the degenerate equilibrium is equal to $w^s = 0.959$. Values higher than $w^s = 0.962$ will result in a degenerate equilibrium and therefore for the range $(0.959 \leq w^s \leq 0.962)$ no equilibrium exists. The range of feasible policies is given by $0.27 < w^s < 0.998$ which is the maximal observed wage before the introduction of the guaranteed wage. The fraction of firms that offer b and the fraction of workers who are employed by these firms L_e^s are increasing in w^s , as illustrated in the figure below.

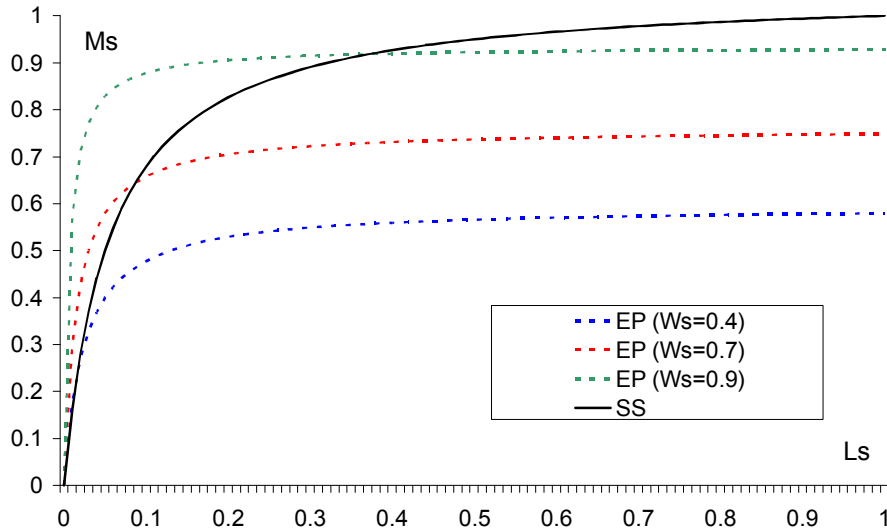


Figure 3: The steady-state (SS) and equal profits (EP) curves for various policies that result in a non-degenerate equilibrium

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