Israel Economic Review Vol. 7 No. 2 (2010), 113–115

BOOK REVIEW: "THE ECONOMIC THEORY OF ANNUITIES" by EYTAN SHESHINSKI

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In his praise of the book, Jerry Green wrote: "Eytan Sheshinski has written the definitive book on the economics of annuities." Indeed, having read this book, Green's praise is a good summary of my overall impression. The analysis is self-contained and rigorous, yet crystal clear and supplemented with simple economic intuition.

An annuity is an asset that pays to its holder a periodic (monthly, annual) return until death.[1] Given uncertain longevity, Sheshinski explores in-depth the role of these assets in enhancing efficient allocation of resources over time, in particular the efficient transfer of resources from the working period of an individual to her retirement period.

The book starts with a benchmark illustration of the consumption smoothing motive for retirement savings. At this stage the model does not include annuities or uncertain longevity, their very raison d'être. The book then goes on to discuss elementary statistical features of survival distribution functions which represent uncertainty about longevity. These features are also illustrated by the commonly used exponential survival function. The concepts of the probability of death at a certain time, the hazard rate (that is, the probability of death at a certain time, conditional on surviving to this time), and life expectancy are all formally and intuitively derived from the survival function. Bearing on the statistical theory of *stochastic dominance*, the notion that one survival function has a "longer life span" than another is formally defined and graphically illustrated. In particular, this definition implies that a longer life span also means a longer life expectancy.

The crux of the analysis of the main theme of this book—the role of annuities in enhancing efficient resource allocation—begins in Chapter 4. It is first established that in a competitive market the predetermined rate of return on an annuity paid in a certain year is equal to the risk-free annual interest rate, plus the probability of death in that year, conditional on surviving up to that year (the hazard rate). This is essentially a "no-arbitrage" or "zero profit" condition. Sheshinski then goes on to show that if uncertain longevity is the only uncertainty that an individual faces (for instance, there is no income uncertainty), then a competitive market for annuities can support the first-best allocation. Furthermore, individuals annuitize all their savings. Indeed, this is what happens when individuals invest their savings in a large pension fund that every year distributes all its returns among its surviving pensioners. (Recall that because of the "law of large numbers" the pension fund does not face any longevity risk.) Numerical simulations suggest also that

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[1] This is termed a long-run annuity in the book

the existence of a competitive market for annuities entails substantial welfare gains, compared to the no-annuity equilibrium.

Things become more complicated when individuals have different life spans (that is, different survival probability distributions), and this is revealed to them before retirement, but not early in life. In this case two different equilibria can emerge, depending on whether the information about the individual survival probabilities is private or public. Suppose for simplicity that there are only two classes of individuals—one "long-lived" (that is, with a low hazard rate) and the other "short-lived" (that is, with a high hazard rate).

If the information whether a certain individual belongs to the long-lived or short-lived class is public, that is, it is revealed also to the potential issuers of annuities, then a *separating equilibrium* will emerge. Early in life there is only one type of annuity which is offered in the market, as no one has any information yet about the longevity class an individual belongs to. The annual return on this annuity is equal to the annual risk-free interest rate, plus the two-class weighted average of the hazard rate. This return is *actuarially fair*. However, as the information about longevity classes evolves over time, there will be two class-dependent annuities, each yielding an annual return which exceeds the annual risk-free interest rate by the relevant longevity-class hazard rate. More specifically, one annuity will be offered only to individuals of the short-lived class and the other one will be offered only to individuals of the long-lived class; and the former annuity will yield a higher return than the latter (by the hazard rate differential). In this equilibrium all individuals purchase the same class-independent annuity early in life. They fully insure against longevity-class risk. Furthermore, the market for class-dependent annuities later in life is inactive. The separating-equilibrium allocation is a first-best allocation in which consumption is independent of longevity.

A separating equilibrium with longevity class-dependent annuities cannot exist when the information about longevity is private, that is, revealed to the individuals but not to the issuers of annuities. In this case there will emerge a *pooling equilibrium* in which all individuals (indistinguishable to the annuity issuers, early as well as later in life) are offered the same annuity. Long-lived individuals over-invest in this annuity, whereas short-lived individuals under-invest in it; this is known as *adverse selection*. The uniform return on this annuity is less than *actuarially fair*, representing the average return weighted by the "distorted" equilibrium quantities of annuities purchased by the various longevity classes. The pooling-equilibrium allocation is inefficient.

Sheshinski offers several interesting real-world extensions of the aforementioned basic theoretical model. First, the inefficiency (and the adverse selection problem) of the pooling equilibrium may be somewhat alleviated by the existence of period-certain annuities, which provide additional payments after death for a predetermined length of time. Second, it is well known that the practice of "bundling" of commodities and/or services cannot ordinarily survive in competitive markets. Nevertheless, in the case discussed above of asymmetric information about longevity which gave rise to a pooling equilibrium, Sheshinski explains how bundling can emerge in a competitive market: "Bundling may reduce the extent of adverse selection and, consequently, tends to reduce prices" (p. 131). For instance, in a separating equilibrium with full information, firms will offer long-lived individuals annuities at a relatively high price (that is, with a relatively low return) but life

insurance policies at a relatively cheap price. Thus, taking these two prices together, the variation in the combined price over the various individuals is smaller than the variation of each individual price. Thus, in the absence of full information, a pooling equilibrium which bundles together annuities with life insurance may approximate the aforementioned combined price of the separating equilibrium, and may well exist in a competitive environment. Third, the utilitarian-optimal pricing (returns) of annuities when individuals differ in their incomes, in addition to their longevities, is investigated. When income is negatively correlated with life expectancy (which is the case when considering all males as one class and all females as another class), utilitarianism may call for a subsidization of the low-income, long-lived class by the high-income, short-live class – a practice followed by many old-age social security systems.

Summarizing: Eytan Sheshinski has written an elegant and concise treatise which provides a thorough analysis of annuities and insurance. Employing modern tools of economic theory, it aims nevertheless at shedding light on pressing real-world issues concerning old-age social security, life insurance, medical insurance, and social insurance in general. Researchers and practitioners in the field of pensions and social insurance will greatly benefit from this treatise.