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**Estimating the Expected Natural Interest Rate
Using Affine Term-Structure Models:
The Case of Israel**

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Estimating the Expected Natural Interest Rate Using Affine Term-Structure Models: The Case of Israel

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Abstract

This study implements affine term-structure models (ATSM) using indexed bonds data, and derives the dynamics of two macroeconomic factors which determine the expected real rates in the economy. The proposed macro-yields model emphasizes the links between macroeconomic variables and the state variables derived from the ATSMs, by restricting the latent factors so that they behave in accordance with specific macroeconomic variables. Therefore, in this framework, there is no need to integrate the ATSMs with macroeconomic variables in order to get macroeconomic explanations. In addition, the advantages of the macro-yields model against the yields-only model are discussed. The expected short natural real rate in Israel is volatile but the expected long natural real rate is steady at around 3.75 percent.

אמידת הריבית הריאלית הטבעית בעזרת מודל לעקום תשואות

רועי שטיין

תקציר

המאמר מיישם מודל מוכר בספרות המחקרית לניתוח עקומי התשואות, וגוזר בעזרתו את ההתפתחות העתידית של שני גורמים הקובעים את עקום הריבית הריאלית במשק: הריבית הריאלית הטבעית ומידת ההשפעה של המדיניות המוניטרית על הריבית הריאלית במשק, כל אחד לטווחים שונים. המודל המוצע מיושם על נתונין של איגרות חוב ממשלתיות צמודות למדד המחירים לצרכן, ותשואותיהן בטווחים השונים משקפות את הריביות הריאליות במשק. המודל מדגיש את הקשרים שבין משתנים מקרו-כלכליים לבין הגורמים הנגזרים מהמודל, שהם רכיבי הריבית; זאת באמצעות הצבת מגבלות על הגורמים כך שהם יתנהגו בהתאם לשני המשתנים המקרו-כלכליים. לכן, במסגרת זו של העבודה אין צורך לשלב נתוני מקרו לגורמים הנגזרים מהמודל כדי לקבל הסבר מקרו-כלכלי. כן נדונים במאמר יתרונותיו של המודל המוצע ביחס למודל הסטנדרטי המקובל בספרות. נמצא שבישראל הריבית הריאלית הטבעית הקצרה היא תנודתית, ואילו הריבית הריאלית הטבעית הארוכה היא יציבה סביב 3.75 אחוזים. עוד נמצא שההשפעה קצרת הטווח של המדיניות המוניטרית על הריבית הריאלית במשק היא משמעותית, ואילו השפעתה ארוכת הטווח אפסית.

1. Introduction

This study implements a term-structure model using the real rates of interest implicit in indexed bonds and derives the dynamics of two macroeconomic factors which determine the expected real rates in the economy. For that purpose, the real interest rate at the first step is identified by macroeconomic variables, i.e., the expected natural real interest rate and the expected influence of the monetary policy on the real rates. The second step is to link these macroeconomic variables and the factors derived from the term-structure models.

Term-structure models have received renewed attention following the contribution of Duffie and Kan (1996) to the class of affine term-structure models (ATSMs). Following that paper, Dai and Singleton (2000, 2002), Duffee (2002) and Piazzesi (2003) showed how to derive a canonical model with relatively unrestricted assumptions. These ATSMs are based on three basic equations: The dynamics of the factors that determine the one-period rate of interest, the dynamics of the price of risk, and the prices of bonds across different maturities that are derived as exponential affine functions of the factors. The most important assumption in these models is that there is no riskless arbitrage opportunity in bonds of various maturities. However, while these affine no-arbitrage models are extremely popular and provide statistical descriptions of the term structure, they offer little insight into the macroeconomic side. Specifically, the factors derived from the ATSMs are usually not represented by macroeconomic variables and they are known as latent factors. Nevertheless, these ATSMs have significantly improved our understanding of the risk premium dynamics implied by bond yields—a significant breakthrough in the expectations hypothesis. Constructing unrestricted equations to describe the risk premium across time and across maturity enabled us, for the first time, to estimate the whole term structure as well as reasonable and consistent parameters. Furthermore, the existence of forward-looking versus backward-looking dynamics of the term structure is an important issue in macroeconomics and may sharpen our ability to predict future developments.

ATSMs of interest rates start from the general asset pricing condition which is based on the stochastic discount factor. The stochastic discount factor determines the behaviour of the yields on bonds of any maturity, and can be expressed in the case of fixed income as the one-period rate *plus* the price of risk associated with its maturity. Ultimately, the one-period rate and the price of risk are derived as affine functions (linear plus constant) of the state variables.¹ The state variables in the canonical ATSMs are not represented by macroeconomic variables and they are known as latent factors. One of the most important questions, according to the recent literature of the ATSMs, is how many latent factors are compounded in the one-period rate as well as in the term structure. It is commonly presumed that three latent factors characterize the shape of the term structure, namely its level, slope, and curvature (Litterman and Scheinkman, 1991). These three latent factors can describe the whole term structure in each period, but do not have any macroeconomic explanation. Trying to project future developments only by latent factors rather than macroeconomic variables is a very

¹ That assumption means that the yield on a bond of any maturity is affine in the state variables.

demanding task. This is probably the main reason that these models are relatively weak in projections. One way to handle this shortcoming is to introduce macroeconomic variables into the standard ATSMs, e.g., Ang and Piazzesi (2003), Dewachter and Lyrio (2004), Hordahl, Tristani, and Vestin (2006), and Duffee (2006). Ang and Piazzesi (2003) constructed a five-factor term structure model with three latent statistical factors and two macroeconomic factors, which are principal components of a small number of selected economic indicators. Diebold et al. (2006) built a six-factor model with three latent factors and three observable factors, which are the Federal funds rate and two macroeconomic indicators. Ang et al. (2006) constructed a three-factor model with two latent factors and one macroeconomic indicator as the third factor—real GDP growth rates. These studies typically rely on the latent statistical factors to explain the bulk of the term structure movement. Lately, Duffee (2010) checked the Sharpe ratios in ATSMs with high-dimensional Gaussian Models. One clear result is that the Sharpe ratios implied by these affine models are much too high relative to constrained models such as the two-factor model. An alternative method, which is proposed in this paper, and which overcomes the problem of the extreme Sharpe ratios, is to define a few latent factors so each one of them is restricted to behaving like the macroeconomic variable.

This paper focuses on two main objectives. The first is to implement the canonical ATSM on Israeli indexed government bonds, and to examine how this model reconciles the expected real spot rates of interest with the cross-sectional shapes of the term structure. The second objective, which is the main contribution to the ATSMs literature, is to explore the links between macroeconomic variables and the state variables derived from the ATSMs. In fact, the macroeconomic variables which influence the real interest rate are accurately identified in the first stage. Ultimately, the state variables of the ATSM are restricted so that they behave as economic variables. This kind of work can produce not only a statistical description of the term structure but also a reasonable economic explanation for it and a projection of the future spot rates. However, the restrictions on the state variables of the ATSM on one hand improve the macroeconomic explanatory power for the movements in the term structure, but at the expense of the goodness of fit of the model. The question that I intend to deal with in this framework is how much are we ready to pay in the fitness of the model to get economic power in the explanation?

In the empirical literature on term structure modelling, the focus has been on nominal rather than real bond yields, mainly because of the relative scarcity of real debt. The U.S. Treasury first issued TIPS in 1997, but for several years after that initial issuance, the liquidity of the secondary TIPS market was greatly impaired by the small amount of securities outstanding and uncertainty about the Treasury's commitment to the program. Indeed, as described by Roush (2008), secondary TIPS market trading was very low at least till 2002, and D'Amico, Kim, and Wei (2007) estimate that such illiquidity boosted TIPS yields by 1 to 2 percentage points. Accordingly, Christensen, Lopez, and Rudebusch (2008) estimated a term structure model for real zero-coupon U.S. Treasury bond yields derived from TIPS yields by covering the period from January 3, 2003 to March 28, 2008 and considering maturities of five years as a shorter term. However, Israel's Ministry of Finance first issued CPI-(Consumer Price Index-) indexed bonds for 15 years on August 1987 and for shorter maturities even earlier. Thus, the

CPI-indexed bonds in Israel were sufficiently traded from the beginning of the 1990s for all maturities between one and 15 years, but due to the lack of available and reliable data at the outset, the sample period in this paper starts from 1995. In addition, it is notable that the CPI-indexed bonds provide only an approximate indexation, due to the fact that the CPI index is measured discretely with a time delay. However, the linkage to inflation of the Israeli bonds is relatively efficient mainly because the indexation lag is relatively short. Bonds of this type provide a huge advantage in analyzing the real rate in the market.

The advantages of estimating the ATSM on indexed bonds, rather than on nominal bonds, is that the state variables can be readily identified. Changes in the yields on nominal bonds are due to movements in real compensation and to inflation compensation, including the expected rate and the price of risk in each of these compensations. Whereas theoretical research and empirical estimates in the area of ATSMs often assume the real interest rate to be constant, they are overly restricted especially in emerging markets such as Israel. This study avoids dealing with inflation compensation and concentrates only on real rates; it thus has the power to allow the short-term real interest rate and even the long-term real rate to vary over time, as expected from that variable.

One of the critical results in this study, especially for conducting monetary policy, is the estimated factor that reflects the expected natural real interest rate, as it provides an important benchmark for conducting inflation targeting. However, there are alternative definitions of the natural rate, as shown in Amato (2004) and Laubach and Williams (2003).

Definition of (expected) natural rate of interest

For this study, the natural rate is defined as the expected natural fundamental real rate of interest (hereafter, expected natural rate), which equates desired saving with desired investment in an economy without price rigidities. In that case, firms would choose a level of investment such that the marginal productivity of capital is equal to the expected natural rate. The expected rate according to this definition prevails in an economy without nominal pressures and therefore without monetary intervention.² This rate varies over time according to either expected shifts in preferences and technology which mainly affect the long run or unexpected shocks which mainly affect the short run. Furthermore, the short-term expected natural rate is affected by rigidities in the economy and by unexpected shocks, and therefore fluctuates over time, sometimes dramatically. Nevertheless, the long-term expected rate, which is also derived in this study, is consistent both with output equalling its natural rate and with stable inflation. Therefore, the medium- and long-term expected natural rate may provide a good anchor for monetary policy under inflation targeting.

The expected natural rate which is derived in this study differs from the traditional macroeconomic models such as the loanable funds model, which estimate the natural rate directly from macroeconomic variables. The main difference concerns timing. While the rates derived from

² This definition is based on the New Keynesian definition and may also be consistent with Wicksell's definition, as shown in Amato (2004) and Laubach and Williams (2003).

traditional macroeconomic models are based on fundamentals and therefore have backward-looking features, the rates derived from ATSMs are based on yields and therefore are forward looking. The latter concept is more consistent with the monetary policy rule which is based on a forward-looking approach, e.g., the Taylor rule (see Clarida, Gali, and Gertler, 1999). Importantly, the expected rate implies uncertainty regarding the fundamentals, which generates a demand for additional yields as part of the risk premium. This component is taken fully into account in the ATSMs.

The paper is organized as follows. The first part of Section 2 briefly describes and estimates the ATSM on Israeli indexed government bonds—the yields-only model. The second part of the section, 2.2, expands the model by linking the factors derived from ATSM and macroeconomic variables—the macro-identified model. The estimation process is described in Section 3, including tests for the proposed macro-identified model against yields-only models with a classification scheme in the form of $A_M(N)$. The data and the sample period are described in Section 4. Section 5 presents the results, including the evaluations of the two macroeconomic factors of the macro-yield model, i.e., the influence of the monetary stance on the real rate and the expected (short-term and long-term) natural rate. The expected natural rate is examined in a number of case studies over the sample period of this study. Section 6 concludes.

2. The Model

I begin by implementing the canonical ATSM, using zero-coupon real yields data for Israel. The specification of this model is based on the framework of Dai and Singleton (2000), who developed a classification scheme in the form of $A_M(N)$, where N is the number of factors in the ATSM, M is the number of factors that are restricted to follow a square-root process, and $N-M$ is the number of factors that follow the Gaussian process. Gaussian and square-root processes are the best-known examples of affine diffusions. The two classes differ with respect to their assumptions about the variance matrix. Gaussian processes, known as the Vasicek (1977) model, have a constant variance matrix and do not depend on the state itself and thus can take negative values with positive probability. Square-root processes, known as the CIR³ model, introduce conditional heteroskedasticity by allowing its variance to depend on the square root of the state, and thus cannot take negative values. While Duffie and Kan (1996) provided a multidimensional and complete characterization of models with affine bond yields, Dai and Singleton (2000) provided a classification of affine models and established the most general representative example of each class of affine models.

An important preliminary step is to characterize the number and general form of the latent state variables. Researchers have typically found that three factors, often referred to as level, slope, and curvature, are sufficient to account for the time variation in the cross section of nominal Treasury yields (e.g., Litterman and Scheinkman, 1991). Nevertheless, in the indexed bonds the findings are less clear and some argue that a two-factor model is sufficient (see Christensen, Lopez, and Rudebusch, 2008). The last part of this section elaborates this issue and introduces a test to resolve the dilemma.

This chapter describes the three-factor essentially Gaussian, $A_0(3)$, model, which Duffee (2002) finds to be most accurate in forecasting future bond yields, together with two types of two-factor models: $A_0(2)$ and $A_1(2)$. Such models may provide a baseline for comparison with the extended model—the macro-yields affine term structure model, which is introduced in the second part with two factors only. In fact, this study adopts a macro-restriction $A_0(2)$ essentially affine model as its proposed framework. The short real rate in the extended model is determined by two factors, whose dynamics are restricted so that they behave in accordance with specific macroeconomic variables.

2.1. Yields-only two- or three- factor ATSMs

This sub-section describes affine term structure models. Specifically, it briefly describes three types of canonical essentially affine models of the real interest rate. The short real interest rate is determined by two or three latent factors, which follow different diffusion processes. Specifically, the implemented yields-only models will be: $A_0(3)$, $A_0(2)$, and $A_1(2)$.

³ Cox, Ingersoll and Ross (1985)

The model is set in discrete time, and the sampling frequency of the yields of any maturity is monthly but the instantaneous real interest rate r_t is taken to be a year rate. Firstly, the instantaneous real interest rate is assumed to be an affine function of three- or two-state variables and is described by the dynamics of the short rate under the risk neutral measure and thereafter the risk premium specifications:⁴

$$r_t = \delta_0 + \delta' \cdot X_t, \quad (1)$$

where δ_0 is a scalar, δ is an z -vector, and X is a dynamic evolution of z state variables, which under risk-neutral measures can be written as

$$X_{t+1} = \tilde{\kappa} \cdot \tilde{\theta} + (1 - \tilde{\kappa}) \cdot X_t + S_t \cdot \tilde{\varepsilon}_{t+1}, \quad (2)$$

$$\tilde{\varepsilon}_{(t+1)} \sim N(0, \tilde{\Sigma}^2),$$

while θ denotes the long-run mean of the factors ($z \times z$ diagonal matrix), the positive term κ denotes the speed of mean reversion (z vector), and S_t is a diagonal $z \times z$ matrix with element of the square root of the factor when describing the square root process or a constant one when describing the Gaussian process.

The specifications of the real short rate (1) and the evolution of the state variables (2) form a discrete-time model based on a two- or three-factor essentially ATSM of the real interest rate.

From the general asset pricing condition the stochastic discount factor, M , is such that

$$P_t^n = E_t [M_{t+1} \cdot P_{t+1}^{n-1}], \quad (3)$$

where P_t^n is the price at time t of an indexed bond with n periods to maturity, and with terminal condition $P_t^0=1$. The stochastic discount factor is assumed to be conditionally log-normal, so after taking logs (3) gives

$$p_t^n = E_t [m_{t+1} + p_{t+1}^{n-1}] + \frac{1}{2} Var(m_{t+1} + p_{t+1}^{n-1}). \quad (3.1)$$

The lowercase letters denote the logs of the corresponding uppercase letters.

If P_t^n represents the price of a bond with n months to maturity, which in the ATSM class is determined by an exponential-affine (constant plus linear) function of the state variables, then

$$- P_t^n = e^{(A_n + B_n \cdot X_t)}. \quad (4)$$

⁴ The maturities of the bonds range between one year and fifteen years. Even though the shortest bond is one year to maturity and there are no observed yields for three month to maturity, the instantaneous real interest rate $r_{t(0)}$ can be defined for one month.

These ATSMs have coefficients that depend on maturity, n , and make the bond prices consistent with each other for different maturities and with the state variables. This type of model imposes the cross-equation restrictions implied by non-arbitrage and allow yields to be non-normal (as is empirically sustained in the data and shown in Section 4). Taking logs of (4) generates

$$-p_t^n = (A_n + B_n \cdot X_t) . \quad (4.1)$$

At this stage, by equating 3.1 and 4.1 and after calculating the expected value and the variance of the log SDF, one can write the functions of the factor loadings. The next sub-section presents the factor loadings in an specific case of the $A_1(2)$ essentially affine model.

In empirical studies of ATSMs, however, the dynamics of the factors are estimated under the actual probability measure. Thus, to estimate the term structure of the yields one needs to define the market prices of risks, for which this study adopts the essentially affine specifications:

$$\Lambda_{i,t} = \lambda_{1(i,1)} + \lambda_{2(i,2)} \cdot X_{i,t} , \quad (5)$$

in the case of the Gaussian process, and as

$$\Lambda_{i,t} = \lambda_{1(i,1)} \cdot \sqrt{X_{i,t}} ,$$

in the case of the square root process,

and the dynamics of the factors under the actual probability measure are specified as

$$X_{i,t+1} = \kappa_i \cdot \theta_i + (1 - \kappa_i) \cdot X_{i,t} + S_i \cdot \varepsilon_{i,t+1} , \quad (6)$$

where

$$\kappa_i = \tilde{\kappa}_i - \sigma_i \lambda_{2(i,2)} , \quad \kappa_i \theta_i = \tilde{\kappa}_i \tilde{\theta}_i + \sigma_i \lambda_{1(i,1)} ,$$

in the case of the Gaussian process, and where

$$\kappa_i = \tilde{\kappa}_i - \sigma_i \lambda_{1(i,1)} , \quad \kappa_i \theta_i = \tilde{\kappa}_i \tilde{\theta}_i ,$$

in the case of the square root process.

The definition of the stochastic discount factor (SDF), M , is assumed to be conditional log normal of the form:

$$\log(M_{t+1}) = -r_t - \frac{1}{2} \Lambda' \Lambda_{t+1} - \Lambda_{t+1} \omega_{t+1} , \quad (7)$$

The SDF depends on the short rate as defined by the factors (eq. 1) and their innovations, represented by ω . When all of the λ 's coefficients are non-zero, the model is specified as essentially affine. However, if $\lambda_{2(i,2)}$ is restricted to zero, the model is specified as completely affine. In that case the variations in expected excess returns are driven by the volatility of the yields. In the completely ATSMs, the volatility of the state variable is zero at the boundary; however, with Duffee (2002), the

condition that the risk premium goes to zero as volatility goes to zero breaks down and still does not offer arbitrage opportunities. This means that the market price of risk in the essentially ATSM is determined not only by the volatilities of the factors, but also by the factors directly. This kind of specification enables the market price of risk to be significant even when the volatilities of the factors go to zero.

The assumption of no arbitrage opportunities allows us to estimate the stochastic discount factors under an objective probability measure and equivalent martingale measure (see Harrison and Kreps, 1979). The objective probability measure has the interpretation of an expected value under the physical (historical) probability measure of discounted future values, where the discounting is subject to uncertainty. However, the martingale measure has the interpretation of an expected value under a risk-neutral measure, which is artificially constructed. The change in the probabilities of the historical and risk-neutral measures enables us to define the price of risk for each one of the bonds (see Dai and Singleton, 2002).

2.1.1. Yields-only two-factor ATSM $A_0(2)$

This paper elaborates the specification of the $A_0(2)$ essentially affine model and explicitly describes the functions of the factor loadings. The other specifications can be found in Duffee (2002) and many others.

When two of the factors are Gaussian process, the matrix parameters are defined as follow:

$$S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_{1(1,1)} + \lambda_{2(1,1)} \cdot X_1 & 0 \\ 0 & \lambda_{1(2,1)} + \lambda_{2(2,2)} \cdot X_2 \end{pmatrix},$$

and the dynamics of the factors under the actual probability measure are specified as

$$X_{1,t+1} = \kappa_1 \cdot \theta_1 + (1 - \kappa_1) \cdot X_{1,t} + \sigma_1 \cdot \varepsilon_{1,t+1}, \quad (8)$$

$$X_{2,t+1} = \kappa_2 \cdot \theta_2 + (1 - \kappa_2) \cdot X_{2,t} + \sigma_2 \cdot \varepsilon_{2,t+1}, \quad (9)$$

where:

$$\varepsilon_{i,t+1} \sim N(0,1).$$

Thus, the short rate is defined as

$$r_t = X_{1,t} + X_{2,t}. \quad (10)$$

By equating (3.1) and (4.1), and implementing all those definitions, the absence of arbitrage is imposed and the following difference equations for the factor loadings can be derived:⁵

⁵ See the techniques in Campbell et al. (1997).

$$A_n - A_{n-1} = B_{1,n-1}\kappa_1\theta_1 + B_{2,n-1}\kappa_2\theta_2 - \frac{1}{2}B_{2,n-1}^2\sigma_2^2 - B_{2,n-1}\sigma_2\lambda_{1(2,1)} - \frac{1}{2}B_{1,n-1}^2\sigma_1^2 - B_{1,n-1}\sigma_1\lambda_{1(1,1)} \quad (11.1)$$

$$B_{1,n} = B_{1,n-1}(1 - \kappa_1) - B_{1,n-1}\sigma_1\lambda_{2(1,1)} + 1 \quad , \quad (11.2)$$

$$B_{2,n} = B_{2,n-1}(1 - \kappa_2) - B_{2,n-1}\sigma_2\lambda_{2(2,2)} + 1 \quad . \quad (11.3)$$

These recursive equations of the factor loadings impose cross-sectional restrictions to be satisfied by the following parameters: $\kappa_1, \kappa_2, \theta_1, \theta_2, \sigma_1, \sigma_2, \lambda_{1(1,1)}, \lambda_{1(2,1)}, \lambda_{2(1,1)}$, and $\lambda_{2(2,2)}$. These equations are completely determined by specification of the physical dynamics of the short real interest rate.

Thus, the bond yields for different maturities can be identified as follows:

$$y_t^n = -\frac{p_t^n}{n} = \frac{A_n}{n} + \frac{B_{1,n}}{n}X_{1,t} + \frac{B_{2,n}}{n}X_{2,t} \quad . \quad (12)$$

Equation (12) enables the entire yield curve to be modelled and this allows us to extract information from the whole term structure. Notwithstanding the insight that can be gained from this kind of approach for understanding asset price behaviour, this is still basically explaining yields with yields. It is interesting to be able to interpret and identify what underlies these latent factors in term of macroeconomic variables.

2.2. The extended model

The short real rate in the extended model is determined by two factors, whose dynamics are restricted so that they behave in accordance with specific macroeconomic variables. Thus, this study has to adopt a macro-restriction $A_1(2)$ essentially affine model as its proposed framework. In this model, the first factor follows a square-root diffusion process as possibly reflecting the expected natural rate and the second follows a Gaussian model as possibly reflecting the influence of monetary policy on the real rate. This specification is essential for our proposal, since while Gaussian factors can take negative values, as appropriate to the influence of the monetary stance on the real rate, the square-root model cannot. The first factor, which follows the square-root diffusion process, allows its latent factor to get only non-negative values, as typically expected for the natural rate. However, in practice the two-factor Gaussian model produced very similar factors to those produced from the proposed model. Thus, this paper examines the two-factor Gaussian model and checks if its factors can describe the macro-variables. Nevertheless, these restricted specifications might contribute less to explaining the yields than do the unrestricted classical model such as a three-Gaussian-factor $A_0(3)$. Thus, a comparable test is implemented on different models, including the proposed specification. However, there is a trade-off between the restrictions which improve the macroeconomic explanatory power of the movements in the term structure, and the restrictions which cost in the fitness of the

model. The question that I intend to deal with relating to this point is how much of the fitness of the model are we ready to sacrifice to increase its macroeconomic explanatory power?

2.2.1. The real interest rate and macroeconomic factors

The one-period real interest rate, as reflected in the yields on one-period indexed bonds, can be identified as a sum of two different macroeconomic components: one reflecting the expected natural rate, as defined in the introduction, and the other reflecting the temporary influence of monetary policy actions on the real rate. It is convenient to show these two components in an identity equation as follows:

$$r_t^b \equiv r_t^e + [r_t^b - r_t^e], \quad (13)$$

where r^b is the yield on the one-period indexed bond, and r^e is the one-period expected natural rate. This identity-equation approach is employed by Sargent (1969) in his model for modelling the interest rates by macroeconomic fundamentals, among others.⁶ The one-period expected natural rate, r^e , would prevail in an economy without nominal rigidities. This rate varies over time according to expected shifts in preferences and technology, which affect the long run as well as the short run, and according to unexpected shocks, which affect mainly the short run. The second component in Equation 13 represents the gap between the real market rate and the natural rate, which reflects the temporary influence of monetary policy on the real rate, henceforth denoted by r^m . The one-period rate factor, r^m , reflects nominal interventions by the central bank to control inflation. In the long term this factor is expected to be negligible since inflation is expected to be under control over time. Therefore, there is only one factor determining the long-term real interest rate, which is defined as the expected natural fundamental real rate of interest. The compensations for risks are implied in the rates and they are identified with the volatility of the factor and its level. Even though the long-term factor, r^m , is negligible, the compensation for risk is expected to be non-negligible.

2.2.2. Macro-state variable in the ATSM framework

Following the yields-only model which was introduced in the first part, the model is extended by restricting the factors so that they behave in accordance with the macroeconomic variables. As explained above, the one-period real interest rate is determined by two factors. While one of the factors is restricted to reflect the influence of monetary policy on the real rate, the other factor stays unrestricted as a plug factor (residual). I decide to restrict the factor that reflects the influence of the monetary stance on the real side since it is easier to identify this factor in the dynamic equation than to identify the natural real interest rate. The natural rate is subject to changes according to a large

⁶ For example, see Mehra (1994), who examined empirically the role of economic fundamentals in explaining changes in the long-term interest rates, and Hoelscher (1986) who employed variants of this identity equation. This identity equation consists with Wicksell's definition which distinguishes between money market and natural rate, while the gap between those rates outputs inflation pressures.

number of macroeconomic variables, and considerable controversy surrounds the effects of the variables on the natural rate and even the specification of the variables, e.g., to what extent does Ricardian equivalence prevail.

Thus, the short real interest rate in the market is determined by two macroeconomic factors:

$$r_t^b = \delta_0 + \delta_1 \cdot r_t^e + \delta_2 \cdot r_t^m , \quad (14)$$

where

r^b – the yield on one-period indexed bonds;

r^e – the expected one-period natural real interest rate (including the country-specific risk premium);

r^m – the effect of monetary policy actions on the one-period real rate.

The specification of the extended model has the same form as the model described in the first part; the only difference is that the dynamics of the factors are restricted to behave as macroeconomic variables, as shown in Equation (13).

2.2.2.1. The monetary policy factor

The desired nominal target rate for the following period, which is designed by central banks to achieve the inflation target, is assumed to adjust according to the following variables: the gap between the public's inflation expectations and the inflation target,⁷ and the natural nominal rate. The natural nominal rate reflects the nominal rate consistent with the natural real rate and stable inflation, equal to its target. Following Clarida, Gali, and Gertler (1999), this study adopts a forward-looking Taylor rule as follows:

$$i_t^T = (\theta_e + \pi_t^T) + \alpha(\pi_t^b - \pi_t^T) , \quad (15)$$

where i_t^T is the desired target nominal rate, and π_t^b and π_t^T are the inflation expectations (for one year) and the inflation target for the following year. The term $(\theta_e + \pi_t^T)$ represents the natural nominal long rate which is consistent with stable inflation equalling its target. This specification was found to be the most accurate in an economy with inflation targeting, Gali (2008). This methodology was also found to be accurate in describing Israel's nominal target interest rate based on an empirical examination of a macroeconomic model in the period from 1992, when the inflation targeting framework was first adopted in Israel, Elkayam (2001).

However, by using this version of the Taylor rule the influence which the nominal target rate has on the one-period real rate is derived. For this purpose Equation 15 is converted to real terms, as follows:

$$i_t^T - \pi_t^b \equiv r_t^T = (\theta_e + \pi_t^T) + \alpha(\pi_t^b - \pi_t^T) - \pi_t^b , \quad (15.1)$$

$$r_t^T = \theta_e + (\alpha - 1)(\pi_t^b - \pi_t^T) , \quad (15.2)$$

where r^T is the desired target rate in real terms, and θ_e is the natural real long rate. The additional component in Equation (15.2) reflects the influence of monetary policy on the real rate for one year, and it is denoted by $r^m_{(t)}$.⁸ As demanded by the Taylor principle, the parameter α needs to be more than one, as a rise in expected inflation should be followed by a larger rise in the nominal rate, so that the real rate would rise. This parameter can be estimated in this framework as well but with regards to yearly rates rather than monthly rate.

The target influence of the monetary policy, r^m , can be written as follows:

$$r_t^m = (\alpha - 1)(\pi_t^b - \pi_t^T) . \quad (16)$$

Note that the data regarding the inflation target are published, but inflation expectations in Israel are a break-even inflation rate, derived from the bond markets, so it depends on the yield on indexed bonds, which is itself estimated in this study. This means that these derived inflation expectations cannot be used directly as data. Therefore, in order to implement a reasonable restriction on the dynamics of the monetary policy factor, inflation expectations have been replaced with nominal rate (i^b) minus the real rate of interest (r^b), and equation (16) can be rewritten as follows:

$$r_t^m = (\alpha - 1)(i_t^b - r_t^b - \pi_t^T) . \quad (16.1)$$

According to Sargent (1969), the real market rate for one year (r^b) is identified as a sum of two components (see footnote 9); placing these two components in Equation (16.1) we get as follows:

$$r_t^m = (\alpha - 1)(i_t^b - (r_t^e + r_t^m) - \pi_t^T) , \quad (16.2)$$

$$r_t^m = \alpha(i_t^b - (r_t^e + \pi_t^T)) , \quad (16.3)$$

$$\alpha = \frac{\alpha - 1}{\alpha} .$$

Equation (16.3) can be interpreted as the gap between two variables: the desired nominal target rate for one year ahead, and the expected natural nominal rate which is consistent with expected inflation for the relevant period (one year).

Taking this approach for determining the monetary policy factor in the ATSMs we shall get under the Gaussian model that the monetary policy factor, r^m_{t+1} , is determined by:⁹

$$r^m_{t+1} = \kappa_m \theta_m + (1 - \kappa_m) \alpha (i_t^T - (r_t^e + \pi_t^T)) + \varepsilon_{m,t+1} , \quad (17)$$

⁷ The inflation target in Israel was lowered during the begging of the sample period.

⁸ See Equation (13).

$$\varepsilon_{m,t+1} \sim N(0, \sigma_m^2) .$$

This specification, which no longer describes the dynamics of one of the factors as a latent, is the main contribution of this work; the monetary policy factor will be capable of reflecting the monetary stance (contraction and expansion); its average and volatility should represent these features (see Section 4).

2.2.2.2. The natural rate factor

The real interest rate determines the desired flow of new capital which influences aggregate supply, and also the desired ratio of savings to consumption, which influences the aggregate demand of households. The equilibrium point of these two forces reflects both the marginal cost of capital and its amount only when neither monetary intervention nor rigidities exist in the economy. In other words, if the economy is in monetary equilibrium the real market rate equals the expected natural rate. However, when monetary policy acts to change the inflation rate, it influences the real rate in the market and the two forces described above will not be in equilibrium.

The major part of the ATSM literature estimates the term structure of the nominal rate and assumes the real interest rate to be a constant, at least in the long run. However, changes in macroeconomic variables, such as the marginal productivity of capital, the propensity to save, the level of the labor force educational, population mobility, and even the propensity to leisure, affect the natural rate. Practically, the huge influx of immigrants from the former USSR to Israel in the early 1990s significantly affected the natural rate in Israel, (Justman and Zuscovitch, 2002), and Beenstock, 1997). The expected short natural rate factor derived in this study, r^e , reflects the time-varying expected marginal cost of capital prevailing at the equilibrium point of a macroeconomic model, e.g., the loanable funds approach (for a more comprehensive discussion see Appendix 1). However, the dynamics of this factor in this study stay completely free from macroeconomic restrictions, as follows:

$$r_{t+1}^e = \kappa_e \cdot \theta_e + (1 - \kappa_e) \cdot r_t^e + \varepsilon_{e,t+1} , \quad (18)$$

$$\varepsilon_{e,t+1} \sim N(0, \sigma_e^2) .$$

The expected short natural rate in this model is determined in each period by the AR(1) process and by the long-run mean of the natural real rate, θ_e .¹⁰ This factor is a plug (residual) number in an accurately identified model which is indirectly restricted to reflecting the expected short natural real interest rate. The expected natural rate and the compensations for risks are assumed to be the only component variables in the yields on bonds with very long terms. For that assumption, the ATSMs are

⁹ The implied assumption in this approach is that the factor is still affine with the yields despite the macroeconomic restriction on the dynamics of the factor.

¹⁰ In order to keep the model as simple as possible, I decided to keep the central tendency constant, even though this might be an unreasonable assumption, especially in (and after) the period of the huge influx of immigrants from the USSR.

the appropriate model to reconcile the compensation for risks with the yields on long-term bonds. The information from this kind of model could be useful in getting indicators reflecting the whole term structure of the natural real interest rate. Following this specification, the expected natural rate for any maturity, as one of the macroeconomic variables, can be derived and used as a critical indicator for conducting monetary policy.

2.2.3. Macro-yields term-structure model

In the macro-yields model the yields on bonds with n time to maturity, p_t^n , are determined by an exponential-affine function of the natural rate factor and the monetary stance factor, as described above. Thus, the Equation 4.1 in the finance-only ATSM changes, as follows:

$$-p_t^n = (A_n + B_{1,n} \cdot r_t^e + B_{2,n} \cdot r_t^m), \quad (19)$$

and the short rate is defined as:

$$r_t = r_t^e + r_t^m. \quad (20)$$

By equating (3.1) and (19) and implementing the macro-state variable definitions as described above (eq. 17, 18 and 20), the absence of arbitrage is imposed and the following difference equations for the factor loadings can be derived:¹¹

$$A_{n,t} - A_{n-1,t} = B_{1,n-1}k_e\theta_e + B_{2,n-1}k_m\theta_m + B_{2,n-1}(1-k_m)\alpha'(i_t^b - \pi_t^T) - \frac{1}{2}B_{2,n-1}^2\sigma_m^2 - B_{2,n-1}\sigma_m\lambda_{1(2,1)} - \frac{1}{2}B_{1,n-1}^2\sigma_e^2 - B_{1,n-1}\sigma_e\lambda_{1(1,1)}, \quad (21.1)$$

$$B_{1,n} = B_{1,n-1}(1-k_e) + B_{2,n-1}\alpha'(k_m-1) - B_{1,n-1}\sigma_e\lambda_{2(1,2)} + 1, \quad (21.2)$$

$$B_{2,n} = -B_{2,n-1}\sigma_m\lambda_{2(2,2)} + 1. \quad (21.3)$$

These recursive equations of the factor loadings impose cross-sectional restrictions to be satisfied by the following parameters: κ_e , κ_m , θ_e , θ_m , σ_e , σ_m , $\lambda_{1(1,1)}$, $\lambda_{1(2,1)}$, $\lambda_{2(1,1)}$, $\lambda_{2(2,2)}$, and α . Note that the function of the factor loading A_n does not remain constant as in the yields-only models, and it is volatile over time with the current monetary stance. This specification is unique in the ATSM framework and it implies that part of the monetary policy factor's influence is expressed (also) in the factor loading A_n and not only in its factor loading $B_{1,n}$.

¹¹ See the techniques in Campbell et al. (1997).

2.3. Nested and non-nested testing

Nested testing and encompassing refer to tests based on comparing sets of estimates, where generally one set is consistent under weaker conditions than the others. However, the extended model in this framework is not related by parametric restrictions to the classic ATSMs, models with a classification scheme in the form of $A_M(N)$, and thus is not a nested model. A goodness of fit between the models, for instance, does not help to distinguish between the models, and more rigorous statistical tests must be carried out. Therefore, a likelihood ratio test is chosen to evaluate the quality of the models. Only when maximum likelihood estimations are used can one implement likelihood ratio tests to evaluate the fit of the non-nested models (see Vuong, 1989). These likelihood ratio tests are shown in Table 3.

However, this kind of test does not take into account the importance of representing the factor that determines the yields. In particular, the main purpose of the extended model is to tie the canonical ATSM to macroeconomic variables by the identity-equation approach employed by Sargent (1969) rather than estimating and forecasting the yields more accurately. Indeed, it is common in the affine literature that the three-factor model is the appropriate model to estimate and forecast the yield curve, but the latent factors do not have economical interpretation. According to Sargent, the real interest rate consists of two macroeconomic components: the expected natural rate, which can only be positive, and the temporary influence of monetary policy actions on the real rate, which can be positive or negative. That is why the specification of the two-factor model as elaborated in the previous section is essential for our proposed model and the results of such nested or non-nested tests will not give us the whole picture to enable comparisons of different models. Furthermore, in the extended model, the factors are restricted to serving as macroeconomic variables by using the information contains in the Taylor rule. This kind of restriction probably has a negative influence on the fitness of the model but contributes getting economic power in the explanation of the model. To achieve a better understanding of the preferred model and validate its factors and to examine the dynamic of these factors with the fundamentals, as mentioned in Section 2, I plan to perform statistical tests and case studies. These examinations are proposed to show that the two factors indeed represent the two macroeconomic variables, as shown in the identity Equation 13.

3. The estimation process

3.1. The Kalman filter and Maximum Likelihood

This section describes a familiar methodology for estimating both the parameters and the factors of an ATSM. The proposed models introduced in this paper impose, like any ATSM, restrictions on the interest rate dynamics through time and cross maturities (shape of the yields curve at each point in time). Estimating one of these dimensions is inefficient, since term structure models clearly have implications for both dimensions. Estimating both dimensions leads to more powerful specification tests and the market price of risk can be estimated, which is not possible in either one of the two dimensions separately. Therefore, when estimating an ATSM, we are confronted with a joint estimation: estimating the factors with a Kalman filter procedure and estimating the parameters with a maximum likelihood estimator. This methodology exploits the theoretical affine relationship between yields and state variables to subsequently estimate the parameter set. The strength of this approach is that it allows the state variables to be unobserved quantities. Indeed, the Kalman filter is an algorithm that acts to identify the underlying, and unobserved, state variables that govern yield dynamics. Given a model in state-space form, the Kalman filter can evaluate state variables (which are called latent factors) which are optimal in the MSE sense and imposes normal distributions for all of the innovations. The state-space Kalman filter approach has two basic equations:¹² the first is the state equation and the second is the measurement equation,

$$\alpha_t = T_t \alpha_{t-1} + C_t + R_t \eta_t, \quad (22)$$

$$Y_t = Z_t \alpha_t + D_t + \varepsilon_t, \quad (23)$$

The first equation describes how the latent factors evolve through time, while the second equation describes how the latent factors affect the observed data. The Kalman filter procedure can be used on any model that can be brought to this form.

It is necessary to state that a multi-factor model with at least one CIR factor does not satisfy all of the normality assumptions required for statistical consistency in the maximum likelihood estimation of a state-space model. In this kind of application, there is a non-linear relationship between the observed data and the unobserved state variables. Thus, the Kalman filter for estimating the unobservable state variables requires a modification. One of the most common modifications in the literature is called the approximate Kalman filter in combination with quasi-maximum-likelihood (QML)¹³. However, the QML procedure, which is used in many papers in current literature, is based on an approximation of the latent factors' density that becomes very inaccurate for typical parameter values (see Giuliano De Rossi, 2004).

¹² The notation is the same as used by Harvey (1989).

¹³ This procedure can be carried out by substituting the exact transition density by normal density.

However, I estimated the models using the Kalman filter algorithm and assuming that the yields are observed with errors, ε_Y . In order to do so, the state equation of the proposed model, $A_0(2)$, is placed in eq. 22 and it looks as in eq. 24.¹⁴

$$\alpha_t = \underbrace{\begin{bmatrix} 1 - \tilde{k}_1 & 0 \\ 0 & 1 - \tilde{k}_2 \end{bmatrix}}_{T_t} \alpha_{t-1} + \underbrace{\begin{bmatrix} \tilde{k}_1 \tilde{\theta}_1 \\ \tilde{k}_2 \tilde{\theta}_2 \end{bmatrix}}_{C_t} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{R_t} \underbrace{\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}}_{\eta_t} \quad (24)$$

To complete the state space representation of the model, the measurement equation which relates the yields and the factors describing the model is presented as follows.

$$Y_t = \underbrace{\begin{bmatrix} A_1 \\ \cdot \\ \cdot \\ \cdot \\ A_n \end{bmatrix}}_{D_t} + \underbrace{\begin{bmatrix} \frac{B_1^1}{1} & \frac{B_1^2}{1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \frac{B_n^1}{n} & \frac{B_n^2}{n} \end{bmatrix}}_{Z_t} \underbrace{\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}}_{\alpha_t} + \varepsilon_{Y,t} \quad (25)$$

Notice that the shocks are assumed to have normal distribution so the Gaussian log-likelihood process can be used. I use the fact that the observed variables are distributed as a function of the parameters and the unobserved state variables; while the unobserved state variables are constructed through the Kalman filter.

Where the yields (Y) normally distributed and both the mean and the variance are computed in the filtering process, it can be written as follows:

$$Y_t \sim N(Z_t a_{t|t-1} + D_t, F_t). \quad (26)$$

The goal is to minimize the function F for each time t , as shown in 26.

$$F_t \equiv E_t(\alpha_t - a_t)(\alpha_t - a_t)'. \quad (27)$$

¹⁴ Notice that if one of the factors has a square-root processes (non-linear factor) one should first linearized it by using the square root of the estimated value in time $t-1$, which is privileged as a parameter.

3.2. The likelihood ratio test procedure

Let L_1 be the maximum value of the likelihood of the data without the additional assumption. In other words, L_1 is the likelihood of the data with all the parameters unrestricted and maximum likelihood estimates substituted for these parameters. Let L_0 be the maximum value of the likelihood when the parameters are restricted (and reduced in number) based on the assumption. Assume k parameters were lost (i.e., L_0 has k fewer parameters than L_1).

Form the ratio $\lambda = L_0/L_1$. This ratio is always between 0 and 1 and the less likely the assumption is, the smaller λ will be. This can be quantified at a given confidence level as follows:

1. Calculate $\chi^2 = -2 \ln(\lambda)$. The smaller λ is, the larger χ^2 will be.
2. We can tell when χ^2 is significantly large by comparing it to the upper $100 \times (1-\alpha)$ percentile point of a Chi Square distribution with k degrees of freedom. χ^2 has an approximate Chi-Square distribution with k degrees of freedom and the approximation is usually good, even for small sample sizes.
3. The likelihood ratio test computes χ^2 and rejects the assumption if χ^2 is larger than a Chi-Square percentile with k degrees of freedom, where the percentile corresponds to the confidence level chosen by the analyst.

4. The data

The input of the model is a monthly average of zero-coupon real yields based on an a-parametric method of deriving zero yield curves from daily market prices of indexed government bonds (Pomposhko and Wiener, 2006). In order to minimize concerns about pre-smoothing the data, I decided to use estimated zero-coupon real yields rather than trading data of each bond. This decision was made mainly due to the low liquidity of the traded bonds which cause very noisy observations and partly because these indexed government bonds do not provide full indexation and the yield to maturity on these bonds must be adjusted so that it correctly expresses the appropriate real yield.¹⁵ The a-parametric method for calculation the zero-coupon real yields is based on a forward curve approximated by a linear (or piecewise constant) spline and should be applicable even for markets with low liquidity. The best fitting curve is derived by minimizing the penalty function. The algorithm is applied to CPI-indexed bonds traded in Israel. The main problem is that the low liquidity of some bonds means that the yield curves are unsmoothed. However, using forward curves as the state space for the minimization problem leads to a stable solution that fits the data very well and can be used as an input for this work. This kind of smoothed data is used also by Dufresne, Goldstein, and Jones (forthcoming in *Journal of Finance*), who employed the Fama-Bliss (1987) method. They argued that constructing a panel of constant maturity is an additional advantage over using bond yields.

The indexation lag issue is very important when dealing with real yields to maturity based on bonds and the yields must be adjusted such that they correctly express the appropriate real yields. The adjustment is made by dividing the term to maturity of each bond into three sub-periods in order to treat them differently: (i) the period in which there is a change in the CPI, but it has not yet been published at the trading day; (ii) the period in which the bond affords full compensation for the future change in the CPI; and (iii) the period close to maturity, in which there is no compensation for the change in the CPI. To simplify this dividing idea we look at the following example: When the yield to maturity is calculated for a particular date, e.g., 10 May, the repayment of principal and interest are calculated on the basis of the known index, i.e., the index published on 15 April, which reflects the average price level of the previous month, March. It is reasonable to assume, however, that the investors who determine the price of bonds on 10 May take into account inflation that has occurred until that date even though it has not yet been published. This would be the case especially if in the interim period a sharp change had taken place in one of the important factors that affect prices. The period of the run-up to redemption does not provide linkage to the CPI, i.e., the redemption of the bond takes place according to the known index, which was published on the 15th of the month, and refers to the average price level of the previous month. Hence bonds do not provide full linkage to prices, and this must be taken into consideration when calculating the real yield.

In line with the characteristics of indexation described above, certain assumptions were made in calculating the yields on indexed bonds: the estimated change in the CPI that has occurred but has not yet been published is based on the average forecasts of the economic consultants who provide

their forecasts on a regular basis. In the run-up to redemption, in which there is no indexation to the CPI, the bond can be defined as a nominal one, and its nominal value can be discounted in this short period. This approach was implemented by Pomposhko and Wiener, 2006.

I use 15 different periods to maturity between one year and 15 years to maturity. Since the adjustments that were made to calculate the real yields affect mainly the very short term rate, I decided that the short maturity should be one year. The period of this research starts at the beginning of 1995, according to available and reliable data. During the beginning of the sample period Israel underwent two prominent developments that significantly influenced the real interest rate. The first one was a huge influx of immigrants from the former USSR at the beginning of the 1990s that continued until 1996. This was the largest immigration in many years, and it significantly influenced the real interest rates (Section 4.2.1). The second key development was the liberalization of Israel's foreign exchange market and the introduction of greater flexibility in its exchange-rate regime, both of which engendered larger capital movements. Since capital flows were liberalised gradually during the 1990s, the foreign interest rate became increasingly significant. Alongside this development, monetary policy became based on the inflation targeting approach and at the beginning of the sample period monetary policy acted to reduce the inflation rate (Section 4.2.2).

The macro-finance model uses, in addition to the yields, also two variables as described in eq. 16.3, namely:

1. Since the short rate in the estimated model is yearly, the key rate of the BOI needed to be adjusted from one month to year. Thus, the macro-finance model uses in twelve month nominal yields to maturity based on the "*makam*" (short-term bills issued by the Bank of Israel), as an indicator for the key nominal rate for the relevant maturity.
2. The target inflation rate which is published since 1992.

4.1. The real-yields

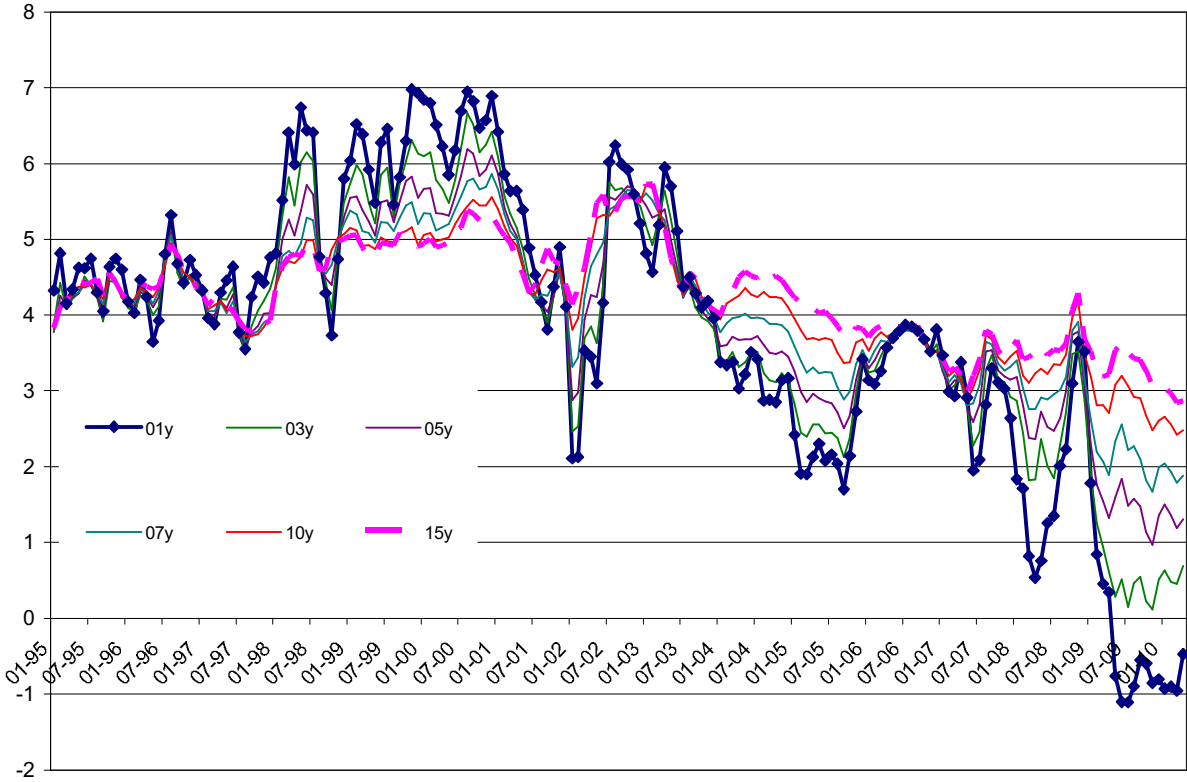
Figure 1 shows the time series plot of the short-, medium- and long-term zero-coupon real rates as derived via the a-parametric zero-coupon yield model, as elaborated above. The length of the sample period was determined by the unavailability of reliable bond data for years prior to 1995. At the beginning of the sample period the real interest rate was high and continued to increase to relatively high levels until 1998–2000¹⁵. During 2001 the real interest rate declined dramatically by more than 2 percentage points and at the end of that year the short rate was reduced by two more percentage points. During the first half of 2002 the yield curve stayed at a relatively low level and increased again during the second half of the year. This change took place against the background of the security problems in Israel during that period. However, since 2003 the real yields, and especially long-term

¹⁵ The feature described above is known as the indexation lag, and affects even more strongly most indexed bonds traded abroad.

¹⁶ Except very short period between 8/1998 and 11/1998, when the BOI sharply reduced the key rate and to control inflation it was forced to increase it again.

yields, decreased monotonically, as shown in Figure 1. Later this section discusses the main developments in Israel during the sample period and describes the dynamic of the fundamentals, which are reflected in the results section - 5.

Figure 1
The Zero-Coupon Real Rates in Israel
monthly average, 1/1995-4/2010 (percent)



01y – one year zero-coupon real rate
...
15y – fifteen years zero-coupon real rate

Table 1
Descriptive Statistics: Zero-Coupon Real Rates in Israel
monthly average, 1/1995-4/2010 (percent)

Maturity	Mean	Median	Std. Dev.	Skewness	Kurtosis -3	Jarque-Bera	AR(1)
Panel A: Yields							
1 year	3.837	4.075	1.95	-0.68*	0.18	14.29*	0.965
2	3.844	4.000	1.72	-0.59*	0.09	10.80*	0.968
3	3.885	3.985	1.53	-0.51*	-0.07	8.04*	0.969
5	3.983	4.050	1.21	-0.39*	-0.32	5.37	0.971
10	4.194	4.240	0.78	-0.14	-0.68	4.21	0.973
15	4.291	4.330	0.67	-0.02	-0.72*	4.02	0.969
Panel B: Monthly changes in yields							
1 year	-0.026	-0.05	0.52	-0.16	2.04*	32.6*	0.267
2	-0.021	-0.05	0.43	-0.06	2.39*	43.6*	0.262
3	-0.017	-0.05	0.38	0.02	2.37*	42.8*	0.265
5	-0.013	-0.03	0.29	-0.01	2.12*	34.3*	0.310
10	-0.007	0.00	0.18	0.02	1.17*	10.5*	0.343
15	-0.005	-0.02	0.17	0.14	0.87*	6.4*	0.311

* Significantly different from zero.

Descriptive statistics for yields and monthly yield changes for the entire sample of this model are given in Panels A and B of Table 1, accordingly. Panel A shows that the average yield curve is an increasing and not so concave function of maturity, but the median yield curve shows a negative hump in the short term (3 years), and is slightly upward sloping for the remaining of the maturity range. This statistical result is very unusual when examining the nominal yield curve but not so when examining the real one (see, Christensen, Lopez, and Rudebusch, 2008). Yields are highly persistent in levels for all maturities, with autocorrelations above 0.96 increasing with maturities. Other stylized facts can be observed in Table 1. The average volatility curve of yield levels decrease with maturity, sharply in the shorter maturities and moderately in the longest. Yields are significantly negatively skewed at short and medium maturities. On the other hand, yield levels show kurtosis insignificantly different from zero except at the long end of the curve. Kurtosis can be described as the degree to which, for a given variance, a distribution is weighted towards its tails. In this way, kurtosis is different from the variance, which measures the dispersion of observations from the mean, in that it captures the probability of outcomes that are highly divergent from the mean, i.e., extreme outcomes. At the long end of the curve the kurtosis shows negative values, which implies that the probability for extreme outcomes is low relative to the normal distribution. Panel B teaches us that monthly changes are small on average over the sample period, mainly in long maturities, and on average are negative. This statistical result tells us that the curve decreases slightly during the sample period and implies strong prominent developments in Israeli's economy (elaborated in the following section – 4.2). The average volatility decreases sharply over the short maturities and declines more slowly in medium and long maturities. They are not skewed at all but have significant positive excess kurtosis and thus their distributions are

significantly different from normal. Monthly changes in yields are persistently low for all maturities—with autocorrelations around 0.3 and slight increases in maturities.

4.2. Prominent developments in the Israeli's economy

4.2.1. The immigration

The large influx of immigrants from the former USSR to Western countries in the 1990s assumed exceptional proportions in Israel. In seven years Israel's population increased by 18 percent. The rate of immigration was very high in the first two years, increasing the 1989 population by 10 percent. Subsequently, immigration settled down to about 3 percent per year. This was the biggest wave of immigration for many years; however, the quantitative importance of immigration in the aggregate labor market is larger than its proportion in the population. Immigration was translated into economic growth mainly because the immigrants had a high participation rate and were willing to work longer hours (see Hercowitz and L. Meridor, 1993). In addition, real wages in Israel were relatively flexible (Beenstock and Fisher, 1997)—they ceased to increase and even fell during the first half of the 1990s, helping to promote output growth. Nevertheless, the expectations related to for the immigrants in addition to their expected gradual assimilation had caused an initial investment boom, especially in the first years, even before they had assimilated in the labor market. Therefore, at first the real interest rate was relatively low, that indicates that the level of the price of new capital was high relative to the level of the marginal productivity of capital. Subsequently, and especially between 1995 and 1998-99, the marginal productivity of capital started increasing along with the gradual assimilation of the immigrants,¹⁷ implying that the changes in the real interest rate during the 1990s should be positive, and so they were.

4.2.2. Liberalization of capital flows and greater flexibility in the exchange-rate regime

Between 1983 and 2001, Israel gradually abolished all foreign exchange controls, and moved from a fixed exchange-rate regime into a floating exchange rate. The change in the exchange-rate policy took place over a few steps during the period sample: first, a horizontal exchange-rate band was adopted, which then changed to an upward-sloping diagonal band and then to an ever-widening band (see, Stein 2003). In addition, Israel's foreign trade grew during the 1990s, following a series of agreements with the EU and the US. Both of these two key developments allowed more flexibility in capital movements. In addition, at the end of 2003, Israel abolished tax discrimination against foreign investment, equalizing the tax rates on foreign and local investment. As a consequence of these gradually implemented policy reforms, the effects of the foreign interest rate became increasingly significant. Financial globalisation can provide significant benefits to developing countries but at the same time poses significant risks. There is strong evidence to suggest that developing economies

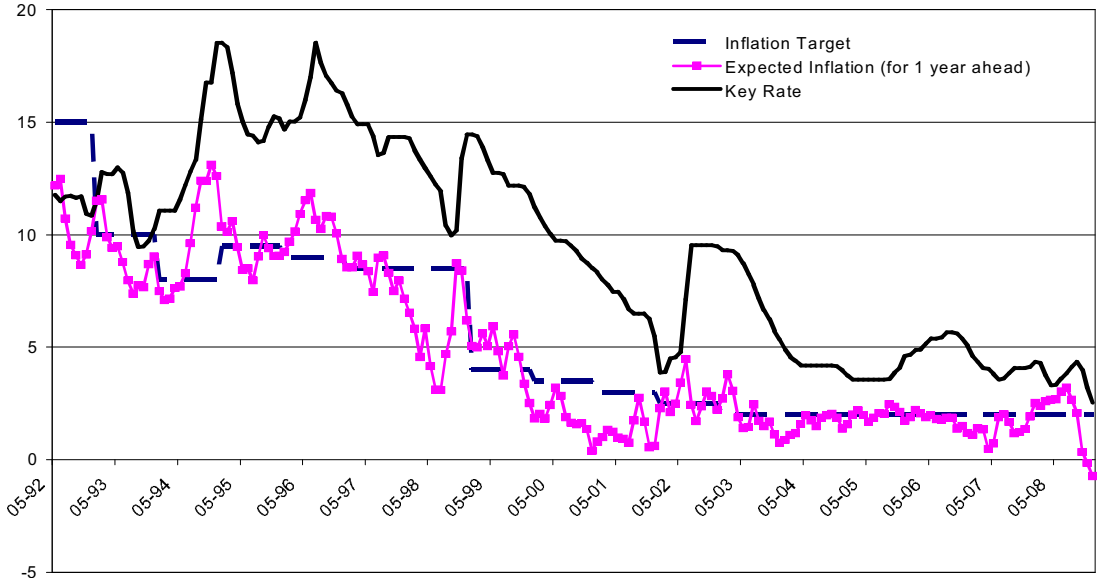
¹⁷ See for example, Beenstock and Fisher (1997).

could benefit from financial globalisation, given certain conditions (Saxena, 2008). He finds that domestic short-term interest rates are significantly affected by foreign interest rates. The link between domestic and foreign interest rates is also in line with Moreno (2008) who finds that the foreign long-term interest rate affects the domestic long-term interest rate more than does the domestic policy rate. Thus, the impact of the liberalization of capital flows in Israeli exchange rate on the natural real rate is not straightforward and depends on the foreign interest rate developments.

4.2.3. Monetary policy framework

Replacing the exchange rate by an inflation target as the nominal anchor was the main change in the monetary policy in the late 1980s and in the 1990s. Hyperinflation in the first half of the 1980s was followed by inflation of about 20 percent at the beginning of the 1990s. Monetary policy continued to reduce inflation and kept the interest rate relatively high during most of the sample period. Ultimately, the inflation rate was reduced from 20 percent per year at the beginning of the sample period to about 2 percent at the end (see Figure 2).

Figure 2
Basic monetary data 1992-4/2010
(percent)



In particular, during the sample period there were four different monetary policies: in the period from January 1992 to June 1996, inflation in general was high, and monetary policy was adaptive. In that period the exchange rate was still administered and therefore inflation targeting was not the main policy framework. The second period, from July 1996 to the end of 2003, is characterized as a period with tight monetary policy except for two occasions when the central bank unexpectedly reduced the nominal rate, and was then forced to raise the rate a few months later to control inflation. In the third period, from 2004 to the end of 2008, inflation was generally around the target and the monetary

stance was geared to achieving the target. However, at the end of the sample period, with the world financial crises, the monetary policy acted strongly to prevent deflation in Israel and reduced the key rate to a historically low level—half a percent. These developments should be reflected in the results *infra*.

4.2.4. Reforms in capital markets

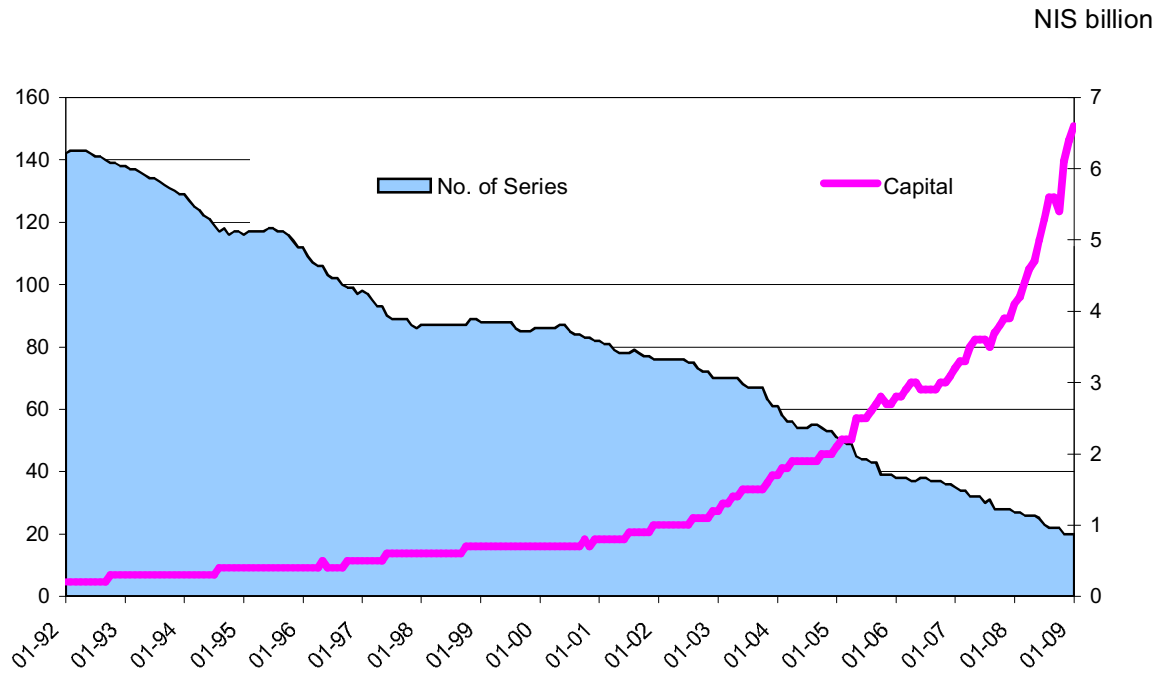
During the 2000s and especially between 2003 and 2007 there occurred an intensive and strong process of reforming the capital and money markets for generating an effective and competitive capital market in Israel. The authorities made a number of structural changes in the market, and established new frameworks for expediting the reforms in the capital market in the coming years. The main reforms were: abolition of the discriminations between the capital and labor markets and between financial assets; reduction of the activity of the pension funds in nontradable assets; a new and advanced underwriting law; the development of private bonds and commercial assets; enhancement of the Bank of Israel's ability to carry out its policy via market tools; a system of market makers in private and government securities; a clearing system for large payments in real time—RTGS (Real Time Gross Settlement); enhancement of the Tel Aviv Stock Exchange clearing houses; insurance for bank deposits (supporting the stability of the banking system).

The reforms are intended to enhance the sophistication of the local capital market, improve the efficiency of securities trading and minimize the risks regards transferring and clearing money and assets. These reforms increased the markets' tradability and liquidity and reinforced the financial infrastructure, as expected, and these exert downward pressure on the natural real rate in the market.

An additional substantial development that took place during the sample period was related to the microstructure of the bond market in Israel: the number of bond series has been reduced over the decade, as is shown in Figure 3. During the last decade, the number of series of government bonds in general, and specifically indexed bonds, has continued to be reduced, and the focus has switched to a small number of large benchmark series. The number of series went down from 150 at the beginning of the 1990s to 60 in 2000 and to about 20 in 2009. At the same time the average size of a series rose from \$200 million in 1990 to \$700 million in 2000 and to more than \$6 billion in 2009. The main goal of this development was to improve the level of tradability and liquidity in the government bond secondary market and to reduce the government's funding costs.

Figure 3

No. of series and par value of capital



5. Results

I use monthly zero-coupon real yields from January 1995 to April 2010 for 15 different maturities between one and fifteen years. Table 2.1 reports the estimated parameters for all finance-only models: three Gaussian factors, $A_0(3)$, and two kinds of two-factor models, $A_0(2)$ and $A_1(2)$. Table 2.2 shows the estimated parameters for the macro-finance $A_0(2)$ model.

Table 2
Summary of Results

Parameter	2.1. yields-only models			2.2. macro-yields models
	$A_0(3)$	$A_0(2)$	$A_1(2)$	$A_0(2)$
θ_1	0.021 (0.2)	0.03 (0.26)	0.034 (0.0034)	0.0375 (0.0013)
θ_2	0.006 (0.051)	0.004 (0.16)	0 -	0 -
θ_3	0.007 (0.0072)	-	-	-
κ_1	0.08 (1.29)	0.11 (0.15)	0.033 (0.0035)	0.075 (0.005)
κ_2	0.15 (6.6)	0.07 (0.12)	0.053 (0.009)	0.5 (0.005)
κ_3	0.06 (1.5)	-	-	-
$\lambda_{1(1,1)}$	0.5 (4.8)	0.71 (20.8)	-1.02 (0.018)	0.51 (0.102)
$\lambda_{1(2,1)}$	0.44 (5.6)	-0.37 (5.3)	-0.28 (0.16)	0.06 (0.06)
$\lambda_{1(3,1)}$	0.84 (26.2)	-	-	-
$\lambda_{2(1,1)}$	-26.1 (17.8)	-29.9 (6.2)	-	-19.0 (0.65)
$\lambda_{2(2,2)}$	59.9 (28.2)	29.9 (3.7)	32.6 (0.87)	-46.6 (1.2)
$\lambda_{2(3,3)}$	27.3 (16.9)	- -	- -	- -
σ_1	0.002	0.003	0.016	0.003
σ_2	0.007	0.007	0.007	0.015
σ_3	0.011	-	-	-
σ_m	0.0004	0.0007	0.0007	0.0007
Maximum likelihood	16372.2	15352.4	15352.3	15129.5
Goodness of fit	2.9E-04	5.5E-04	5.5E-04	5.5E-04
α	-	-	-	0.92
Sample period	1/1995–4/2010			

* Parentheses show asymptotic standard errors.

The κ_i and θ_i parameters, along with the σ_i parameters, control the time series properties of the yields ($i \in e, m$); whereas the κ_i , which are the speed of mean reversion parameters, determine the dynamics of convergence to the rates of long-run mean parameters, θ_i . The λ_i parameters, on the other hand, control the risk adjustments when moving from the real world distribution to the risk neutral distribution for asset pricing.

Table 2.1 shows the estimated results for the yields-only models. The estimated parameters reflect the fact that the factors indeed do not represent macro variables: for instance, the persistence of the mean reversion, κ_i , is too small to reflect the features of macro-variables, especially in the case of monetary policy which is expected to change its stance much more frequently than a measure of 7 percent speed of mean reversion each month. Thus, the unexpected shocks are expected to impact on the short rate yields for a relatively too long period—an unreasonable outcome.¹⁸ The parameters derived from the ATSM reflect the dynamics of the two factors under actual probability measure. However, in Table 2.2, which shows the estimated results for the macro-yields models, the parameters are more economically reasonable and the factors indeed represent macro economic variables. Figures 4-8 support this statement.

The factor loadings implied by the estimated parameters provide an insight into how each factor translates into movements across the yield curve. However, in the macro-yields model the factor loading A_n is volatile over time, an unfamiliar outcome in the canonical AFTMs. Thus, Figure 4 shows the factor loadings driving the yield curve, but in the case of the factor loading A_n presents the average for each time to maturity, n , along with its standard deviation. Figure 5 displays the natural rate factor and the monetary policy factor. The average yield curves during the sample period, the dynamic of the risk premiums and an example of one current yield curve are shown in Figures 6 to 8. Several features, reflected from these figures, are worth noting:

1. The second Gaussian factor's sensitivity (B_2 , see Figure 4) fell significantly over time to maturity, as is expected from a factor that represents the influence of the monetary policy on the real rates. This means that the effect of unexpected shocks on inflation has a relatively short life—after three months 12.5 percent of the unexpected shock is left and after a year only 6 percent of the unexpected shock is left. This result can be interpreted as evidence of a short-term effect of monetary policy during the sample period.¹⁹
2. The first Gaussian factor's sensitivity (B_1 , see Figure 4) does not fall significantly over the term to maturity, as expected from a factor that represents the natural real rate. This means that the effect of unexpected shocks on the real side of the economy has a relatively long life—after one year 40 percent of the unexpected shock is left and after 3 years 7 percent of the unexpected shock is left.

¹⁸ In the case of the $A_1(2)$ model, the unexpected shocks are estimated to impact for many years since the k parameters are smaller than in all other models, between 3 and 5 percent, in each of the factors.

¹⁹ This evidence is seemingly contrary to Ber, Brender and Ribon's (2004) evidence. They argued that the effect of the monetary policy on long-term yields is not inconsiderable mainly due to the disinflation process that took place in their sample period, 1990-2002.

The first factor (shown in Figure 5) which reflects the natural real rate moves in a narrow band, with a minimum of two percent in May 2007 and maximum of 6 percent in June 2002 and February 2003. However, in the first half of the sample period the natural real rate fluctuated around 4 percent but from 2003 it decreased to a lower level and it fluctuated around 3 percent. The natural short rate was volatile but its long-run rate was steady around 3.75 percent.

3. The influence of monetary policy on the real rates factor was more volatile than on the natural real rate, and the dramatic changes around October 1998 and in the first half of 2002 can be seen clearly in Figure 5. For instance, it can be seen that during the first seven months of 1998, the monetary policy kept the rates extremely high and the effect on the real rate was correspondingly high—and indeed the inflation rate decreased in that period to 4 percent, compared with the inflation target which in that year was 8 percent. On August, however, the monetary policy reduced the key rate by 2 percent and the influence shrank from 3 percent to one percent and two months later the LTCM crisis unfolded and the shekel depreciated at a sharp rate. As a result, the natural real rate increased slowly and the influence of the monetary policy continued to shrink and even roll over to a measure of minus 1.4 percent.²⁰ By the end of the year, the monetary policy was forced to increase the key rate again by more than two percentage points due to inflation targeting. Another example that can be seen clearly from Figure 5 is at the end of the sample period when the monetary policy acted to reduce the short rate to prevent deflation in Israel; its influence in that period was at a historical high. The monetary policy succeeded to reduce the short real rate by more than 3 percent and so it offset the short real natural rate.
4. The average yield curve has a positive slope but the curve without the risk premium is completely flat, as expected. The average risk premium for the natural rate factor has a positive and monotonic slope and the average risk premium for the monetary policy factor has a positive slope but it moderates with time to maturity, n . This means that the risk premium of the natural rate factor increases with time to maturity and the risk premium of the monetary policy factor increases but in the long run the rise is reduced and converge to zero. In addition, the risk premium of the monetary policy factor is volatile especially in the short run; sometime it is negative and sometime it is positive. Thus, at the end of the sample period, when the monetary policy acted strongly to reduce the short rate, the risk premium was negative and indicated that the bond holders priced the risks on this factor at about one percentage point. Thus, the yield curve without risk premiums was higher (and less negative) especially in the short term. Nevertheless, the risk premium always converges with the time to maturity to a low positive value. Thus, the standard deviation of the monetary stance risk premium decreases with time to maturity.

²⁰ It is helpful to look at eq. 16.2, which clearly shows the relations between the key rate, the natural rate, and the inflation target.

Figure 4
The loading factors driving the yield curve without risk premium

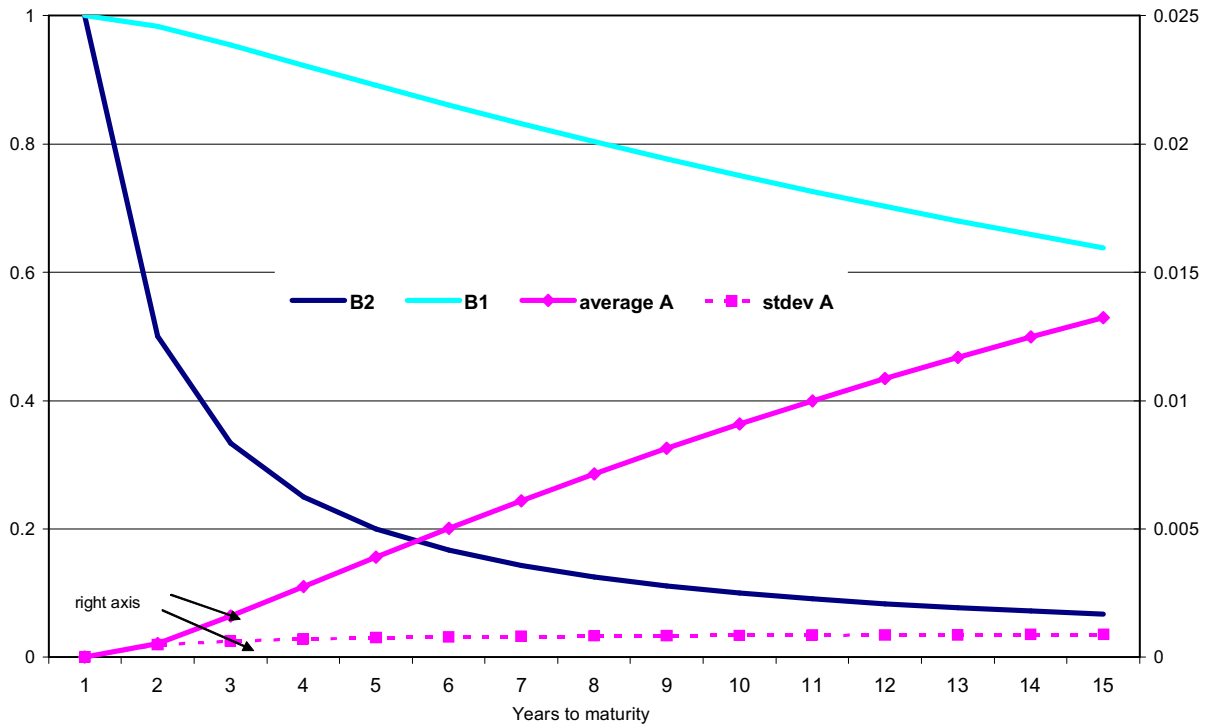


Figure 5
The real interest rate and its components:
The Natural real rate and the monetary stance policy

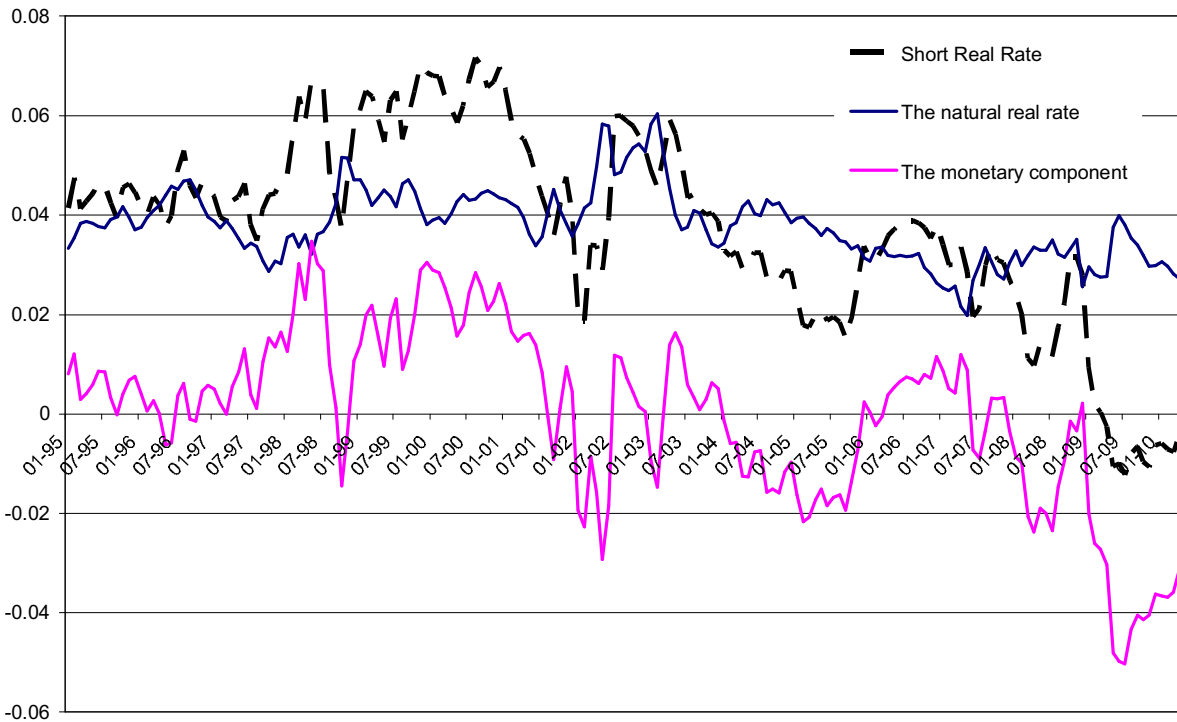


Figure 6
Average yield curves – 01/1995-4/2010

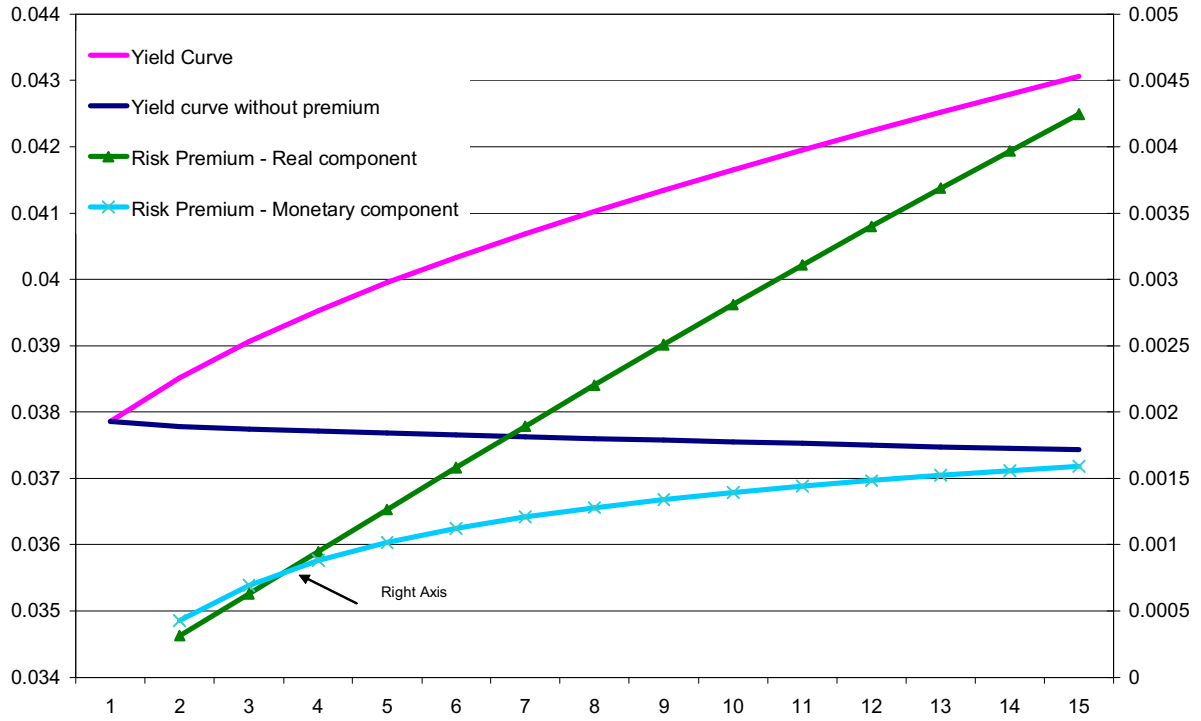


Figure 7
Risk Premiums over time for two and ten years

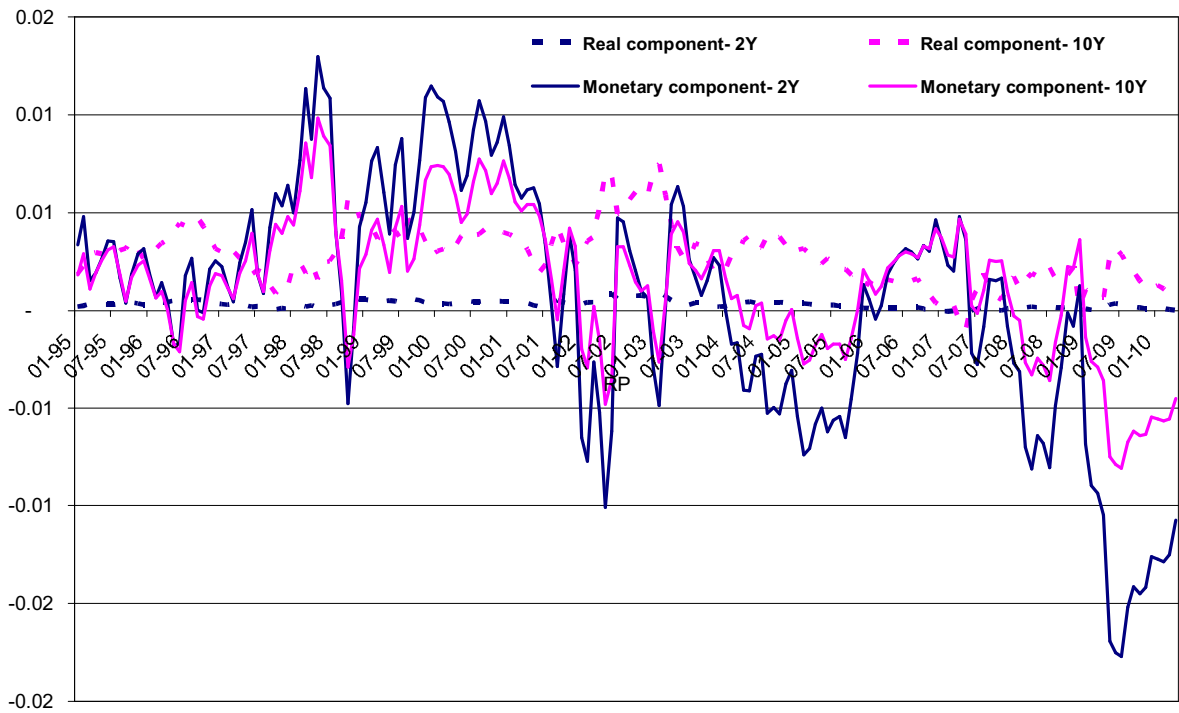
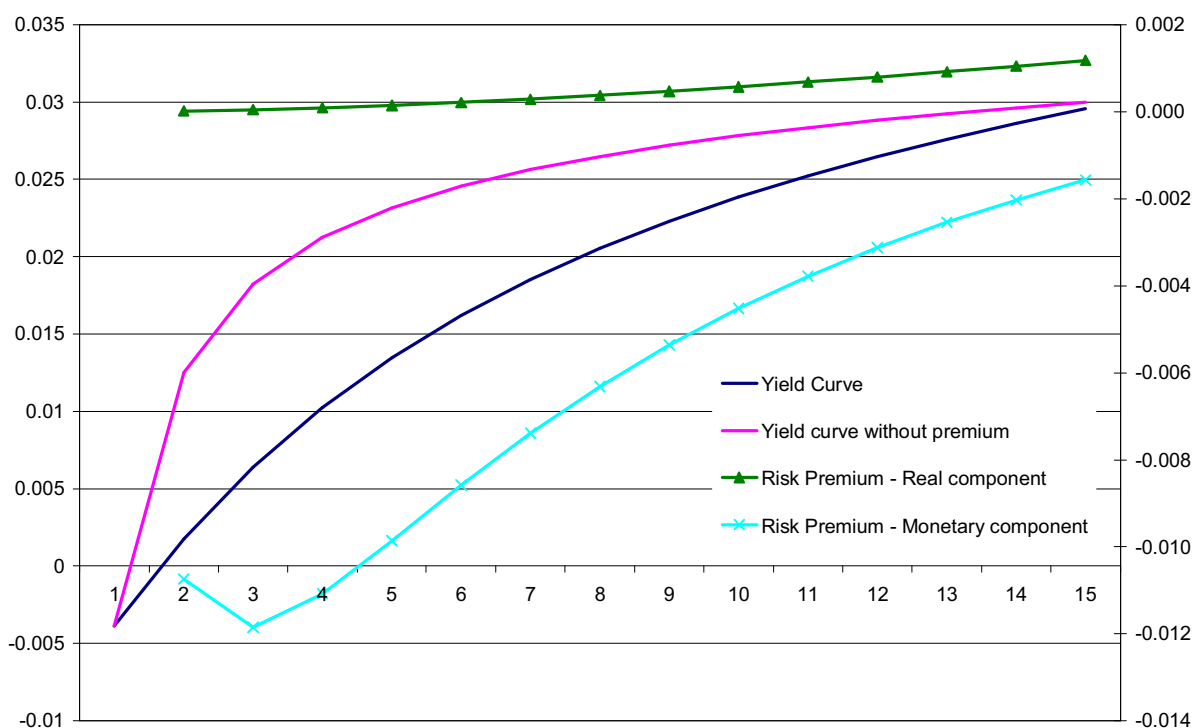


Figure 8
The Yield Curve 04/10



5.1. A comparison between the models—validating the factors

A likelihood ratio test is chosen for a statistical evaluation of the quality of the models.²¹ The likelihood ratio tests, as shown in Table 3, compare the different models to the $A_0(3)$ finance-only-model as representing an unrestricted model.

Table 3
Likelihood Ratio Tests for Nested and Non-Nested Models

The Model	Maximum Likelihood	Likelihood Ratio Statistic	Degrees of Freedom	Critical Value	
yields-only				0.95	0.99
$A_0(3)$	16372.2				
$A_0(2)$	15352.4	0.129	6	1.635	0.872
$A_1(2)$	15352.3	0.129	7	2.167	1.239
Macro-Yields					
$A_0(2)$	15129.5	0.158	5	1.145	0.554

* The benchmark for all models is the $A_0(3)$ model.

²¹ The likelihood ratio test computes the statistic, χ^2 , and rejects the assumption if χ^2 is larger than a critical value, which is based on degrees of freedom and the confidence level chosen by the analyst.

According to the results in Table 3 the maximum likelihood differences between the models are insignificant, which means that the macro-yields model with two factors has insignificant cost in fitness of the model. Even so, to approve the proposed macro-yields model as the preferred model, the results must be examined rigorously. In particular, the differences between the factors and parameters of the two—the yields-only and the macro-yields of the $A_0(2)$ models—should be examined, and whether those of the macro-yields model yield a better economic explanation. Afterwards, a number of case studies are implemented to show that the factors obtained indeed reflect macroeconomic variable, as identified in this work.

The latent factors in the $A_0(2)$ yields-only model and those obtained from the macro-yields model are very similar, with correlations of 0.7 and 0.99 for the first factor (the natural rate) and second (the monetary stance) respectively (see Appendix 2). However, when we elaborate the comparison between the two models (Table 4) we infer a few important differences:

1. While there is not much difference in average and in standard deviation between the first factors and between the second factors of the two models, the skewness and especially the kurtosis for the first factors (which reflect the natural real rate) are very different: in the yields-only model the first factor has significant positive excess kurtosis and thus the probability of extreme outcomes is high and not suitable to the expected natural real rate. In contrast, in the macro-yields model the first factor has insignificant excess kurtosis and thus the probability of extreme outcomes is relatively low, as expected from this factor.
2. The correlation between the two factors in the yields-only model is significantly negative whereas the correlation between the two factors in the macro-yields model is insignificantly positive. This implies that the monetary policy stance as reflected in the macro-yields model and the short-real natural rate are uncorrelated.

Table 4

Descriptive data for the estimated factors

The Model	Average	Standard deviation	Skew	Kurt	Correlation
yields-only - $A_0(2)$					
First factor	0.03	0.006	0.61*	1.42*	-0.25*
Second factor	0.008	0.02	-0.75*	0.33	
Short rate	0.038	0.02	-0.62*	0.11	
Macro-Yields—$A_0(2)$					
Natural rate (first factor)	0.038	0.007	0.47*	0.53	0.12
Monetary stance (second factor)	0	0.018	-0.64*	0.27	
Short rate	0.038	0.02	-0.62*	0.11	

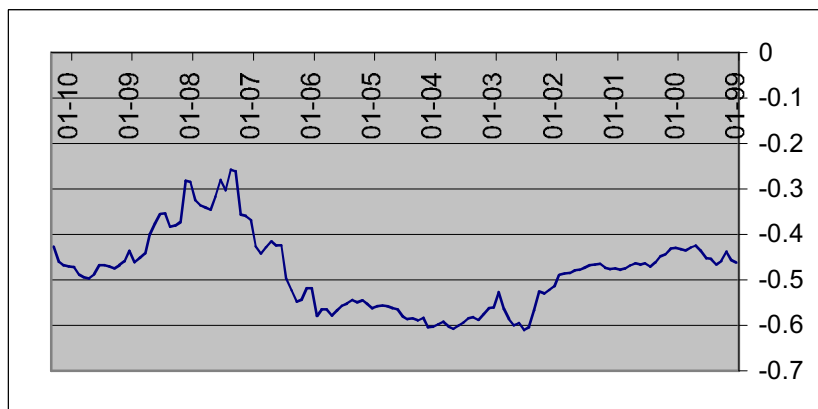
* Significantly different from zero.

5.2. Tests for the monetary stance factor—validating the factor

The correlation between the current monetary stance factor and the inflation rate is significantly negative, as expected and appropriate. However, this kind of examination may be biased due to common innovations typical of macroeconomic variables. It is conventional to avoid this bias by calculating the correlation between the changes in these variables, see Figure 9. The correlation estimated for the whole sample period is minus 47 percent—a satisfactory measure which implies about the strong connection between them. While this result indicates that the monetary stance factor is indeed connected to the inflation rate, it is not enough to indicate the direction of the connection, which would be a more accurate examination.

Figure 9

The correlation between the changes in monetary stance factor and in cumulative 3 month inflation (moving average of 48 observation points)



An alternative and neater way to validate the monetary stance factor is to apply pair-wise Granger Causality Tests. The Granger (1969) approach to the question of whether the monetary stance factor causes the inflation rate is to measure to what extent the current inflation rate can be explained by past values of the inflation rate and then to measure whether adding lagged values of the monetary stance factor can improve the explanation. The inflation rate is said to be Granger-caused by the monetary stance factor if it helps in the prediction of the inflation rate. Note that this test is more powerful and rigorous than correlation, which does not necessarily imply causation.

The regressions for the Granger causality are estimated as follows:

$$1. \pi_t = \alpha_0 + \alpha_1 \pi_{t-1} + \dots + \alpha_j \pi_{t-j} + \beta_1 r_{t-1}^m + \dots + \beta_j r_{t-j}^m + \psi_t$$

The null hypothesis, H_0 : the monetary stance factor does not Granger-cause the inflation rate.

$$2. r_t^m = \alpha_0 + \alpha_1 r_{t-1}^m + \dots + \alpha_j r_{t-j}^m + \beta_1 \pi_{t-1} + \dots + \beta_j \pi_{t-j} + \psi_t$$

The null hypothesis, H_0 : the inflation rate does not Granger-cause the monetary stance factor.

In general, the null hypothesis is that: $\beta_1 = \beta_2 = \dots = \beta_j = 0$, for each regression.

When the lag, j , is more than 6 months the null hypothesis is rejected for the first regression and is not rejected for the second. This means that the monetary stance factor, as estimated in the proposed model, affects the inflation rate, as expected from this kind of factor, and in the opposite direction it is not affected by the inflation rate. Hence, the monetary stance factor cannot be explained by the inflation rate, as appropriate to this kind of variable. In contrast to this result, while replacing the monetary factor with the nominal key rate of the central bank we get that the key rate affects the inflation rate (with no high level of significance) and in the opposite direction it is affected by the inflation rate; this is a problematic result for an indicator that is supposed to reflect the monetary stance policy. Furthermore, when replacing the inflation rate with the expected one-year inflation rate we get that the monetary factor on one hand negatively affects the expected inflation rate, and on the other hand it is positively affected by the expected inflation rate. These results validate the monetary factor, i.e. when the expected one-year inflation rate rises the monetary stance factor tends to rise and only then the expected one-year inflation rate tends to fall.

The natural real rate factor is very difficult to validate, since this factor cannot be measured by any macroeconomic model with satisfactory precision (see, Weber, Lemke, and Worms, 2007, and Giammarioli and Valla, 2004)). However, the results of the Granger causality tests give an important insight about the two estimated factors: the monetary policy as well as the natural real rate factor. The natural real rate factor together with the monetary stance factor composes the real rates in the markets and hence the natural real rate is validated indirectly in these kinds of tests. Nevertheless, in the next sub-section (5.3.) a number of case studies documenting the natural real rate and the fundamentals are analyzed.

Before the next section, that analyzes the natural real rate factor, it is worthwhile to present an example of the kind of information one can derive from the proposed model. The following quote is from the Bank of Israel Annual Report for 2004: "A retroactive analysis of price developments raises the question as to whether monetary policy in 2004 was expansionary and if so, to what extent." The authors tried to deal with this question based on a number of arguments that take into account the developments of two indicators: the expected real interest rate in the economy determined by Bank of Israel—the key rate of the Bank of Israel less inflation expectations for one year; and the slope of the curve of indexed bonds. However, that analysis of the monetary policy did not provide a measurement of the extent of the expansion. In contrast, the proposed model can provide not only a satisfactory assessment of the monetary stance but can also quantify it.

5.3. Case studies—validating the natural real rate factor

The case selections:

1. Jun-1996 to Dec-1997: In this period the natural real short rate declined from 4.5 percent to 3 percent.
2. Apr-2003 to May-2007: In this period the natural real short rate declined first sharply and then monotonically from 6 percent to 2 percent.
3. March-2009 to end of 2010: in this period the natural real short rate first jumped and then declined from 4 percent to 2.7 percent, while the short real rate in the market stayed negative without any specific trend.

1. In the period from Jun-1996 to Dec-1997 GDP growth was moderate. It was right after the period of immigration influx diminished. Developments in unemployment in previous years show that after a gradual decline in unemployment from more than 11 percent in 1992 to 6.5 percent in the second quarter of 1996 (mainly due to the assimilation of the new immigrants in the labour market), the trend reversed, and unemployment began to rise, reaching 8 percent in the third and fourth quarter of 1997. The increase in unemployment was mainly the result of an easing of business-sector labour demand caused by a slowdown in economic activity; this can be seen from real-side analysis which shows that the slowdown in product demand was the dominant cause in the deceleration of economic activity whereas the effect of supply factors was secondary (Bank of Israel Annual Report, 1997.). These developments reflected demand shock and pushed the natural real short rate down to 3 percent, a low level relative to the benchmark.

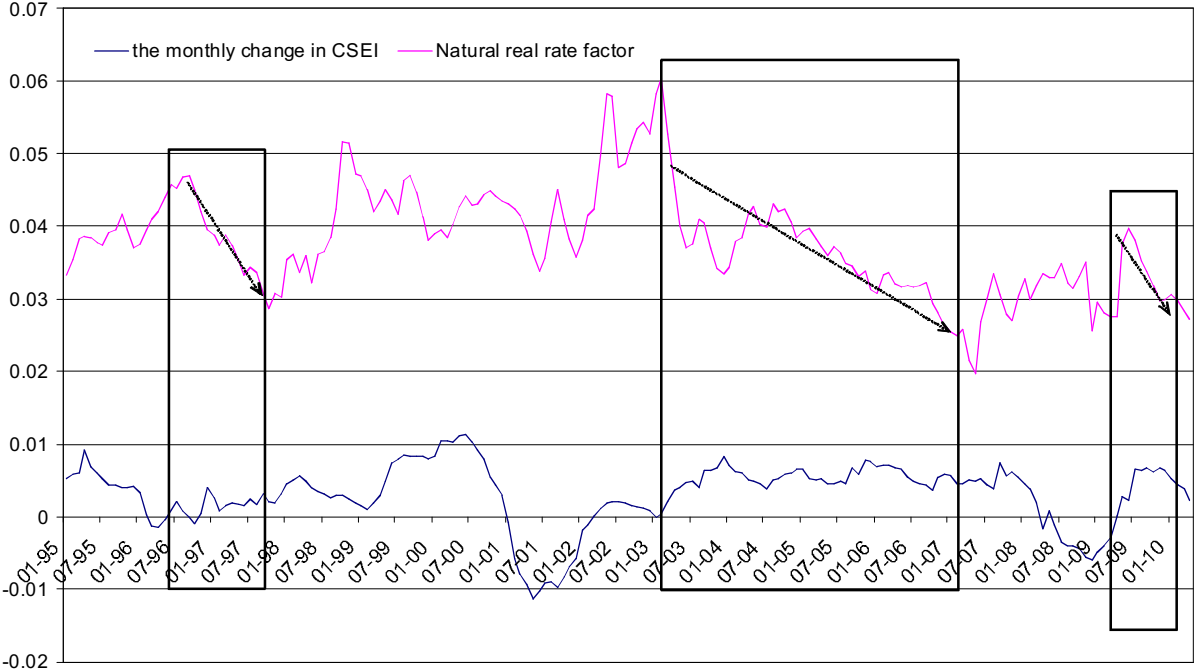
2. In the period from Apr-2003 to May 2007 GDP growth was relatively high and stable. The downturn in the natural real rate despite rapid growth was the result of a combination of several variables: the increase in government saving, which reduced the need to finance the government deficit; the upturn in private saving, which boosted demand for financial assets; globalization and capital inflows, which strengthened the relationship between the real domestic interest rate and the lower global rate; and last but not least the structural reforms.²² The structural reforms in those years—chiefly those relating to the pension industry, the tax system (which abolished most tax distortions), the liberalization of institutional investors' activities, the bond market—have transformed the domestic financial system and helped it advance toward greater competitiveness, efficiency, and stability. Apparently, the structural reforms together with the fundamentals mentioned above pushed down the natural real short rate.

3. In the period from March-2009 to the end of 2010 GDP growth was relatively high at the beginning of the period until the end of 2009, but shrank slowly to zero during 2010. From mid-2009 on, the economy began to recover gradually along with the developments abroad, thanks to good

²² See details in Subsections 4.2.2 and 4.2.4

fundamentals and an expansionary monetary policy. However, the forecasts of the global recovery in real activity after the world financial crises were moderate mostly due to restrained private consumption and the heavy public debt in some developed countries. In light of those forecasts and together with the first signs of the slowdown in growth, the natural real short rate decreased to a historically low level—1.5 percent.

Figure 10
The natural real rate factor and the Composite State of the Economy Index (CSEI)



In general, one can conclude as follow: there is no correlation between the business cycle and the natural real rate since the growth can stem from demand or supply shocks. However, at the starting point of the business cycle, when the first signs of growth appear, the natural real rate is inclined to increase dramatically and thereafter it decreases slowly with continuing growth. This is explained by the Real Business Cycle Model,²³ in which the expectations of renewed growth cause two effects: consumers increase their consumption and reduce the saving ratio since they expect more income in the near future; and firms increase their investment. These two effects increase the natural real rate. Nevertheless, at the end of the business cycle the natural real rate does not change dramatically. One possible explanation for that finding is that the recession usually comes as a surprise.

²³ According to Gali (2003), the natural real rate consists of two factors: the rate of time preferences and the expected change in capital productivity.

6. Conclusion

This paper estimates affine term structure models (ATSM) and links between macroeconomic variables and the state variables derived from the ATSMs. The macroeconomic variables that influence the real interest rate are identified from the outset. Then the state variables of the ATSM are restricted to serve as economic variables (macro-identified model). This process produces not only a description of the term structure but also a reasonable economic interpretation of how the term structure is determined and thus produces a projection of the future spot rates. The advantages of estimating the ATSM on indexed rather than unindexed (nominal) bonds is that the state variables can be readily identified as representing two different macroeconomic components, one reflecting the expected natural rate, and the other reflecting the temporary influence of monetary policy measures on the real rate. To do so, the extended model restricts the factors so that they behave in accordance with the macroeconomic components. One of the factors is restricted so that it reflects the influence of monetary policy on the real rate, while the other stays unrestricted as a plug factor (residual). I decided to restrict the factor reflecting the influence of the monetary stance on the real side, since this factor can be identified in the dynamic equation much more easily than can the natural real interest rate. In this step I begin with a version of the Taylor rule,²⁴ and convert this equation into real terms.

The expected natural rate which is derived in this study differs from the traditional macroeconomic models such as the loanable funds model, which estimate the natural rate directly from macroeconomic variables. The main difference concerns timing. While the rates derived from traditional macroeconomic models are based on fundamentals and therefore have backward-looking features, the rates derived from ATSMs are based on yields and therefore are forward looking. The model enables us to observe not only the expectation of the natural real rate but also the expectations of the monetary stance. Therefore, the estimating results, as derived from the proposed model, can use as an indicator for monetary policy decisions. It is useful for monetary policy purposes to decompose real yield into a natural rate, monetary stance, and their risk premiums.

²⁴ That is a monetary policy rule that prescribes how a central bank should adjust its interest rate policy instrument in a systematic manner in response to developments in inflation and macroeconomic activity, see Taylor (1993).

Appendix A. The natural interest rate and the loanable funds approach²⁵

The natural real interest rate determines, on the one hand, the desired flow of new capital which influences the aggregate supply of producers, and on the other, the desired ratio of saving to consumption, which influences the aggregate demand of households. The equilibrium point of these two forces reflects both the marginal cost of capital flow and its amount. Ultimately, the real interest rate is determined by that equilibrium point.

A.1 Demand for investment (I):

$$I_t = \alpha_0 + \alpha_1 \Delta Y_t + \alpha_2 \Delta L_t^e + \alpha_3 R_t^e + \alpha_4 SG_t .$$

A.2 Demand for saving—private sector (SP):

$$SP_t = \beta_0 + \beta_1 R_t^e + \beta_2 Y_t + \beta_3 SG_t .$$

A.3 Capital account (CA)—surplus:

$$IMC_t = \gamma_0 + \gamma_1 (R_t^e - RF_t) + \gamma_2 \Delta RP_t .$$

A.4 Identity equation: $I \equiv SP + SG + IMC$.

The demand for investment, according to the Accelerator Model, depends on output growth, the real interest rate, demographic change of the labor force, and government saving (because of its influence on the labor productivity). The effective labor force (L^e) is a function of (1) new immigrants, (2) their relative wage, (3) the assimilation process, (4) capital inventory, (5) education, (6) and average number of working hours per day.

The demand for saving in the private sector, according to the expended Keynesian saving model, depends on the real interest rate, output growth, and government saving (because of the Ricardian hypothesis).

The capital account depends on 1) the gap between the domestic and foreign interest rates, 2) exchange controls, and 3) the relative country-specific risk premium.

A.5 The natural real interest rate would prevail at the intersection of saving, international flow, and investment desired.

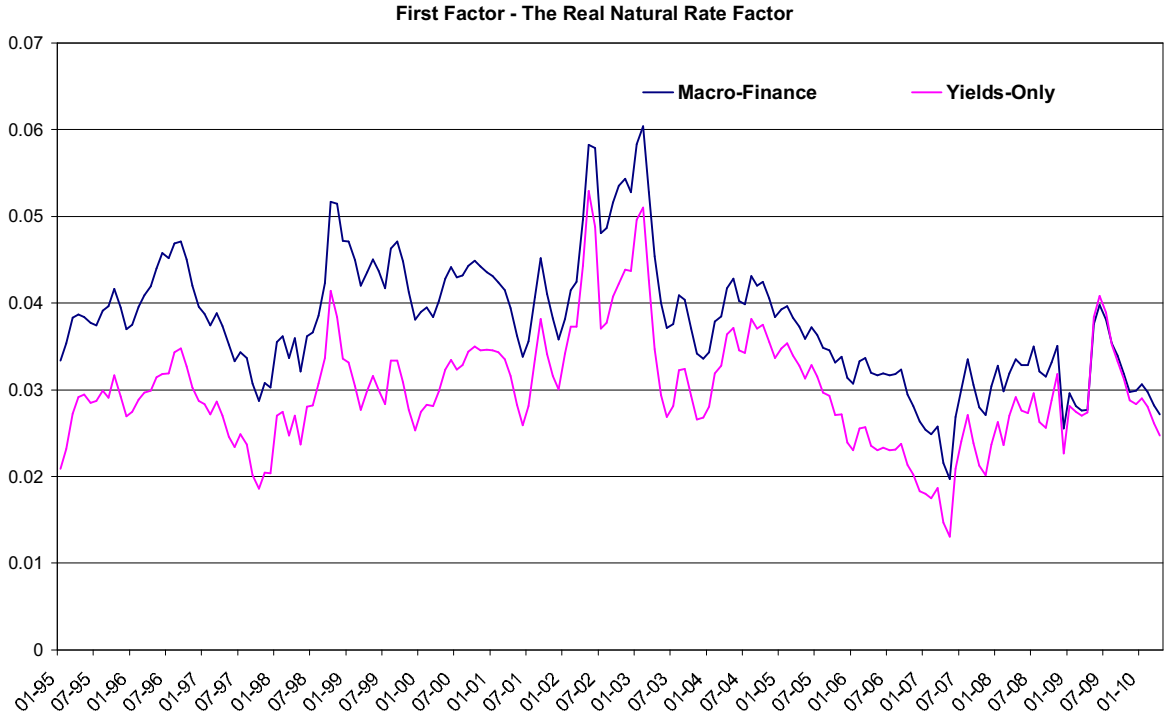
$$R_t^e = f[\Delta Y_t, \Delta L_t^e, RF_t, RP_t, SG_t \dots] .$$

Each one of these variables is affected by many fundamentals. Thus, the natural real interest rate is affected by important key developments over the sample period, e.g., the large influx of immigrants from the former USSR to Western countries in the 1990s, the liberalization in the forex market, the adoption of an inflation targeting approach, the boom in high-tech industries, the dramatic change in the rate of time preferences of the government and the private sector, and the capital and money market reforms.

²⁵ This appendix uses a simple model only to help our understanding of the fundamentals that affect the natural real interest rate.

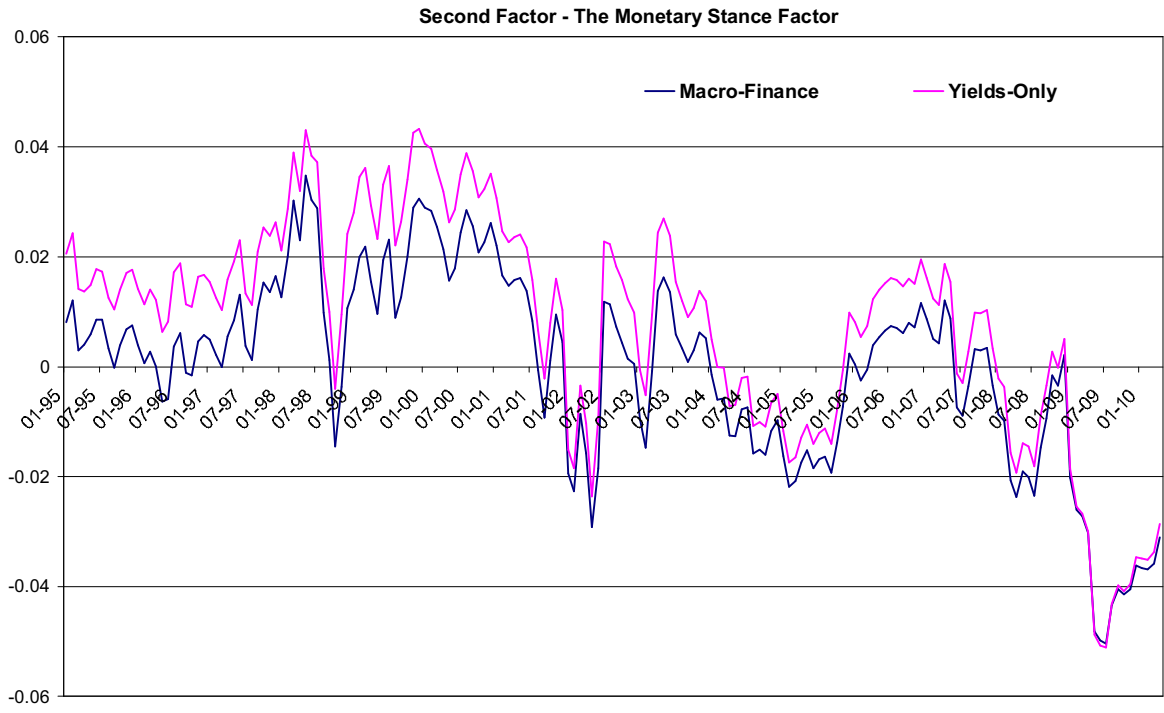
Appendix B

Figure B.1



Correlation = 0.88

Figure B.2



Correlation = 0.99

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