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SMALL OPEN ECONOMY NEW KEYNESIAN PHILLIPS CURVE: DERIVATION AND APPLICATION TO ISRAEL

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A New Keynesian Phillips Curve is derived for a small open economy with the characteristics of the Israeli economy: price indexation to inflation of the Consumer Price Index and to depreciation of the Shekel against the US$. Estimation is then carried out, in order to study the inflation dynamic in Israel under the inflation targeting regime from 1995 to 2006. The main results show that the openness of small economy in general, and the characteristics of the Israeli economy in particular, induce significant channels of influence from the exchange rate in the transmission mechanism of the monetary policy.

1. INTRODUCTION

a. Evolution of the New Keynesian approach

The New Keynesian (NK) approach extends the description of inflation's driving forces: in addition to real causes it includes the influence of future expected inflation, thus providing immunity to the Lucas critique to which the purely empirical Phillips Curve is exposed. The approach has therefore evolved as a theoretical framework, based on optimizing behavior of rational economic agents, that is to say, a micro-founded approach.

A cornerstone of the NK approach is an economic environment characterized by short-run nominal frictions. Such nominal frictions are at the heart of the conviction, well accepted in academic research and by central banks worldwide, that future expected inflation is one of the forcing variables of present inflation. Nominal frictions are also a necessary condition for an otherwise neutral monetary policy.

The description of nominal frictions has evolved gradually. In the first stage, a micro-founded new-classical Phillips Curve was introduced, based upon a subset of producers who predetermine prices one period in advance. Hence, this new-classical Phillips Curve introduced a linkage between real variables and the unexpected component of inflation: $\pi_t - E_{t-1}[\pi_t]$. In a second stage, the New Keynesian Phillips Curve (NKPC) was

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introduced, providing a richer description of the nominal frictions, thus inducing the dynamic forward-looking nature of inflation. The New-Classical one period lagged expectations, $E_{t-1}[\pi_t]$, were replaced by forward-looking expectations, $E_t[\pi_{t+1}]$, as a determinant of the inflation. That yielded the channel of expectation for future policy conduct and highlighted the role of Central Banks as managers of those expectations.

Recently, some extensions have been introduced to the otherwise purely forward-looking NK approach. Those extensions are motivated by the empirically observed inflation persistence worldwide, and they measure part of the residual inertia that the baseline pure forward-looking framework treats as unexplained.[1] These extensions yield the 'hybrid' structure, which consists of both backward- and forward-looking behavior. The literature suggests a wide range of sources for inflation persistence; some of them are discussed in the coming chapter. Apart from improving the fit to the data, inflation inertia treated by the hybrid version also enables the humped shape of inflation response to monetary policy shock, rather than the immediately declining response of the purely forward-looking models.

b. The open economy

Big open economies are usually treated as closed ones. Examples of such treatment include Smets and Wouters (2003), who estimate a *closed* economy model for the Euro area, and Rotemberg and Woodford (1998), who do the same for the United States.

The literature on the small open economy NKPC and its policy implications is diversified. Gali and Monacelli (2005) show that under certain assumptions, the small open economy NKPC is isomorphic to its closed economy counterpart. In contrast, Monacelli (2005) discusses influences of openness, assuming incomplete exchange rate pass-through to prices of imported goods. Razin and Yuen (2002) and Loungani and Razin (2005) show how open trade account and capital flow weaken the influence of domestic activity on inflation; that is, they show how openness flattens the NKPC. Engler (2007) and Razin and Binyamini (2007) extend this analysis, showing how an open labor market farther flattens the NKPC. The present work follows the same theoretical framework as the above-mentioned papers, and discusses a NKPC that captures characteristics of the Israeli economy.

c. Econometric identification

In the literature, the NKPC is derived and estimated, either in a limited information single-equation framework or as part of a complete Dynamic Stochastic General Equilibrium (DSGE) framework. Among other empirical issues, some attention has been paid to weak econometric identification of the NKPC inertia, both under the limited as well as under the full information framework. Jondeau and Bihan (2003) compare the properties of ML vis-à-vis GMM estimators of hybrid NKPC. Mavroeidis (2005) investigates problems that arose

[1] See Fuhrer and Moore (1995), Roberts (1997), Gali and Gertler (1999), Leith and Malley (2002) and Christiano et al. (2005), among others.

from a limited information NKPC framework. In the debate between Gali et al. (2005) and Linde (2005), the former favor the use of the single equation GMM method, while the latter prefers the Full Information ML approach. The advantages and disadvantages of both estimation methods are discussed in the empirical chapter of this paper, together with some Monte-Carlo results, which are related to the NKPC derived here.

d. The present work

The present work presents theoretical and empirical treatment. Theoretically, it presents the derivation of NKPC for a small open economy. Using micro-foundations it captures some characteristics of the Israeli economy: indexation to past CPI inflation, indexation to the contemporaneous exchange rate against the US$ and the role of the CPI (rather than domestic prices) as a general price deflator. The derived NKPC and its deviations from the case of closed economy will then be employed as a basis for a short discussion about policy implications. Empirically, the work studies the inflation dynamic in Israel, using the above-mentioned theoretical framework, while also referring to the issue of weak econometric identification.

The rest of the paper is organized as follows: Section 2 presents a step by step derivation of the small open economy NKPC. Section 3 presents empirical discussion and results and section 4 concludes.

2. THEORETICAL FRAMEWORK

a. Households

The representative household maximizes a standard Constant Relative Risk Aversion (CRRA) utility function from the expected paths of consumption and leisure:

$$(1) \qquad Max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - \int_0^1 v[h_t(j)] \cdot dj \right],$$

where E is the expectations operator; the parameter β is a constant time discount factor; the period utility function is additively separable in the consumption composite, C_t, and in labor. [2] There is an infinite continuum of differentiated goods (indexed by j), uniformly distributed over the unit interval, so that $j \in [0,1]$. Production of each differentiated good uses specialized labor inputs; thus, $h_t(j)$ denotes labor supply of type j. The composite index, C_t, is a sub-utility index with Constant Elasticity of Substitution (CES) between composites of imported and domestically produced goods:

[2] It is assumed that there is no utility from holding real cash balances. Woodford (2003) offered the interpretation of a bookkeeping cashless economy.

$$(2) \qquad C_t \equiv \left[\psi^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} + (1-\psi)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} ,$$

where the parameter ψ is inversely related to the home bias in consumption.[3] Subscripts F and H indicate foreign and home country variables, respectively. Thus, the variables $C_{F,t}$ and $C_{H,t}$ are utility indices from total consumption of foreign produced goods (the total import) and of domestically produced goods, respectively. As is the common practice in this literature, I simplify by assuming that all goods are tradable. The parameter $\eta > 0$ is the CES between composites of home and foreign goods.

The sub-utility indices $C_{F,t}$ and $C_{H,t}$ are further decomposed, using standard Dixit-Stiglitz utility function:

$$(3) \qquad C_{F,t} \equiv \left(\int_0^1 c_{F,t}(j)^{\frac{\theta-1}{\theta}} \cdot dj \right)^{\frac{\theta}{\theta-1}} \quad ; \quad C_{H,t} \equiv \left(\int_0^1 c_{H,t}(j)^{\frac{\theta-1}{\theta}} \cdot dj \right)^{\frac{\theta}{\theta-1}} ,$$

where the variable $c_{i,t}(j)$ denotes the consumption level of variety j produced by country $i = F, H$. The infinite continuum of goods enables monopolistic competition, which in turn enables sticky prices. The parameter $\theta > 1$ is the CES between any two goods produced in the same country of origin, assumed to be common for all countries.

The budget constraint is given by:

$$C_t + \frac{1}{(1+i_{H,t})P_t} B_{H,t} + \frac{S_t}{(1+i_{F,t})P_t} B_{F,t} =$$
$$\frac{1}{P_t} B_{H,t-1} + \frac{S_t}{P_t} B_{F,t-1} + \frac{1}{P_t} \left[\int_0^1 w_t(j) \cdot h_t(j) \cdot dj + \int_0^1 D_t(j) \cdot dj \right] ,$$

where the variable P_t is the CPI; the variable $B_{H,t}$ $(B_{F,t})$ is the amount of domestic (foreign) riskless bond holdings at the end of period t, denominated in the currency of the issuing country; the variable S_t is the import-based nominal-exchange rates basket; the variable $w_t(j)$ is the nominal wage paid for each labor unit of type j, $h_t(j)$; and the variable $D_t(j)$ is the profit paid as a dividend by the household-owned $j'th$ producer.

Optimal goods allocation

First-order conditions of the optimal solution to the household problem yield the standard result for optimal allocation between imported and locally produced goods:

[3] This is a common practice in the literature, in order to control for the steady state share of imported goods in total private consumption.

(4) $$C_{F,t} = \psi \left[\frac{P_{F,t}}{P_t} \right]^{-\eta} C_t \quad ; \quad C_{H,t} = (1-\psi) \left[\frac{P_{H,t}}{P_t} \right]^{-\eta} C_t \ ,$$

where $P_{F,t}$ and $P_{H,t}$ are price indices of imported and domestically produced goods, respectively, in both cases denominated in the currency of the home country. In the equation for the consumption of imported goods, complete pass-through is assumed: the local price of imported good is simply its price in the manufacturing country, multiplied by the nominal exchange rate. It is straightforward that in steady state with Purchasing Power Parity (PPP), ψ would be the import share in consumption.

Analogically to the derivation of the inter-country allocation, the allocation in equation (4) can be further decomposed into the optimal allocation between goods from the same country of origin:

(5) $$c_{F,t}(j) = \left[\frac{p_{F,t}(j)}{P_{F,t}} \right]^{-\theta} C_{F,t} \quad ; \quad c_{H,t}(j) = \left[\frac{p_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C_{H,t} \ ,$$

where $p_{i,t}(j)$ is the price of variety j produced by country $i = F, H$. Thus, the demand for variety j is a negative function of its real price.

I will assume that $\theta > \eta$. That is to say that the monopolistic power, while facing a competitor from the same country of origin, is smaller than the monopolistic power while facing a foreign competitor.

The price indices

The CPI, P_t, is defined as the cost of obtaining one unit of the sub-utility maximizing bundle:

$$P_t \equiv \frac{\int_0^1 p_{F,t}(j) \cdot c_{F,t}(j) \cdot dj + \int_0^1 p_{H,t}(j) \cdot c_{H,t}(j) \cdot dj}{C_t} \ .$$

Substituting equations (5) and (4) and rearranging, we get a standard open economy CPI:

(6) $$P_t = \left[\psi \cdot P_{F,t}^{(1-\eta)} + (1-\psi) \cdot P_{H,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \ .$$

The CPI is therefore a function of the CES between home and foreign goods, the home bias and the domestic and foreign price indices. The last one means that in the open economy, there are at least two additional sources of inflationary pressure: inflationary shocks abroad and shocks to the nominal exchange rate.

In a similar way, the price index of imported goods, $P_{F,t}$, and the domestic price index, $P_{H,t}$, can be derived. Those are functions, among other things, of the CES between any two goods produced in the same country of origin.

Optimal time allocation

Assuming a competitive labor market, we get the standard labor supply, according to which households equate the marginal utility of time to marginal (indirect) utility from labor:

$$(7) \quad v_h(h_t(j)) = u_c(C_t) \cdot \frac{w_t(j)}{P_t} \ .$$

b. Producers

Marginal cost

Domestic firms produce using the following technology:

$$(8) \quad y_{H,t}(j) = A_t f(h_t(j)) \ ,$$

where $y_{H,t}(j)$ is the total output level of the domestic $j'th$ producer and A_t is an exogenous technological shock, common to all producers.[4]

The marginal cost is the ratio between the unit cost of production input and its marginal productivity. Using this measure and considering the labor supply (7) and the production technology (8), we get the real marginal cost expressed in terms of the output level, the aggregate consumption and the technology shock:[5]

$$(9) \quad MC_t(j) = \frac{1}{A_t} \cdot \frac{v_h\left(f^{-1}(y_t(j)/A_t)\right)}{u_c(C_t)} \cdot \frac{1}{f'\left(f^{-1}(y_t(j)/A_t)\right)} \ .$$

Output demand

Assuming consumers in different open economies face the same goods space, it follows that the elasticity of substitution between goods is the same across different open economies.[6] Thus, preferences of the foreign representative consumer can be expressed

[4] It is a common simplification in the NK literature that the production technology excludes capital, assuming that the stochastic technology shock captures (among other things) shifts in the capital/labor ratio. Admittedly there is an empirical sacrifice, to the degree that this ratio is not exogenous.

[5] The expression $f^{-1}(\cdot)$ denotes the inverse function of $f\ (*)$.

[6] This is not to say that $\theta = \eta$. It does mean, however, that θ measures the substitutability between varieties produced in the local economy, in the view of both local and foreign consumers.

identically to those of the local consumer.[7] That means that the total demand for the goods produced by the domestic $j'th$ producer is given by:

$$(10) \quad y_{H,t}(j) = \left[\frac{p_{H,t}(j)}{P_{H,t}} \right]^{-\theta} \cdot Y_{H,t} \ ,$$

where the aggregate domestic output is the Dixit-Stiglitz composite of all domestically produced goods, both for the domestic market as well as for export:

$$Y_{H,t} \equiv \left\{ \int_0^1 \left[y_{H,t}(j) \right]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} .$$

Following Dixit and Stiglitz (1977), the single producer is assumed to be small enough and therefore treats the aggregates as exogenously given.

Optimal staggered pricing

The forward-looking pricing behavior is introduced, using the staggered contracts setup of Calvo (1983), which has became the leading approach in the NK literature. The producers find it too costly, and hence suboptimal, to continuously revise prices. Hence they optimally update prices only upon receiving an exogenous idiosyncratic signal, which shows up in stochastic time intervals. In every period there is a constant probability of $(1-\alpha) \in (0,1)$ of receiving such a signal. This probability is assumed to be independent of the time that has elapsed since the last update and of the current price level. Thus, by the law of large numbers, a share of $(1 - a)$ of all firms receives the signal every period. The implied expected time interval between any two subsequent price optimizations is therefore $1/(1 - a)$.

To capture the widely observed inflation persistence, some deviations from pure micro-foundations are sometimes introduced.[8] The present paper extends the Smets & Wouters (2003) mechanism for partial price indexation: whenever a producer does not get a signal to optimally revise prices, the price—in this case indicated $p_{H,t}^{ind}(j)$—is mechanically updated as follows:

$$(11) \quad p_{H,t}^{ind}(j) = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_\pi} \cdot \left(\frac{S_t^{\$}}{S_{t-1}^{\$}} \right)^{\gamma_\$} \cdot p_{H,t-1}(j) \ ,$$

[7] This assumption simplifies the analysis, by introducing unique solution for the problem of the local producer, both with regard to the local market and to export (no price discrimination).

[8] Clarida et al. (1999) allow for a serially correlated inflationary shock. Roberts (1997) allows for adaptive expectations by a subset of producers. Gali & Gertler (1999) and Leith and Malley (2002), among others, extend the baseline theory by allowing for a subset of producers who use a backward-looking rule of thumb (rather than optimization) when setting prices. Mash (2004) uses a time-dependent Calvo parameter.

where the parameter $\gamma_{\pi} \in [0,1]$ is the degree of price indexation to past inflation; the parameter $\gamma_{\$} \in [0,1]$ is the degree of price indexation to contemporaneous depreciation against the US$; and $S_t^{\$}$ is the nominal exchange rate against the US$. At the macro level, the parameters γ_{π} and $\gamma_{\$}$ can be interpreted as the share of producers who index their prices to past inflation and to contemporaneous depreciation, respectively, whenever not optimizing.

There are two principal differences between optimization and indexation. The first difference is that under optimization producers react to the real state of the economy, while under indexation they react to nominal changes only. The second difference is that under optimization producers are forward-looking, while under indexation they react only to past and present states of the economy.

Indexation can be justified both on normative and on positive bases. On a normative basis: Woodford (2003) claims that in the first place, prices are revised only occasionally not as a constraint, but as a characteristic of economic environment which seeks to save the costs associated with continuous market and prices review. But a simple enough interim rule of thumb can strike a balance between that desire to save such managerial costs and the need to respond to a dynamic economic environment. On a positive basis: such practice has existed for many years in Israel, since the period of hyperinflation. However, one may question the choice of nominal variables employed by the indexation rule. While there might be more than a single normative choice, the common practice in Israel includes one of two choices: indexation to past CPI inflation or indexation to contemporaneous depreciation against the US$. Indexation to the US$ is common in Israel, particularly in the real estate sector. A possible justification for such practice is that the nominal exchange rate is observed on a daily basis, while CPI inflation is measured with a delay, making indexation to present inflation impossible. Such consideration was relevant during the hyperinflation period. Though causing unnecessary exposure in times of low inflation, it is still very common.

In order to enable both types of indexation, on the macro level, the indexation rule (11) combines both, subject to the restriction that $\gamma_{\pi} + \gamma_{\$} \leq 1$.

Upon receiving the price update signal, the producer selects a price level, $p_{H,t}^{opt}(j)$, so as to maximize the discounted dividend flow:

$$(12) \quad \underset{p_{H,t}^{opt}(j)}{Max} \; E_t \sum_{s=0}^{\infty} \alpha^s \cdot Q_{t,t+s} \left[p_{H,t+s}(j) \cdot y_{H,t+s}(j) - w_{t+s}(j) \cdot h_{t+s}(j) \right].$$

Here, the nominal profit in every period is weighted by the probability that another optimization will not take place earlier, α^s, and is discounted by the corresponding nominal interest rate:

$$E_t Q_{t+s} \equiv E_t \prod_{i=0}^{s-1} \frac{1}{\left(1 + i_{H,t+i}\right)} \quad .$$

Substituting equations (10) and (11) into equation (12), eliminating $h_{t+s}(j)$ by substituting also the inverse of the production function (8), and finally, deriving the result with respect to $p_{H,t}^{opt}(j)$, we get a first-order condition from which it follows that upon receiving a price update signal, the local producer selects the optimal price, $p_t^{opt}(j)$, to satisfy:

(13)
$$E_t \sum_{s=0}^{\infty} \alpha^s Q_{t+s} y_{H,t+s}(j) p_{H,t}^{opt}(j) \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\gamma_\pi} \left(\frac{S_{t+s}^{\$}}{S_t^{\$}} \right)^{\gamma_\$} = \frac{\theta}{\theta - 1} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t+s} y_{H,t+s}(j) P_{t+s} MC_{t+s}(j) .$$

This suggests that the expected weighted average of future prices is optimally marked-up over the expected weighted average of future marginal costs. The optimal markup, $\theta/(\theta-1) > 1$, grows with the monopolistic power. Expected prices and marginal costs are weighted by the probability of not re-optimizing, α^s, by nominal interest rates captured by Q_{t+s}, and by the producer output level $E_t[y_{H,t+s}(j)]$. Only the limiting case of perfect flexibility $(\alpha \to 0)$ makes the future irrelevant. The unique output of that limiting case is the *normal* output—the output of the completely flexible prices. It follows that under nominal rigidity $(\alpha > 0)$, the average markup deviates from its optimal level.[9] In a small open economy, just as in a closed one, the *normal* output is independent of the monetary policy. Since in this particular case—of no nominal frictions—the nominal endogenous variables flexibly adjust themselves, so real variables immediately converge to their steady-state level.

The-first order condition (13) can be expressed in real terms by substituting:

$$E_t Q_{t+s} = \beta^s \frac{P_t}{E_t P_{t+s}} \cdot \frac{E_t u_c(C_{t+s})}{u_c(C_t)} .$$

This equality follows from recursive substitution of the standard inter-temporal Euler condition, that maximizes (1) with respect to the saving decision. In addition, assuming symmetry among producers the producer index (j) is dropped:

(13')
$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s u_c(C_{t+s}) y_{t+s} \frac{P_{H,t}^{opt}}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\gamma_\pi} \left(\frac{S_{t+s}^{\$}}{S_t^{\$}} \right)^{\gamma_\$} = \frac{\theta}{\theta - 1} E_t \sum_{s=0}^{\infty} (\alpha\beta)^s u_c(C_{t+s}) y_{t+s} MC_{t+s} .$$

[9] Note that as we approach the case of perfect competition, the optimal markup approaches zero, and the optimal price converges to the marginal cost. In such case, rigid prices may occasionally lead to a price which is lower than the marginal cost. This is why rigid prices are analyzed in a framework of monopolistic competition.

The interpretation is similar to that given to equation (13). However, there are differences in the periodic weight. It now includes the time discount factor, the marginal utilities from future consumptions and the future CPIs as the price deflator. With the new weights, the price-setting first-order condition can be interpreted as maximizing the contribution of the nominal dividends to the utility of the households (the owners of the firms). To see that, note the shadow price included in the periodic weight—the ratio of marginal utility from consumption to the CPI of the relevant period.

This micro-founded result is consistent with utilizing the CPI as a general price deflator, which is customary in Israel.[10]

Equation (13') augments equation (25) of Smets & Wouters (2003) to case of an open economy. The augmentation includes the effects of indexations (considering future expected indexations), to the US$ and to the CPI. This augmentation induces two additional transmission mechanisms to domestic inflation. The first is from inflation abroad, which is purely exogenous to the small open economy. The second is from the nominal exchange rate.

Hence, the optimal price evolves in a way that satisfies the condition in equation (13'), which accounts for the indexation mechanism described by equation (11). Together with the process of the prices of imported goods, they are integrated into the evolution of the CPI (6), which by the law of large numbers becomes:

$$(14) \qquad P_t^{1-\eta} = \psi \cdot P_{F,t}^{(1-\eta)} + (1-\psi) \cdot \left[(1-\alpha) \cdot P_{H,t}^{opt} + \alpha \cdot P_{H,t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_\pi} \left(\frac{S_t^{\$}}{S_{t-1}^{\$}} \right)^{\gamma_\$} \right]^{1-\eta} .$$

c. Log-linearized representation

The steady-state inflation is not necessarily zero. A credible inflation targeting regime, for instance, anchors the expectations of future inflation to the target. In such an environment, it is natural to refer to the target as steady-state inflation. Nevertheless, log linearization around a *zero* inflation steady state is a common simplification in the NK literature. Woodford (2003) claims that such approximation is satisfactory, as long as long-run inflation is closed enough to zero. The common practice of log linearization around zero inflation steady state is employed here.

Appendix A presents step by-step-log linearization of the first order condition (13') and of the CPI law of motion (14) around a zero inflation steady state with PPP, integration of the results and extraction of the domestic inflation. The result augments the Hybrid NKPC of Smets & Wouters (2003) and of Woodford (2003) for the case of small open economy with partial indexation to nominal depreciation:

[10] Until as late as 2004, financial statements in Israel were adjusted to inflation. As the price deflator for this adjustment, the financial accounting standard set the CPI inflation, or—under special circumstances only—the exchange rate changes.

$$(15)\qquad \hat{\pi}_{H,t}=\frac{1}{1+\beta\gamma_{\pi}(1-\psi)}*\begin{Bmatrix}\gamma_{\pi}\hat{\pi}_{t-1}+\gamma_{\$}\Delta\hat{S}_{t}^{\$}\\ \beta E_{t}\left(\hat{\pi}_{H,t+1}-\psi\gamma_{\pi}\hat{\pi}_{F,t}-\gamma_{\$}\Delta\hat{S}_{t+1}^{\$}\right)\\ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\hat{mc}_{t}+\psi\cdot\hat{\tau}_{t}\right)\end{Bmatrix}\ .$$

Here, hatted variables denote logarithmic deviations from the steady state and the variable $\hat{\tau}_t \equiv \hat{P}_{F,t} - \hat{P}_{H,t}$ is the logarithmic deviation of the terms of trade from their "normal" level, that is—their level under the hypothetic case of flexible prices.

The inflationary influence of the terms of trade is brought about by the price-setting first-order condition which employs the CPI as the nominal profits deflator.

Domestic inflation is therefore driven by three sets of forcing variables: The first line in the curly parentheses in equation (15) captures the influence of nominal indexations; the second line captures the forward-looking influence after compensation for future indexations; the third line captures the influence of the real state of the economy, which increases with price flexibility and with time impatience.

There are two sources of deviation from the closed economy case. The first is the employment of indexation mechanisms, which induces the influence of nominal depreciation and of imported goods inflation.[11] The second is the deflation of nominal profits by the CPI, which induces the influence of the terms of trade. Openness may also lead to changes in structural parameters, such as the degree of price rigidity. Such possible influences however are beyond the scope of the present work.

As to the marginal cost—similarly to Razin & Yuan (2002), the marginal cost (9) is log-linearized around a zero inflation steady state with normal output and consumption levels, and with technological shock normalized to one.[12] Integrating for all producers, we get the logarithmic deviation of the average marginal cost, and therefore drop the producer index, j. It follows that the marginal cost is negatively related to the productivity shock, and positively related to output and consumption, by substitution and income effects on labor supply:

$$(16)\qquad \hat{mc}_t=\frac{1}{1+\varpi\theta}\left[\varpi\cdot\hat{y}_t+\sigma\cdot\hat{c}_t-(1+\varpi)\hat{a}_t\right]\ .$$

[11] Note that inflation of imported goods is also hidden in the lagged CPI inflation.

[12] In order to express the producer marginal cost in terms of aggregate output level—rather than in terms of own output—a log linear approximation of the producer output, equation (10), is substituted. Thus, the marginal cost will include a term that captures the degree of strategic complementarity between producers, $1/(1+\varpi\theta)$. See section 1.4 in chapter 3 of Woodford (2003) for a detailed discussion about strategic complementarity.

Equation (16) expresses the logarithmic deviation of the real marginal cost from its steady-state value, that is—from the inverse of the optimal markup, $[\theta/(\theta-1)]^{-1}$. Hatted variables denote logarithmic deviations from the steady state and:[13]

$$\sigma \equiv -\frac{u_{cc}}{u_c}\bar{c} \quad ; \quad \varpi \equiv \omega_w + \omega_p \quad ; \quad \omega_w \equiv \frac{v_{hh}}{v_h \cdot f'}\bar{y} \text{ and } \omega_p \equiv -\frac{f''}{(f')^2}\bar{y} .$$

The marginal cost is influenced by input costs and their marginal productivity. The influence of labor cost: σ measures relative risk aversion and ω_w is the elasticity of the desired real wage with respect to the output level (through the required labor input).[14] The influence of marginal productivity of labor (MPL): ω_p is the elasticity of the desired price with respect to the output level (for a given level of real wage).[15] Assuming Constant Return to Scale (CRS) technology, the production function is homogenous of degree one in its single factor of production, labor. It follows that the production technology takes the form of a linear function, that is, $\omega_p = 0$.

When a closed economy is concerned, the marginal cost can be expressed in terms of the output gap exclusively. But with an open trade account and with capital flow, the correlation between domestic consumption and domestic output is no longer perfect. Hence, it is not longer possible to express the marginal cost in terms of the output gap alone; it has to be expressed in terms of the consumption gap as well.

The log linear approximation of the marginal cost, equation (16), is substituted in the NKPC, equation (15). We can thus obtain the Phillip Curve, expressed in terms of the output and consumption gaps:

$$(17) \qquad \hat{\pi}_{H,t} = \frac{1}{1+\beta\gamma_\pi(1-\psi)} * \left\{ \begin{array}{l} \gamma_\pi \hat{\pi}_{t-1} + \gamma_\$ \Delta\hat{S}_t^\$ \\ \beta E_t\left(\hat{\pi}_{H,t+1} - \psi\gamma_\pi\hat{\pi}_{F,t} - \gamma_\$\Delta\hat{S}_{t+1}^\$\right) \\ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\frac{\varpi}{1+\varpi\theta}\cdot\hat{y}_t + \frac{\sigma}{1+\varpi\theta}\cdot\hat{c}_t + \psi\cdot\hat{\tau}_t\right) \end{array} \right\} + \varepsilon_t \quad ,$$

[13] Being trending variables, consumption and output do not converge to any steady state. Therefore, functions with constant elasticities are employed: the utility function is of the CRRA type with CES, while the production function has constant elasticity with respect to its sole production function. Thus, the log linear approximation around the steady state does not require solving for the steady state itself.

[14] σ and ω_w measure the curvature of the households utility function with respect to the consumption and output gap, respectively.

[15] ω_p measures the curvature of the production function. It is the inverse of the elasticity of the labor marginal productivity with respect to the output level.

where the residual is a cost push shock, which is inversely related to the technology shock gap:

$$\varepsilon_t = -\frac{(1-\alpha)(1-\alpha\beta)}{[1+\beta\gamma_\pi(1-\psi)]}\cdot\frac{(1+\varpi)}{\alpha(1+\varpi\theta)}\cdot\hat{a}_t \quad .$$

It inherits serial correlation from the technology shock, to the extent that such serial correlation does exist.

It can be seen now that the small open economy NKPC introduces an additional challenge to the assessment of monetary policy, that is—the estimation of the unobservable natural path of the terms of trade. As will be clarified shortly, the role of this unobservable variable is much more important, quantitatively speaking, than the role of the output and consumption gaps.

3. ESTIMATION

a. Methodology

Estimation method

The forcing variables of the NKPC are not predetermined to inflation itself.[16] Ignoring the way they evolve over time may cause endogenity bias.

In the NKPC literature, there is a dispute regarding the appropriate estimation method. The common methodologies are ML and GMM. The ML, as a full information method, requires a complete specification for the endogenous forcing variables. Some efficiency can be sacrificed however, in favor of the limited information framework of GMM, which does not require specifying the complete data-generating process (DGP) for the forcing variables. It provides both computational simplicity as well as robustness to miss-specifications of the rest of the economy. Mavroeidis (2005) suggests that this is the reason for the popularity of GMM in the NKPC literature.

Another merit of the GMM comes from uncertainty regarding the distribution of inflationary shocks. To the extent that the latent technology shock is serially correlated, this serial correlation is inherited by the cost-push shock—the residuals of the NKPC. GMM addresses this issue, using the Heteroskedasticity and Autocorrelation Consistent (HAC) method, by incorporating the estimated auto-covariances of the orthogonality conditions into the estimation of the over-identification weighting matrix.

In contrast, there are two drawbacks for the GMM. The primary one involves the choice of Instrumental Variables (IV): while lags of the forcing variables make *relevant* IVs, serially correlated shocks may make them *invalid*. That is, lags of the forcing variables may

[16] That includes the inflation's lag, which may be correlated with the residuals, to the extent that the later is serially correlated.

be correlated with the contemporaneous shock by its serial correlation. A secondary drawback is the poor small-sample properties of the GMM.[17]

The NKPC, equation (17), was therefore estimated by both methods, which will be presented and compared.

The instrument set for the GMM estimation includes constant, contemporaneous values of the Federal Reserve interest rate, lags of the monetary interest rate and of the forcing variables—with the exception of the inflation's lags.[18]

Data

The estimation utilized 46 quarterly observations from time series of the Israeli economy for the period 1995:Q1–2006:Q2, taken from the Bank of Israel's series database.

Domestic inflation is derived from the GDP deflator.[19] Imported inflation is derived from the price index of imported consumer goods. The inflation expectations are market based. Output and consumption gaps are logarithmic deviations from trends of per capita business sector output and of per capita private consumption, respectively, using HP filter with 1,600 as the smoothness parameters.[20] The terms-of-trade gap is the logarithmic deviation from the HP trend of the ratio between the price index of imported consumption goods and the GDP deflator. The expected depreciation is market based.[21]

Inflation is expressed in terms of seasonally adjusted logarithmic deviations from the target. Although officially adopted in 1992, inflation targeting has actually been implemented in Israel only since 1995.[22] Hence, the period studied begins in that year.

All nominal variables are seasonally adjusted. Intercept is allowed, accounting for possibly asymmetric distribution of the inflationary shocks during the short period studied. Appendix B discusses related issues in greater detail.

[17] The estimated optimal weighting matrix consists essentially of estimators of fourth moments. Such estimators require long series so as to yield unbiased results.

[18] Again, since the residuals may be serially correlated, they may not be orthogonal to the lags of the forcing variables. Such a problem is particularly relevant when it comes to lags of the inflation, as inflation is the dependent variable itself. As a result, inflation's lag might be invalid IV for orthogonality condition.

[19] A shortcoming of this price index comes from the fact that it refers to the entire business sector output, not only to the output consumed by domestic households. An advantage of this index, however, is the fact that it is not directly influenced by the price of imported inputs, because it refers to the *added value* of local producers only.

[20] The value suggested by Hodrick and Prescott (1997) for quarterly data.

[21] Based on expectations implied from shekel-dollar financial derivatives, as explained by Hecht and Pompushko (2005).

[22] During the first years after inflation targeting was introduced, the actual monetary regime was one of a crawling peg exchange rate. The inflation targets for those years were set only to serve the decisions regarding the desirable crawling slope. During that time, the inflation target was determined mainly by the actual observed inflation, rather than by long-term policy guidelines. For a more detailed discussion of inflation targeting in Israel under the crawling peg regime, see Blejer et al. (2000, chap. 9).

Calibration

Some of the parameters in equation (17) were calibrated. For reasons discussed below, the time discount factor, β, and the import share, ψ, although identifiable, were calibrated. Table 1 presents the choices of the calibration values.

Rotemberg and Woodford (1998) as well as Smets and Wouters (2003) argue that although theoretically identifiable, the time discount factor, β, should not be treated as a free parameter under the constraints of the final structure, since it is directly observed by the first moment of the data. By the standard inter-temporal Euler condition, β is the inverse of the gross long-term quarterly real interest rate. It was therefore calibrated to 0.99.

The import share, ψ, was calibrated to 0.15, based on estimation of a log linearized version of equation (6), the CPI. Such value is also consistent with findings of other studies of the Israeli economy.[23]

Table 1
Calibrated values

Parameter	*Interpretation*	*Calibration*
σ	Relative risk aversion	1.00
ω_w	Elasticity of real wage with respect to output	2.50
ω_p	Elasticity of the MPL inverse with respect to output	0.00
β	Time discount factor	0.99
θ	CES in consumption.	10.00
ψ	Long term import share of private consumption	0.15

Following the discussion regarding the economic interpretation of the parameter ω_p, it was calibrated to zero.

The estimation results are very robust to a wide range of parameterization for σ and ω_w, which were therefore arbitrarily calibrated to 1.0 and 2.5, respectively.

Calibration of the CES in consumption, θ, is a different story. Firstly, simply because the estimation results are quite sensitive to the choice of calibrated value for θ; secondly, because it seems to be unstable over time – while treated as a free parameter, the point estimates for this parameter are sensitive to different sub-samples choices. The later the employed sub-sample, the higher is the CES, which basically varies from 9 to 24. This finding is consistent with the hypothesis that the Israeli economy becomes more competitive over time. The calibration value for the CES is 10, suggesting average markup of 11%.

[23] Leith and Malley (2002) treat this issue differently. They claim that using constant value for the entire sample period would imply too great (small) a weight on open economy effects, during periods where the actual import share is smaller (bigger) then the constant value in use. They account for this by using time varying import share, based on the actual data. For Israel, there is no data on the import share of the private consumption.

b. Results and robustness

Results

Table 2 presents the estimation results. The adjusted R^2 indicates that while taken to the data, the inflation dynamic derived in the theoretical part of this paper accounts for about 40% of the seasonally adjusted deviations of inflation from its target. This is a satisfactory result, considering that the maximization of R^2 was neither the sole, nor the most decisive, criterion of the GMM estimation. With the seasonality and the target incorporated, the explained share of inflation volatility is even larger.

Table 2
Estimates results

No.	*Method*	α *(t-stat)*	γ_π *(t-stat)*	$\gamma_\$$ *(t-stat)*	*D.W.*	*Adjusted* R^2	*J-stat [Prob]*
1.	ML	0.47 (3.2)	0.72 (1.4)	0.23 (1.1)	2.7	0.39	
2.	GMM	0.47 (5.3)	0.88 (3.3)	0.20 (1.7)	2.3	0.44	0.26 [0.70]

The estimation of α, the Calvo parameter, is 0.47, suggesting price revisions twice a year.

The estimation of the partial indexation parameter, γ_π, suggests that about 80% of the producers index prices to past inflation whenever not optimizing. The estimation of the partial indexation parameter, $\gamma_\$$, suggests that about 20% of the producers index prices to contemporaneous depreciation whenever not optimizing.[24]

Neither of the indexation parameters is stable over time and their relatively low t-statistics reflect this. The point estimation of both is lower, the later the sub-sample employed. The point estimate of the indexation to past inflation, γ_π, falls from 0.9 in the first half of the sample period to 0.6 in the second half. As will be shown later, this parameter suffers from weak econometric identification. Therefore its instability should be interpreted with caution. Yet, the above mentioned behavior of that estimator is consistent with a hypothesis that as the disinflation process in Israel took place, indexation decreased.

In the NK literature, the structural Calvo parameter, α, is estimated under various specifications and using data from different economies. Nevertheless, the point estimates are usually of similar magnitude to the one presented here, and are robust to a wide range of parameterization (for the other parameters). For Israel, Ribon (2004) estimated a value of approximately 0.5. Gali and Gertler (1999) used various specifications and methods to estimate values of around 0.80 for the US economy. Christiano et al. (2005) estimated a

[24] In the present issue of the Israeli Economic Review, Lavi and Sussman (2007) find that when the housing component is excluded, the nominal exchange rate does not have econometrically significant explanatory power in explaining the inflation of the CPI.

value of 0.6 for the US as well. Smets and Wouters (2003) estimated a value of 0.9 for the euro area, while admitting that their estimate is biased upward due to their theoretical structure.

As for inflation inertia, Smets and Wouters (2003) treated γ_π as a free parameter and estimated a value of 0.48. Leith and Malley (2002) estimated an open-economy NKPC for the G7, using a subset of producers who use a backward-looking rule of thumb, rather than optimization. They estimated Calvo parameter, ranging from 0.54 in some countries up to as high as 0.87 in others. But they also found that the higher this parameter (that is, the lower the optimization frequency), the larger is the subset of backward-looking producers. Since the backward-looking behavior is incorporated in different ways, there is no structural parameter to compare across different works. The implied reduced form can be compared, however. To some extent, the forward-looking element usually dominates the backward looking one.[25]

The reduced form of equation (17), while substituting the values $\alpha = 0.47$, $\gamma_\pi = 0.80$ and $\gamma_\$ = 0.20$, is:

$$(18)\qquad \hat{\pi}_{H,t} = \left\{ \begin{array}{l} 0.48 \cdot \hat{\pi}_{t-1} + 0.12 \cdot \Delta\hat{S}_t^\$ \\ \\ + 0.59 \cdot \hat{\pi}_{H,t+1} - 0.07 \cdot \hat{\pi}_{F,t} - 0.12 \cdot \Delta\hat{S}_{t+1}^\$ \\ \\ + 0.03 \cdot \hat{y}_t + 0.01 \cdot \hat{c}_t + 0.05 \cdot \hat{\tau}_t \end{array} \right\} + \varepsilon_t \quad .$$

Similarly to the case of a closed economy, the direct influences of the output and the consumption gaps are econometrically significant but quantitatively small.

The NKPC in its reduced form, equation (18), suggests that the primary monetary policy transmission mechanism in the Israeli economy works through the different channels of the nominal exchange rates. The transmission mechanism of the output and consumption gap, which is the only one in the closed economy, becomes secondary and negligible.

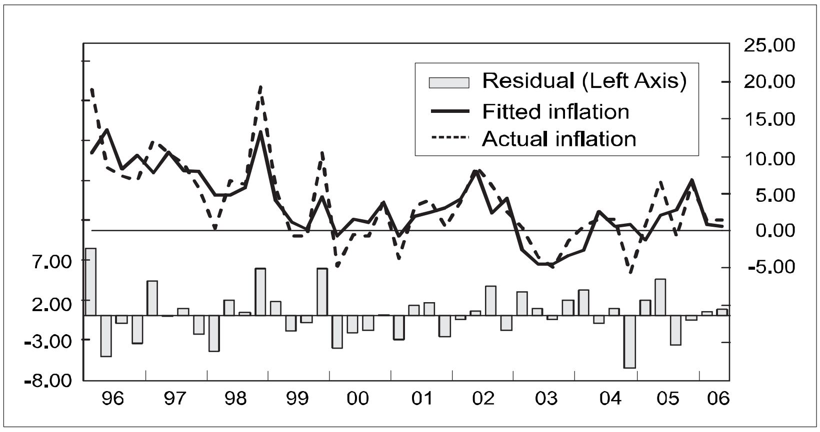
The quantitatively meaningful role of the terms of trade induces at least two new challenges for the monetary policy maker. The first challenge is the need to achieve and maintain credibility, due to the quick response of the nominal and the real exchange rate to weakening credibility. The second challenge is the need to estimate the terms-of-trade gap from its unobservable natural path; for this gap—not the level per se—is the inflationary factor that enters the NKPC. This state uncertainty, with respect to the terms of trade, is

[25] Gali & Gertler (1999) got weights of 0.59 and 0.38 respectively. Christiano et al. (2005), who assume a full indexation mechanism (analogously to calibrating $\gamma_\pi = 1$ in the present paper), implicitly impose approximately equal weights for both components. Studying the Israeli economy (using different NK structure) Elkayam and Argov (2006) and Ribon (2004) got approximately equal weights for the forward- and backward- looking components.

added to the state uncertainty with respect to the output gap that already exists in the closed economy.[26]

Equation (18) also demonstrates the meaningful effects of the indexation mechanisms to the exchange rate; considering both its coefficient and its volatility, this effect is even greater than that of the terms of trade. Figure 1 depicts in-sample one-step-ahead forecast.

Figure 1
Annualized quarterly inflation: actual versus fitted, 1995–2006



Stability of the estimators

Many of the papers dealing with hybrid NKPC estimation address the issue of the weak identification of the backward-looking effect. Jondeau and Bihan (2003) point out that the ML estimation of hybrid NKPC tends to give more weight to forward-looking element, in contrast to the GMM estimation which gives more weight to the backward-looking one. They use Monte-Carlo simulations to find that miss-specification, rather than finite sample biases, accounts for those biases in opposite directions. Mavroeidis (2005) shows similar findings and discusses the conditions required for identification, under a limited-information GMM framework.

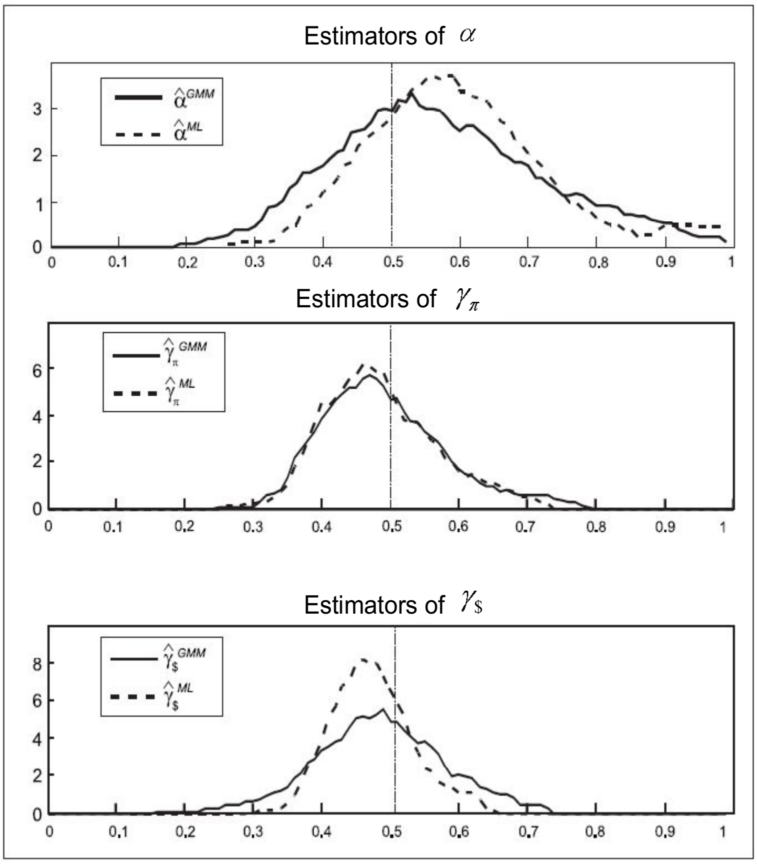
Confronting the identification issue of the NKPC derived here, Monte-Carlo simulations were carried out as follows. Time series for the variables involved were simulated. Inflation was simulated using the NKPC derived here. To avoid bias toward the ML estimation, the forcing variables were simulated using estimation results from a structural VAR, while inflation was subject to shocks drawn from a heteroskedastic and serially correlated distribution. The shocks, hitting the rest of the economy, were drawn from normal and standard distributions with the estimated standard deviations. The parameters α, γ_{π} and $\gamma_{\$}$ were calibrated to 1/2. At the next step, ML and GMM estimations took place in the

[26] Cukierman and Lippi (2005) discuss the meaningful effect of this type of uncertainty on monetary policy and its consequences.

same manner as was carried out with the actual data. One hundred repetitions of that procedure yielded two sets of estimators: one set of 100 ML estimators and another set of 100 GMM estimators for each one of the parameters of interest. Implementing conclusions from the debate between Gali et al. (2005) and Linde (2005), I used the estimated VAR as the DGP rather than theoretically driven equations, while both estimation methods were implemented under a single-equation framework. Using the estimated VAR as the DGP for the Monte-Carlo simulations on one hand, and the limited-information approach of single equation estimation on the other, avoids the bias towards the ML approach.

The distributions of the estimators, depicted in figure 2, indicate the undesired properties of both ML and GMM estimators. The GMM estimators for α and for $\gamma_{\$}$ are less biased than their ML estimators, but are characterized by higher variance. Both estimators for γ_{π} seem equally biased (downward).

Figure 2
Kernel Distribution of the Estimators
(The true Value is Represented by the vertical line)



Conclusions from the above Monte-Carlo study should be treated cautiously, as long as its sensitivity to different statistical environments has not been tested. It can be stated with some degree of skepticism however, that both estimation methods discussed suffer from undesired properties.

4. CONCLUDING REMARKS

The paper presents a derivation of a New Keynesian Phillips curve for small open economy, with emphasis on some characteristics of the Israeli economy. The derived Phillips curve was estimated using Israeli data, under the inflation targeting regime.

Openness influences inflation through more than one channel. The direct and trivial channel is the influence of prices of imported goods and services. This paper describes three indirect channels which characterize the Israeli economy: indexation to the nominal exchange rate, indexation to CPI inflation (which includes the prices of imported goods) and the employment of the CPI as a general price deflator.

These channels of influence induce a transmission mechanism for monetary policy in addition to that which already exists in the closed economy. This additional transmission mechanism enhances the role of the central bank as manager of expectations and therefore, it enhances the importance of the credibility. Furthermore, in a small open economy, the central bank faces the challenge of a state uncertainty with respect to the natural level of the terms of trade (the equilibrium level under the hypothetical case of complete price flexibility). Both the theoretical discussion and the empirical results stress the important role of the terms-of-trade gap among the determinants of inflation. Further, the empirical analysis suggests that the New Keynesian Phillips curve presented here provides satisfactory description of the dynamic of the domestic inflation in Israel, under the inflation targeting regime, from 1995 to 2006.

One of the merits of the New Keynesian approach is that, as it is based on micro-foundations, it enables us to analyze—qualitatively and quantitatively—the consequences of structural changes. Thus, for example, it supplies a friendly framework for analysis of the consequences of increased openness or of reduced use of the various indexation mechanisms (both to the CPI and to the exchange rate), due to the lower inflation environment that has characterized the Israeli economy in recent years.[27] Consideration of such structural changes is essential, for analyzing the inflation dynamic in general, and for monetary policy management in particular.

[27] Direct estimates of the Israeli Central Bureau of Statistics, for instance, suggest that there is a gradual decline in the share of US$ denominated contracts in the real-estate sector, due to the continuous appreciation of the Israeli Shekel with respect to the US$ during 2006 as well as in the first half of 2007.

Appendix

a. The NKPC

Rearranging the log linearization of equation (13') around a steady state with zero inflation and PPP, we get:

(A.1)
$$\frac{1}{1-\alpha\beta}\widehat{P}^{*}_{H,t}+E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\left[\gamma_\pi\left(\widehat{P}_{t+s-1}-\widehat{P}_{t-1}\right)+\gamma_\$\left(\widehat{S}^{\$}_{t+s}-\widehat{S}^{\$}_{t}\right)-\left(\widehat{P}_{t+s}-\widehat{P}_{t}\right)\right]= E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\,\widehat{mc}_{t+s},$$

where hatted variables denote logarithmic deviation from the steady state and $\widehat{P}^{*}_{H,t}\equiv\ln(P^{opt}_t)-\ln(P_t)$. Equation (A.1) can be rewritten as:

(A.2)
$$\frac{1}{1-\alpha\beta}\widehat{P}^{*}_{H,t}+E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\left[\sum_{i=1}^{s}\left[\gamma_\pi\widehat{\pi}_{t+i-1}+\gamma_\$\Delta\widehat{S}^{\$}_{t+i}-\widehat{\pi}_{t+i}\right]\right]=E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\,\widehat{mc}_{t+s}\quad .$$

The second element of the left hand side in equation (A.2) can be rearranged by writing it explicitly for every $s\in(0,\infty)$ and for every $i\in(0,s)$; then it can be collapsed down to:

(A.3)
$$\frac{1}{1-\alpha\beta}\widehat{P}^{*}_{H,t}+\frac{1}{1-\alpha\beta}E_t\sum_{s=1}^{\infty}\left[(\alpha\beta)^s\left(\gamma_\pi\widehat{\pi}_{t+s-1}+\gamma_\$\Delta\widehat{S}^{\$}_{t+s}-\widehat{\pi}_{t+s}\right)\right]=E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\,\widehat{mc}_{t+s}\,.$$

This, after extracting the variable $\widehat{P}^{*}_{H,t}$, becomes:

(A.4)
$$\widehat{P}^{*}_{H,t}=E_t\sum_{s=1}^{\infty}\left[(\alpha\beta)^s\left(\widehat{\pi}_{t+s}-\gamma_\pi\widehat{\pi}_{t+s-1}-\gamma_\$\Delta\widehat{S}^{\$}_{t+s}\right)\right]+(1-\alpha\beta)E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\,\widehat{mc}_{t+s}\quad .$$

Separating the first period from the other periods, it can be rewritten as:

(A.5)
$$\begin{aligned}\widehat{P}^{*}_{H,t}&=E_t\left\{\alpha\beta\left(\widehat{\pi}_{t+1}-\gamma_\pi\widehat{\pi}_{t}-\gamma_\$\Delta\widehat{S}^{\$}_{t+1}\right)+(1-\alpha\beta)\widehat{mc}_t\right\}\\&+E_t\left\{\sum_{s=2}^{\infty}\left[(\alpha\beta)^s\left(\widehat{\pi}_{t+s}-\gamma_\pi\widehat{\pi}_{t+s-1}-\gamma_\$\Delta\widehat{S}^{\$}_{t+s}\right)\right]+(1-\alpha\beta)\sum_{s=1}^{\infty}(\alpha\beta)^s\left[\widehat{mc}_{t+s}\right]\right\}\end{aligned}.$$

By rational expectations, equation (A.5) can be written as:

(A.6)
$$\widehat{P}^{*}_{H,t}=\alpha\beta\left(E_t\widehat{\pi}_{t+1}-\gamma_\pi\widehat{\pi}_t-\gamma_\$E_t\Delta\widehat{S}^{\$}_{t+1}\right)+(1-\alpha\beta)\widehat{mc}_t+\alpha\beta\cdot E_t\widehat{P}^{*}_{H,t+1}\quad .$$

Now, after log linearization of the CPI equation (6) and rearrangement, we get: $\hat{P}_{H,t} = \frac{1}{1-\psi}\hat{P}_t - \frac{\psi}{1-\psi}\hat{P}_{F,t}$. Adding to the right hand side the expression $\left[-\frac{\psi}{1-\psi}\hat{P}_t + \frac{\psi}{1-\psi}\hat{P}_t = 0\right]$, rearranging and taking one period backward we get:

(A.7) $$\hat{P}_{H,t-1} = \hat{P}_{t-1} - \frac{\psi}{1-\psi}\hat{q}_{t-1} ,$$

where the variable $\hat{q}_t$ denotes the logarithmic deviation of the real exchange rate from its steady-state level, formally defined as: $\hat{q}_t \equiv \hat{P}_{F,t} - \hat{P}_t$; both price indices are denominated in local currency. It is straightforward that the (PPP) steady-state level is $\hat{q}_{s.s.} = 0$.

Substitute equation (A.7) into the log linearization of equation (14) around steady state with zero inflation and PPP. After subtraction of $\hat{P}_t$ from both sides and rearrangement, we get:

(A.8) $$\hat{P}^*_{H,t} = \frac{\alpha}{1-\alpha}\left(\hat{\pi}_t - \gamma_\pi\hat{\pi}_{t-1} - \gamma_\$\Delta\hat{S}^\$_t\right) - \frac{\psi}{(1-\psi)(1-\alpha)}\left[\hat{q}_t - \alpha\hat{q}_{t-1}\right] .$$

The last equation of annex B.4 of Woodford (2003) is a private case of equation (A.8), where $\gamma_\pi = \gamma_\$ = \psi = 0$ (closed economy without indexation).

Now substitute equation (A.8) into equation (A.6) to eliminate both $\hat{P}^*_{H,t}$ and $E_t\hat{P}^*_{H,t+1}$. After rearrangement we get the NKPC in terms of CPI inflation:

(A.9) $$(1+\beta\gamma_\pi)\hat{\pi}_t = \begin{Bmatrix} \gamma_\pi \cdot \hat{\pi}_{t-1} + \gamma_\$ \cdot \Delta\hat{S}^\$_t + \frac{\psi}{(1-\psi)}\cdot\Delta\hat{q}_t \\ + \beta\left(E_t\hat{\pi}_{t+1} - \gamma_\$ E_t\Delta\hat{S}^\$_{t+1} - \frac{\psi}{(1-\psi)}\cdot E_t\Delta\hat{q}_{t+1}\right) \\ + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\cdot\left[\hat{mc}_t + \frac{\psi}{1-\psi}\hat{q}_t\right] \end{Bmatrix} .$$

The first difference of the log linearized CPI, equation (6), yields:

(A.10) $$\hat{\pi}_t = (1-\psi)\hat{\pi}_{H,t} + \psi\Delta\hat{\pi}_{F,t}$$

Subtracting $\hat{\pi}_t$ from both sides and rearranging we get:

(A.11) $$\Delta\hat{q}_t = (1-\psi)(\hat{\pi}_{F,t} - \hat{\pi}_{H,t}) \ .$$

Now, substitute equations (A.10) and (A.11) into equation (A.9). After rearrangement we get the NKPC in terms of domestic inflation:

(A.12) $$\hat{\pi}_{H,t} = \frac{1}{1+\beta\gamma_\pi(1-\psi)} * \left\{ \begin{array}{l} \gamma_\pi\hat{\pi}_{t-1} + \gamma_\$\Delta\hat{S}_t^\$ \\ \beta E_t\left(\hat{\pi}_{H,t+1} - \psi\gamma_\pi\hat{\pi}_{F,t} - \gamma_\$\Delta\hat{S}_{t+1}^\$\right) \\ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\hat{mc}_t + \psi\cdot\hat{\tau}_t\right) \end{array} \right\} ,$$

where the variable $\hat{\tau}_t \equiv \hat{P}_{F,t} - \hat{P}_{H,t}$ is the terms of trade gap. Equation (A.12) is actually equation (15) from the text.

b. The data

Inflation expectations

Expectations of future inflation are taken with respect to CPI inflation one year ahead, derived from the difference between market yields of nominal and real Treasury Bills. Three compromises are involved in this choice:

The first compromise concerns the price index to which the expectations refer. The relevant price index is the GDP deflator, but since the forecast of domestic inflation is not available, I employ market-based expectations with respect to CPI inflation. I assume however, that for the one-year horizon, this makes a satisfactory proxy to the inflation of the GDP deflator. Admittedly, the CPI contains imported prices as well, but for long enough horizons it is reasonable to assume that expectations of CPI inflation and of domestic inflation converge.

The second compromise concerns the expectations horizon. Equation (17) contains the expectation for inflation in the following quarter, while the market-based expectations refer to inflation for the next four quarters. However, although data of inflation expectations for two months ahead is available, these short-term expectation series are derived by a limited group of forecasters. In contrast, the longer term expectations are market based and therefore better reflect the opinion of the broad market participants. Note that estimation based on the shorter expectation produced similar results, although less efficiently (less significant estimators).

The third compromise concerns the "rationality" of the expectations. Although market-based expectations might be rational broadly speaking, they are not necessarily "model consistent" with the model employed here.

Output and consumption gaps

Output and consumption gaps are logarithmic deviations from trend, extracted by an HP filter. Admittedly, there is no theoretical ground for using a smoothed line as proxy to the unobservable levels of normal output and consumption. For those normal levels are driven by various real shocks and therefore should not be represented by smoothed lines. In a framework of complete DSGE models, it is possible to extract an estimation for the unobserved normal levels, using multivariate methods such as the Kalman Filter.[28] In the single-equation framework employed here, it is common to use a univariate method, with the most popular one being the HP filter.

Terms-of-trade gap

In this work, the terms of trade are defined as the ratio between the price index of imported goods and the GDP deflator. Assuming that the PPP principle is satisfied in the long run, the terms-of-trade gap should be deviations from zero. In contrast, it can be assumed that deviations of the terms of trade from PPP are not only caused by nominal frictions, but from real factors as well—factors such as productivity differences between economies. Thus, logarithmic deviations from HP trend are taken here as well. Yet again, similarly to the case for the output and consumption gaps, this method yields estimation errors, since here also the long-term trend is not necessarily a smoothed line.

[28] See Bjornland et al. (2006) for comprehensive discussion of multivariate versus univariate methods, for estimating unobserved variables. Estimation of output gap in Israel, under variety of methods, are presented by Fridman and Suchoy (2005), Elkayam et al. (2002) and Menashe and Yachin (2004). In another work on Israel, Lavi and Sussman (1999) treat the state uncertainty attributed to the unobserved natural variables (the NAIRU in their work), by estimating difference equation of the Phillips curve. In so doing, they minimize estimation errors, assuming that first order differences of the NAIRU are neglected, relatively to first order differences of the actual rate of unemployment.

REFERENCES

Bjornland, C. Hilde, Leif Brubakk and Sofie J. Anne (2006). *Forecasting inflation with an uncertain output gap*, Norges Bank Working Paper 2006/02.

Blejer, Mario, Ize Alain, Leone M. Alfredo and Werlang Sergio (2000). *Inflation Targeting in Practice: Strategic and Operational Issues and Application to Emerging Market Economies*, International Monetary Fund.

Calvo, Guillermo (1983). "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics 12,* 383–398.

Christiano, J. Lawrence, Martin Eichenbaum and Charles L. Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy 113,* No.1, 1–45.

Clarida, Richard, Jordi Gali and Mark Gertler (1999). "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature XXXVII,* 1661–1707.

Cukierman, Alex and Francesco Lippi (2005). "Endogenous Monetary Policy with Unobserved Potential Output", *Journal of Economic Dynamic and Control 29,* 1951–1983.

Dixit, Avinash and Joseph Stiglitz (1977). "Monopolistic Competition and Optimum Product Diversity", *The American Economic Review 67,* No.3, 297–308.

Elkayam, David and Eyal Argov (2006). *An Estimated New-Keynesian Model for a Small Open Economy: an Application for Israel*, Bank of Israel, Monetary Discussion Paper 2006.02. (Hebrew).

Elkayam, David, Yaakov Elashvily and Mimi Regev (2002). *Estimation of the Output Gap and Examination of its Influence on the Israeli Inflation in Recent Years*, Bank of Israel, Monetary Discussion Paper 2002.01. (Hebrew).

Engler, Philipp. (2007). *Gains from Migration in a New-Keynesian Framework*, mimeo, Freie Universität Berlin, Germany.

Fridman, Amit and Tanya Suchoy (2005). "The NAIRU in Israel: An Unobserved Componenets Approach", *Israel Economic Review 2,* No. 2, 125–154.

Fuhrer, Jeffrey and George Moore (1995). "Inflation Persistence", *The Quarterly Journal of Economics110,* No.1, 127–159.

Gali, Jordi and Mark Gertler (1999). "Inflation Dynamic: A Structural Econometric Analysis", *Journal of Monetary Economics 44,* 195–222.

Gali, Jordi, Mark Gertler and David Lopez-Salido (2005). "Robustness of the Fstimates of the Hybrid New Keynesian Phillips Curve", *Journal of Monetary Economics 52,* 1107–1118.

Gali, Jordi and Tommaso Monacelli (2005). "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies 72,* 707–734.

Hecht, Yoel and Helena Pompushko (2005). *Normality, Model Risk Level, and Exchange-Rate Jumps*, Bank of Israel, Monetary Discussion Paper 2005.01.

Hodrick, J. Robert and Edward C. Prescott (1997). "Postwar U.S. Business Cycles: An Empirical Investigation", *Journal of Money, Credit and Banking 29,* No.1. 1–16.

Jondeau, Eric and Herve Le Bihan (2003). *ML vs. GMM Estimates of Hybrid Macroeconomic Models (With an Application to the "New Phillips Curve")*, Banque de France Working Paper 103.

Lavi, Yaacov and Nathan Sussman (1999). *Phillips Curve and Shifts in its Ties 1965 till 1996*, in L. Leiderman (Eds.) *Inflation and Disinflation in Israel* (pp. 315–346) Bank of Israel.(Hebrew).

Lavi, Yaacov and Nathan Sussman (2007). "The Phillips Curve in Israel", *Israel Economic Review 5,* No. 1, 1–18.

Leith, Campbell and Jim Malley (2002). *Estimated Open Economy New Keynesian Phillips Curve for the G7*, Department of Economics, University of Glasgow Working Paper.

Linde, Jesper (2005). "Estimating New-Keynesian Phillips Curve: A Full Information Maximum Likelihood Approach", *Journal of Monetary Economics 52,* 1135–1149.

Loungani, Prakash and Assaf Razin (2005). *Globalization and Inflation-Output Tradeoffs*, NBER Working Paper 11641.

Mash, Richard (2004). *Optimising Microfoundations for Inflation Persistence*, Economic Series Working Papers 183, University of Oxford.

Mavroeidis, Sophocles (2005). "Identification Issues in Forward-Looking Models Estimated by GMM, with an Application to the Phillips Curve", *Journal of Money, Credit and Banking 37,* No.3, 421–448.

Menashe, Yigal and Yossi Yakhin (2004). "Mind the Gap: Structural and Nonstructural Approaches to Estimating Israel's Outpu Gap", *Israel Economic Review 2,* No.2, 79–106.

Monacelli, Tommaso (2005). "Monetary Policy in a Low Pass-Through Environment," *Journal of Money, Credit and Banking 37,* No.6, 1047–1066.

Razin, Assaf and Alon Binyamini (2007). *Flattened Inflation-Output Tradeoff and Enhanced Anti-Inflation Policy: Outcome of Globalization?*, NBER Working Paper 13280.

Razin, Assaf and Chi-Wa Yuen (2002). "The 'New Keynesian' Phillips Curve: Closed Economy Versus Open Economy", *Economics Letters 75,* 1–9.

Ribon, Sigal (2004). *A New Philips Curve for Israel*, Bank of Israel, Research Department Discussion Paper 2004.11.

Roberts, J.M. (1997). "Is Inflation Sticky?" *Journal of Monetary Economics 39,* 173–196.

Rotemberg, Julio and Michael Woodford (1998). *An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version*, NBER Technical Working Paper 233.

Smets, Frank and Raf Wouters (2003). "An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area", *Journal of the European Economic Association 1,* 1123–1175.

Woodford, Michael (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy,* Princeton: Princeton University Press.