

## FISCAL POLICY DYNAMICS WITH A PUBLIC-DEBT GUIDELINE

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This paper analyzes the implications of a cost of deviating upwards from a public debt/output guideline, such as the 0.6 ratio in the Maastricht Treaty, in the context of a fiscal policymaking model. Given a preannounced timetable for enforcement, the dynamic paths of the tax rate and government spending, which depart from smoothing over time, and the public debt are characterized. The model's predictions are that the tax rate is high and government spending is low prior to the date of starting enforcement, and that at this date the tax rate begins to decline and government spending begins to increase. The model is used to interpret fiscal evidence during the 1990s from three countries with large public debt: Belgium, Italy and Israel.

### 1. INTRODUCTION

This paper focuses on the implications of a public-debt/output guideline — such as the 0.6 ratio in the Maastricht Treaty — for the dynamics of tax rates, government spending and deficits. The model centers on a policymaker who faces tax-smoothing considerations, benefits from providing services to the public, and a cost of deviating upwards from a debt/output guideline. This specification is consistent with existing empirical results, as in Kremers (1989) who found mean reversion of the public debt in the US. Recently, the Maastricht Treaty drew attention to the existence of guidelines as the force driving reversion of the public-debt/output ratio — albeit asymmetrically, only when the ratio is high. In the Maastricht Treaty, the draft of which was signed in 1992, a specific timetable was set, including fines on member countries not complying with the public-debt guideline of 60 percent of GDP. The date stipulated for starting the enforcement of this guideline was 1 January 1999.

The present model incorporates a stylized version of such a guideline into a policymaking problem, where the severity associated with this guideline depends on two factors: the costs of deviating from it, and the closeness of the enforcement date. Both factors are known at the time the guideline is adopted. The model predicts that the tax rate remains high and government spending (as a share of output) remains low, both at constant levels, from the adoption date till the enforcement date. Then the tax rate starts to decline and government spending starts to increase. Over time, the tax rate converges to a level that is lower than the one prevailing

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before the guideline was adopted, while government spending converges to a higher level than previously.

The implications of the model are used to interpret evidence from three countries with public-debt/output ratios higher than 1 during the 1990s: Belgium, Italy and Israel. Belgium and Italy face the formal Maastricht guideline of 60 percent of GDP. Israel has no formal guideline, although policymakers often declare the goal of reducing the public-debt/output ratio.<sup>1</sup>

The paper is organized as follows. Section 2 presents the model. The main text is restricted to exogenous output. An appendix addresses the case where output depends on the tax rate and shows that the main results remain unchanged. Section 3 evaluates the fiscal dynamics in Belgium, Italy and Israel in light of the model, and Section 4 concludes.

## 2. THE MODEL

The policymaker solves a dynamic fiscal problem involving taxation, public debt and government spending paths, facing a public-debt/output guideline. Deviating upwards from this guideline after some future date, known in advance, involves a cost. The case of exogenous spending is presented first, in order to focus on the deviation from tax-smoothing implied by the guideline, and then the framework is extended to endogenous spending.

### a. The basic setup: exogenous government spending

The framework is an extension of Barro (1979). The policymaker faces exogenous flows of spending and determines the dynamic paths of the tax rate and the public debt, in the presence of deadweight losses from taxation and a public-debt guideline.

The deadweight loss from taxation, associated with tax rate  $\tau_t$  and output  $Y_t$ , is denoted by  $z(\tau_t)Y_t$  with  $z' > 0$ ,  $z'' > 0$ . The functional form adopted is  $z(\tau_t) = \frac{1}{2}(\tau_t)^2$ . In addition, there is a cost associated with a high ratio of public debt to output,  $b_t \equiv B_t/Y_t$ , where  $B_t$  is the outstanding public debt at the end of period  $t$ . This cost applies, starting from some future date  $\bar{t}$  (the time of guideline enforcement), if  $b_t$  is higher than  $\bar{b}$ . If  $b_t \leq \bar{b}$ , there is no loss or benefit. This cost is then specified as

$$w(b_t)Y_t I_t, \quad w' > 0, \quad w'' > 0,$$

where  $I_t$  is the indicator function;

$$I_t = 1, \text{ if } b_t > \bar{b} \text{ and } t \geq \bar{t}$$

and

$$I_t = 0, \text{ otherwise.}$$

<sup>1</sup> The official budget documents mention the 60 percent guideline in the Maastricht Treaty as important for Israel as well.

The inclusion of this cost in the policymaker's objective function may reflect either an explicit rule, deviation from which involves a fine as in the Maastricht Treaty, or an implicit guideline, accompanied by a reputation loss. The functional form adopted is

$$w(b_t) = \frac{\lambda}{2} (b_t - \bar{b})^2, \lambda > 0.$$

A deterministic setup seems appropriate to deal with the preannounced nature of the guideline. The objective function of the policymaker, acting in a small open-economy environment, is

$$(1) \quad \text{Min} \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} \frac{Y_t}{2} \left[ (\tau_t)^2 + \lambda (b_t - \bar{b})^2 I_t \right],$$

where  $r$  is the real interest rate and period 1 is the date of guideline adoption.

The marginal deadweight loss of taxation equals  $\tau_t Y_t$ , while the marginal reputation loss from public debt equals  $\lambda (b_t - \bar{b}) Y_t$  if  $b_t > \bar{b}$  and  $t \geq \bar{t}$ . Hence, for  $t \geq \bar{t}$ , the parameter  $\lambda$  determines the loss associated with the public debt, relative to the deadweight loss from taxation.

The periodical budget constraints are

$$(2) \quad \frac{1+r}{1+\mu} b_{t-1} + g_t - b_t - \tau_t = 0 \quad t = 1, 2, \dots, \infty,$$

where  $\mu$  is the growth rate of output and  $g_t$  is real expenditure (net of interest payments) as a fraction of output. The intertemporal budget constraint is

$$(3) \quad \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} Y_t (g_t - \tau_t) + (1+r) Y_0 b_0 = 0.$$

In the present case, the spending/output ratio is exogenous at the constant level  $g$ . The starting public-debt/output ratio,  $b_0$ , is predetermined.<sup>2</sup>

The first-order conditions are

$$(4) \quad \tau_t - \eta_t = 0,$$

$$(5) \quad \lambda (b_t - \bar{b}) I_t - \eta_t + \eta_{t+1} = 0, \quad t = 1, 2, \dots, \infty,$$

where  $\eta_t$  is the Lagrangian multiplier associated with (2).

Equations (2), (3), (4) and (5) characterize the solution. Let us consider two possible cases for the initial debt: (a)  $b_0 \leq \bar{b}$  and (b)  $b_0 > \bar{b}$ . The case  $b_0 \leq \bar{b}$  yields the standard tax-smoothing solution for the entire planning horizon, where the smoothened tax rate is set at the level which satisfies the intertemporal budget constraint. Then, the debt/output ratio remains at the initial value  $b_0$ , implying that  $I_t = 0$  for all  $t$ .

The interesting case in the present context arises when  $b_0 > \bar{b}$ . Prior to  $\bar{t}$ , given that  $I_t = 0$ , it follows from (5) that  $\eta_t = \eta_{t+1}$ , implying  $\tau_t = \tau_{t+1}$  from (4). That is, the tax rate is smoothened prior to  $\bar{t}$ .

The pattern of the tax rate from  $\bar{t}$  onwards can be inferred from (5) given the sign of  $(b_t - \bar{b})$ . Hence, the question here is whether  $b$  reaches the value  $\bar{b}$  at  $\bar{t}$ . The answer is no;  $b_t > \bar{b}$  holds. The reason is the following. If  $\tau$  is set high enough prior to  $t$  so as to get  $b_t \leq \bar{b}$ , then

$I_t = 0$ , implying from (5) that  $\eta_t = \eta_{t-1}$ , and thus  $\tau_t = \tau_{t-1}$ . Iterating this reasoning forward implies that  $\tau$  stays constant forever at a rate high enough to reduce the debt/output ratio. Hence,  $b$  continues to decline without bound. This is inconsistent with the intertemporal budget constraint: surpluses which reduce  $b$  are never reversed. The conclusion is, therefore, that the optimal debt/output ratio at  $\bar{t}$  should satisfy  $b_{\bar{t}} > \bar{b}$ , which triggers  $I_{\bar{t}} = 1$ . Then  $\eta_{\bar{t}+1} < \eta_{\bar{t}}$  follows from (5). Accordingly, the tax rate is reduced from  $\bar{t}$  onwards, while  $b$  declines towards  $\bar{b}$ . As  $b \rightarrow \bar{b}$ , the tax rate converges to a lower smoothened level.<sup>3</sup>

The interpretation of this behavior is the following. Prior to  $\bar{t}$ , the government takes into account the future reputation loss, and therefore it keeps the tax rate high so as to reduce the debt/output ratio towards the critical date  $\bar{t}$ . At  $\bar{t}$ , the tax rate starts to decline, converging to a new smoothened level. The degree to which  $b_{\bar{t}}$  is close to  $\bar{b}$  at the critical date  $\bar{t}$  depends on the value of  $\lambda$ . The higher the value of  $\lambda$ , the larger is the reputation cost, and hence the higher should be the tax rate prior to  $\bar{t}$  and the closer  $b_{\bar{t}}$  should be to  $\bar{b}$ .

*Example 1: Fiscal dynamics with exogenous spending*

The solution of the model, and in particular the dependency of the time profiles of  $\tau$  and  $b$  on the parameter  $\lambda$ , is illustrated by the following example:  $b_0 = 1$ ,  $\bar{b} = 0.6$ ,  $\mu = 0$ ,  $g = 0.4$ ,  $r = 0.05$  and  $\bar{t} = 6$ .

Three alternative cases are considered:

Case 1:  $\lambda = 0$  ("no guideline"),

Case 2:  $0 < \lambda < \infty$  ("guideline of moderate severity"),

Case 3:  $\lambda \rightarrow \infty$  ("guideline of extreme severity").

Figures 1 and 2 plot the simulated  $\tau$  and  $b$  time profiles, respectively, for the three cases.<sup>4</sup> Case 1 yields tax smoothing at the rate 45 percent for the entire planning horizon, while the debt/output ratio remains at the starting value of 1. Case 3 corresponds to an extremely high value of  $\lambda$ . In this case, the tax rate until period  $\bar{t}$  is higher, 50.8 percent, which generates a fast decline of the debt/output ratio, to reach 0.6 at  $\bar{t}$ . At this point, the tax rate jumps to the smoothened lower level 43 percent.

<sup>2</sup> The deterministic nature of the model and the constant real interest rate precludes state-contingent taxation and state-contingent return on the debt as in Chari, Christiano and Kehoe (1994). The absence of assets in the model, as money in Lucas and Stokey (1983), avoids time-inconsistency in taxation.

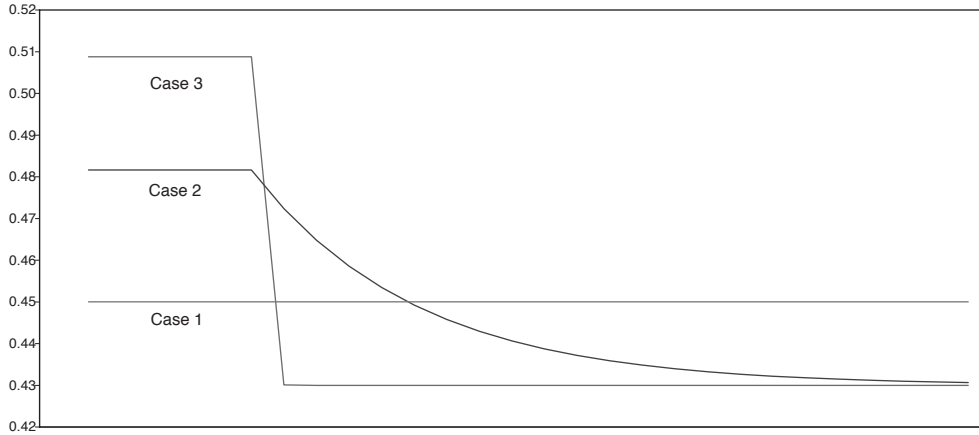
<sup>3</sup> As  $b$  declines, it will not go below  $\bar{b}$ , because if it does, then  $(b_t - \bar{b}) < 0$  in (5) implies that  $\tau_{t+1} > \tau_t$ . Hence, as soon as  $b$  declines below  $\bar{b}$  the tax rate changes direction and begins to increase, reducing  $b$  even further, and so on. This violates the intertemporal budget constraint.

<sup>4</sup> To express the system in a convenient form for the simulation, equations (4) and (5) are combined with the periodical budget constraint (2) to yield

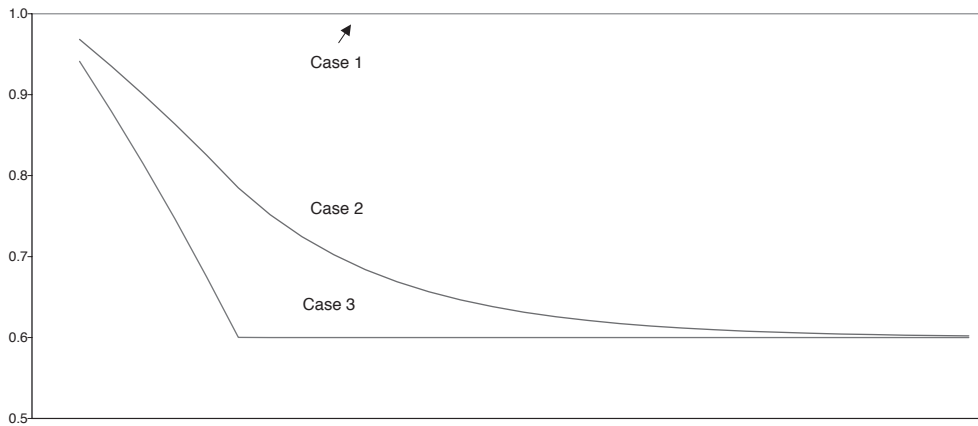
$$-\left(\frac{1+r}{1+\mu}b_{t-1} + g_t - b_t\right) + \left(\frac{1+r}{1+\mu}b_t + g_{t+1} - b_{t+1}\right) + \lambda(b_t - \bar{b})I_t = 0, \quad t = 1, 2, \dots, \infty$$

This is a second order linear equation in  $b_t$  – given the exogenous variables and the indicator function  $I_t$ . The equation is solved given the initial debt  $b_0$  and the terminal condition implied by (3). In the computation, a finite value was postulated for the terminal debt/output ratio at a far enough horizon.

**Figure 1**  
**Tax Rate, Exogenous Government Spending**



**Figure 2:**  
**Debt/Output Ratio, Exogenous Government Spending**



Case 2 is an intermediate one, with a small value of  $\lambda$  (0.05). Given that the fine for  $b_0 < \bar{b}$  is small relative to that in Case 3, the tax rate till date  $\bar{t}$  is lower here, 48.1 percent, implying that the debt/output ratio declines slower. At date  $\bar{t}$  the tax rate begins to decline, reaching asymptotically the level of 43 percent, as  $b$  approaches  $\bar{b}$ .

### b. Endogenous government spending

The previous setup is extended here to include endogenous determination of government spending. Two considerations are involved: (a) The policymaker's utility from providing a real-expenditure/output ratio  $g_t$  is  $u(g)Y_t = \frac{\tau}{2}(g^* - g_t)^2 Y_t$ ,  $\gamma > 0$ . Given that  $u'(g_t) = \gamma(g^* - g_t)$ , the exogenous  $g^*$  determines the level of spending beyond which it provides negative marginal benefit.<sup>5</sup> (b) Reducing the level of  $g$  involves adjustment costs, while increasing it is assumed to be free of this cost. This asymmetric treatment of changing  $g$  captures political or social costs of reducing services provided to the public by the government. Increasing  $g$  always involves the deadweight losses of higher taxation. The adjustment costs function takes the form  $c(g_{t-1}, g_t)Y_t = \frac{\phi}{2}(g_{t-1} - g_t)^2 Y_t I'_t$ ,  $\phi > 0$ , with  $I'_t = 1$  if  $g_{t-1} > g_t$  and  $I'_t = 0$  if  $g_{t-1} \leq g_t$ . This asymmetry is particularly relevant for the initial range of the planning horizon. If the imposition of the debt guideline requires reducing government services, the present setup will lead to a gradual decline. These adjustments costs give a role to  $g_0$  — the level of spending prior to guideline adoption — which becomes a relevant exogenous variable.

The objective function is now

$$\text{Min} \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} \frac{Y_t}{2} \left[ (\tau_t)^2 + \lambda(b_t - \bar{b})^2 I_t + \gamma(g^* - g_t)^2 + \phi(g_{t-1} - g_t)^2 I'_t \right],$$

while the constraints (2) and (3) remain unchanged. The first-order conditions are

$$(6) \quad \tau_t - \eta_t = 0;$$

$$(7) \quad \gamma(g^* - g_t) + (g_{t-1} - g_t)I'_t - \phi(g_t - g_{t+1})I'_{t+1} - \eta_t = 0;$$

and

$$(8) \quad \lambda(b_t - \bar{b}_t)I_t - \eta_t + \eta_{t+1} = 0, \quad t = 1, 2, \dots, \infty.$$

The main implications of the present setup refer to the situation where  $b_0 > \bar{b}$ .<sup>6</sup> Two cases are possible regarding spending at time 0, prior to the planning period: (a)  $g_0 \leq g_1$ , where  $g_1$  is optimal spending in period 1, (b)  $g_0 > g_1$ . Case (b) involves a reduction in government services that triggers adjustment costs, while in Case (a) government spending can be adjusted costlessly.

Let us consider first Case (a). Given the absence of adjustment costs, combining (6) and (7) yields

$$(9) \quad g_t = g^* - \frac{\tau_t}{\gamma}.$$

<sup>5</sup> Expanding regulating agencies beyond a certain level may be an example of such a negative effect.

<sup>6</sup> When  $b_0 < \bar{b}$ , the tax rates and government/output ratios are smoothened for the entire horizon, leaving constant the debt/output ratio at the initial level.

Hence, the path of government spending is linked to the tax rate in a simple way.<sup>7</sup> Given that  $I_t = 0$  until date  $\bar{t}$ , equation (8) implies that  $\eta_t = \eta_{t+1}$ , and hence from (6) it follows that  $\tau_t = \tau_{t+1}$ . Correspondingly,  $g$  is also smoothened until period  $\bar{t}$  from (9). For the same argument presented in Section 2.1,  $b$  starts to decline in period 1, but it is still higher than  $\bar{b}$  at time  $\bar{t}$ . Hence,  $I_{\bar{t}} = 1$ , which implies from (8) that  $\eta_{\bar{t}+1} < \eta_{\bar{t}}$ , and therefore  $\tau$  begins to decline at  $t+1$ . From (9), then, it follows that  $g$  starts to increase at  $t+1$ . In other words, while  $\tau$  is smoothened until  $t$  at a temporarily high level,  $g$  is smoothened at a temporarily low level. Starting from  $\bar{t}+1$ , as  $\tau$  declines approaching its permanent lower level,  $g$  increases to a new permanent level. Hence, the main implication of endogenizing  $g$  is that the lower burden of interest payments following from reducing the debt towards the target is now divided between lower taxes and higher spending.

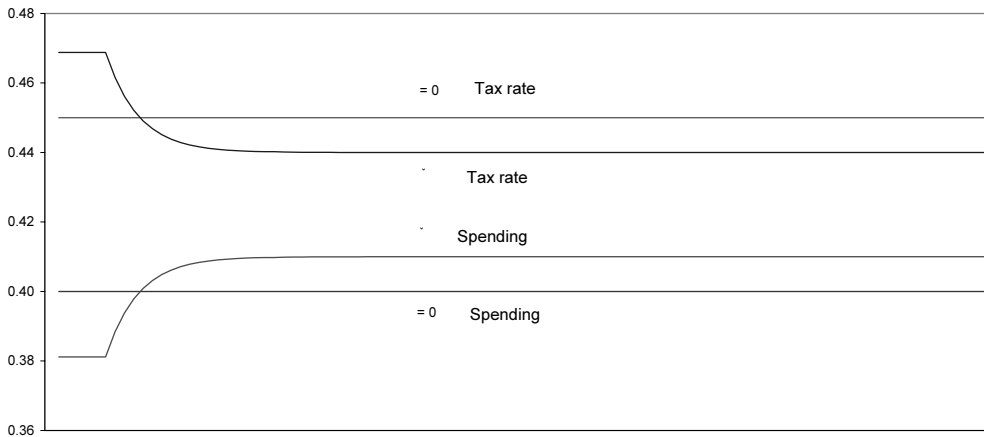
In Case (b),  $g_0$  is high enough so that government spending must be reduced, involving adjustment costs. This case differs from Case (a) only in one aspect: government spending is reduced gradually, rather than immediately, to the level prevailing until date  $\bar{t}$ .

#### *Example 2: Fiscal dynamics with endogenous spending*

Example 1 in Section 2.1 can be modified to illustrate the present case. The exogenous value 0.4 for  $g$  is replaced by the parameters  $g^* = 0.85$  and  $Y = 1$ , which yield  $g_t = 0.4$ ,  $t = 1, 2, \dots$ , endogenously when  $\lambda = 0$ . Figure 3 shows the paths of  $g$  and  $\tau$  in Case (a), i.e., when  $g_0$  is low enough so that there are no adjustment costs, for  $\lambda = 0.05$ . The two variables have mirror-image behavior, as implied by (9). The tax rate follows the same type of pattern as in Example 1, but it declines less since  $g$  increases, thereby allowing for a smaller tax reduction.

The implication of imposing a debt/output guideline with endogenous government expenditures is also illustrated in Figure 3 by comparing the  $\lambda > 0$  case with the  $\lambda = 0$  case: the long-run level  $g$  is higher when  $\lambda > 0$ .

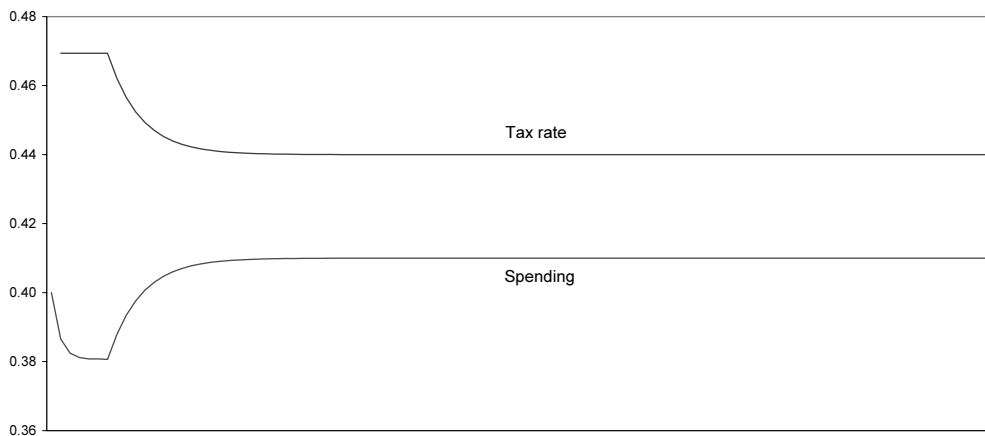
**Figure 3**  
**Government Spending and Tax Rate, Case (a)**



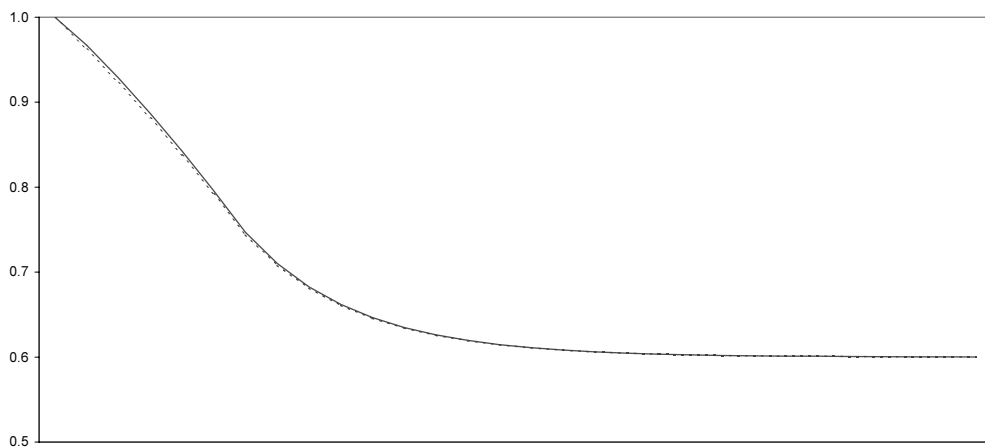
<sup>7</sup> The case of exogenous government spending at level  $g^*$  is represented by  $\gamma \rightarrow \infty$ .

Case (b), corresponding to  $g_0 > g_1$ , does involve adjustment costs. This case is plotted in Figure 4 for  $g_0 = 0.4$ . The main difference between this and Case (a) in Figure 3 is that  $g$  adjusts gradually at the beginning of the planning period, rather than immediately. In other respects, including the debt pattern, the two cases are similar. As shown in Figure 5, the debt/output ratio in both cases starts to decline from period 1, asymptotically approaching the 0.6 level.

**Figure 4**  
**Government Spending and Tax Rate, Case (b)**



**Figure 5**  
**Public Debt/Output Ratio, Cases (a) and (b)**





### c. Endogenous output

So far, output was assumed to be exogenous. A relevant question is whether the optimal dynamic pattern of fiscal variables remain the same when taxation has a negative effect on output. In other words, the question is whether the tax rate remains high at a constant level and spending remains low at a constant level until the critical date  $\bar{t}$ , while the debt/output ratio declines towards the guideline  $\bar{b}$ , and whether only then do they start to adjust to the permanent levels.

It is shown in the Appendix that in the framework of exogenous spending endogenous output does not alter the form of the optimal policy pattern.

## 3. SOME INTERNATIONAL EVIDENCE

The framework developed above is used here to interpret evidence from three countries with high public debt: Belgium, Italy and Israel. Belgium and Italy signed the Maastricht Treaty, and hence they face a formal public-debt guideline. Israel does not have a formal guideline, but the continuous decline in the debt/output ratio during the 1990s suggests the presence of an informal one. The exercise carried out consists in comparing the fiscal behavior in each one of the three countries to the theoretical predictions in Section 2.

In the Maastricht Treaty a specific timetable was set, including fines on member countries violating the public-debt guideline of 60 percent of GDP. Given that the draft of the treaty was signed in February 1992, one may match this year with period 1 in the model (the period of guideline adoption), and  $b_0$  with the debt/output ratio at the end of 1991.<sup>8</sup>

A natural candidate for date  $\bar{t}$  (the period of guideline enforcement) is 1999, since fines on violators were stipulated to be applied starting on January 1st, 1999. The choice  $\bar{b}$  for is 0.6, as specified by the treaty.

Figures 6 and 7 display the public-debt/output ratios for Belgium and Italy, respectively, using the reported gross debt levels until 1991 and a synthetic calculation, using general government expenditure and revenue data, since then.<sup>9,10</sup> Given that for both countries  $b_0 > \bar{b}$ , one could expect, according to the model, that  $b$  declines monotonically from 1992 onwards. However, the debt/output ratio *increases* in 1993 in both countries. This evidence does not necessarily contradicts the relevance of the guideline, given that in this year output growth was very low in both countries (-1.5 percent in Belgium and -1.2 percent in Italy). In the examples in Section 2, output growth is constant over time. Allowing for variable output growth in the model, low growth in a given period increases the debt/output ratio in that period for two

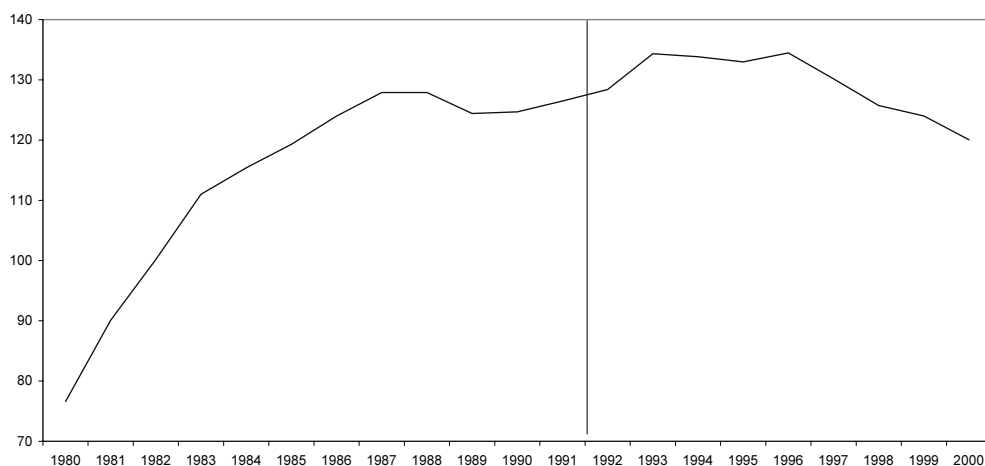
<sup>8</sup> The countries that signed the Treaty are (with the date of referendum approval in parentheses): Belgium (5.11.92), France (23.9.92), Italy (29.10.92), Luxembourg (2.7.92), Holland (15.12.92), Ireland (18.6.92), Greece (31.7.92), Portugal (10.12.92), Spain (25.11.92), Denmark (18.5.93), United Kingdom (23.7.93), Germany (12.10.93), Austria (12.6.94), Finland (16.10.94) and Sweden (13.11.94). Source: Kessing's Records of World Events

<sup>9</sup> The use of a synthetic calculation is aimed at assuring consistency between spending and tax data and the public debt path. The synthetic debt is obtained by adding the general government deficit to the outstanding debt, and it should capture the behavior of the net debt.

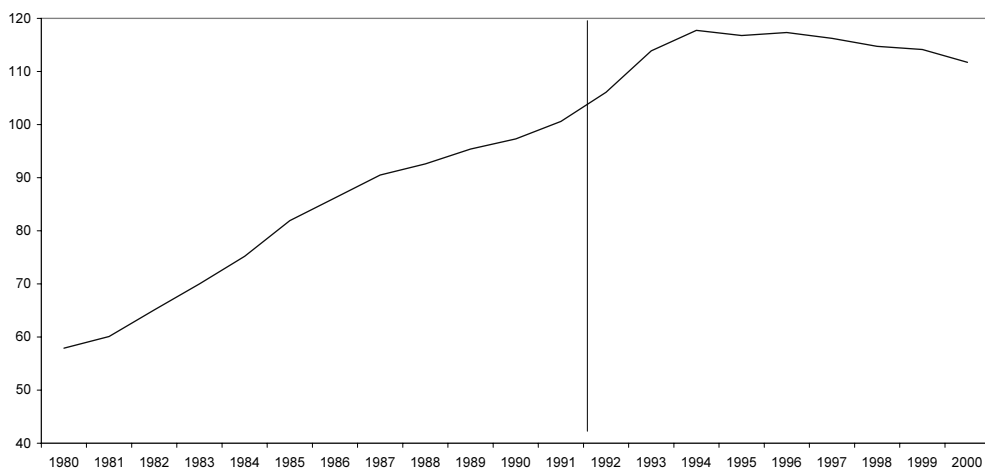
<sup>10</sup> The data for Belgium and Italy are from European Economy (1999). The figures for 2000 are estimates.

reasons: (1) it motivates large deficits for tax-smoothing purposes, and (2) for given deficits it implies a lower denominator of the ratio.

**Figure 6**  
**Belgium: Public Debt (% GDP)**



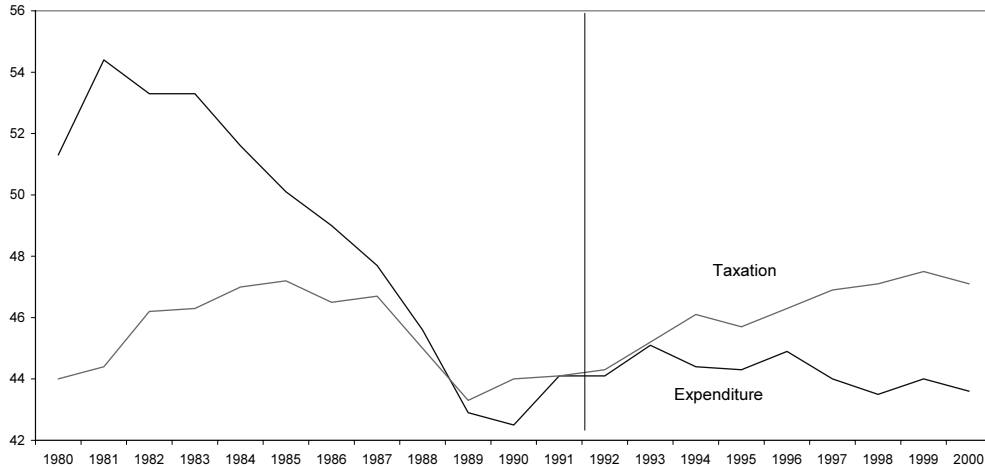
**Figure 7**  
**Italy: Public Debt (% GDP)**



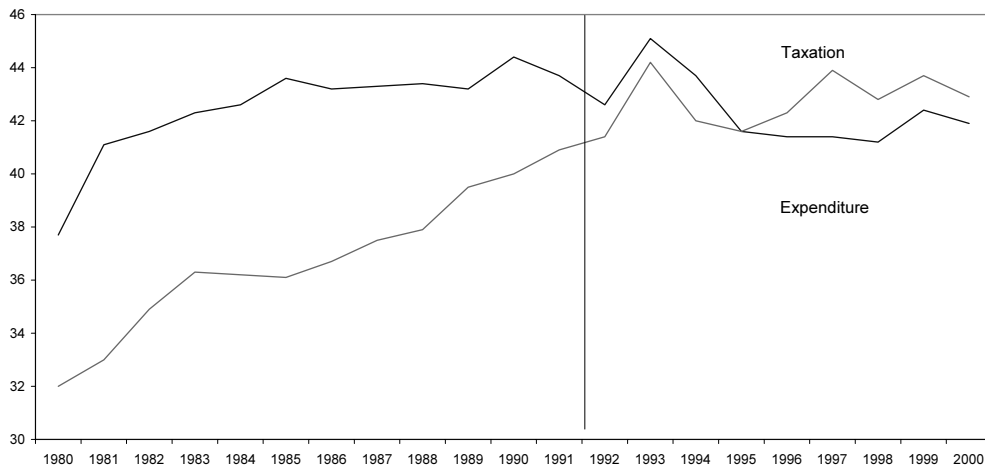
Government spending from 1992 onwards is on a declining path in the two countries, as shown in Figures 8 and 9.<sup>11</sup> The tax rate since 1992 shows no clear trend in Italy, but is rising in Belgium.

<sup>11</sup> The increase in the spending/output ratio in 1993 is likely to be related to the recession in that year.

**Figure 8**  
**Belgium: Government Expenditure and Taxation (% GDP)**



**Figure 9**  
**Italy: Government Expenditure and Taxation (% GDP)**

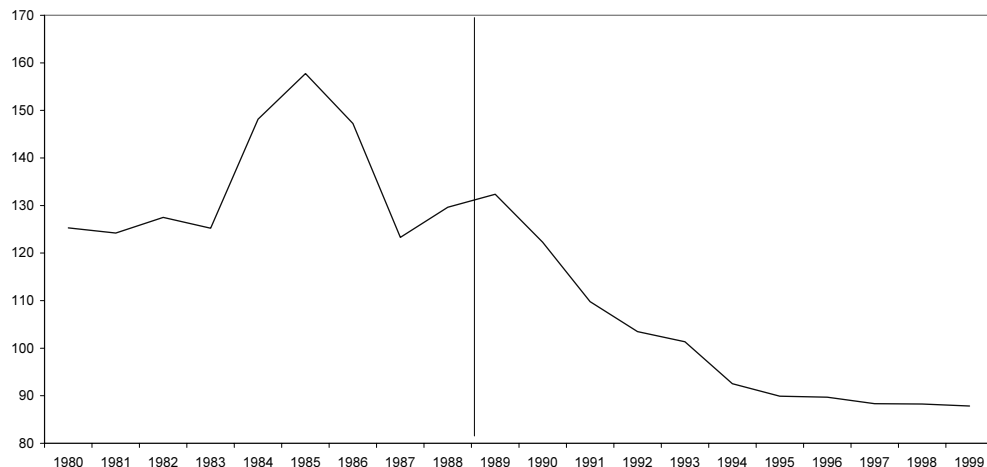


The behavior of the fiscal variables in Italy can be interpreted along the lines of Case (b) in Section 2, as plotted in Figure 4. The adoption of the guideline requires a reduction of the government spending/output ratio, which takes place gradually given adjustment costs in reducing public services, while the tax rate is set at a new smoothened level.

In Belgium, government spending after 1992 has a similar declining path as in Italy, but taxation has a positive trend. This behavior would be consistent with having adjustment costs not only for reducing government services, but also for increasing tax rates.

In the case of Israel there is no formal guideline, and hence there is no specific enforcement date. However, actual fiscal behavior, as shown in Figures 10 and 11, can be interpreted as reflecting an informal guideline.

**Figure 10**  
**Israel: Public Debt (% GDP)**



**Figure 11**  
**Israel Government Net Expenditure and Taxation (% GDP)**



In Israel the first half of the 1980s was dominated by hyperinflation, which came to an end in 1985 with the implementation of a stabilization plan that included drastic spending cuts and a tax hike. Towards the end of the 1980s both the tax rate and the spending/output ratio stabilize relatively to the previous years, and the public debt starts to decline monotonically. If 1989 is taken as period 1, this evidence can be interpreted according to Case (a), as plotted in Figures 3 and 5.<sup>12</sup> The fact that  $g$  and  $\tau$  stabilized around 1989 while  $b$  declined continuously is consistent with the presence of a guideline: the tax rate is kept high and spending is kept low so as to reduce the debt. The stabilization of spending starting from the first period (Figure 11) is consistent with no adjustment costs, given that  $g$  is adjusted upwards after the dramatic cut in the middle 1980s.

#### 4. CONCLUDING REMARKS

This paper introduces a public-debt/output guideline to an optimizing policymaker framework. The guideline may involve a reputation loss and/or a fine — as stipulated in the Maastricht Treaty.<sup>13</sup> The model implies that the policymaker takes into account both the buffer role of deficits — which allow for tax smoothing — and the desire to avoid a high public-debt/output ratio.<sup>14</sup> According to the model, the tax rate remains high at a constant rate from guideline adoption to guideline enforcement, and only then does it start to decline. This result also holds when the negative effect of taxation on output is taken into account.

When government spending is endogenous, the model predicts that the path of spending has a mirror image to that of the tax rate: spending is low while the tax rate is high prior to guideline enforcement. On that date a reversal occurs: as the tax rate starts to decline, the share of government spending in output starts to increase. The long-run share of expenditure in output is higher than prior to guideline adoption, while the tax rate is lower. Hence, the benefits from lower interest payments are divided between lower taxes and higher spending.

The evidence on fiscal behavior in Belgium, Italy and Israel was interpreted along the lines of the present model. The three countries experienced a decline in the public-debt/output ratio. The model predicts that this should have started at the date of guideline adoption. However, in Belgium and even more so in Italy the decline in the debt was delayed, apparently due to the contraction in 1993. The fiscal evidence from Israel until 1999 is consistent with an informal guideline adopted around 1989. The debt/output ratio declined sharply in the 1990s, along with smoothened tax rate and expenditure/output ratio.

<sup>12</sup> The government spending variable is defined here net of non-tax income, which is composed primarily by unilateral transfers from abroad. Important defense items are financed by transfers from the U.S. government. Both defense spending and defense transfers declined together during the 1990s, as fractions of GDP. Hence, netting out transfers fits better the theoretical notion of government expenditure.

<sup>13</sup> An alternative mechanism for generating the reduction of the public debt is the possibility of an unfeasible debt path, as analyzed by Drazen and Helpman (1990).

<sup>14</sup> These two elements are emphasized by Corsetti and Roubini (1993) in their analysis on optimal fiscal rules.

## APPENDIX

**Endogenous output**

This appendix explores the implications of output being a negative function of the tax rate. The model in Section 2 is modified in one respect: the exogenous  $Y$  is replaced by

$$(10) \quad Y = Y(\tau), \quad Y'(\tau) < 0, \quad Y''(\tau) < 0.$$

This specification is seen as a reduced form, reflecting negative effects of taxation on the motivation to produce.

The deadweight loss from taxation is now defined as the loss of output due to taxation:

$$(11) \quad v(\tau) \equiv Y(0) - Y(\tau), \quad v'(\tau) > 0, \quad v''(\tau) > 0.$$

where the signs of the derivatives follow from (10).

The revenue from taxation is given by the function

$$(12) \quad \begin{aligned} R(\tau) &\equiv \tau_t Y(\tau_t), \\ R'(\tau_t) &= \tau_t Y'(\tau_t) + Y(\tau_t), \\ R''(\tau_t) &= 2Y'(\tau_t) + Y''(\tau_t) < 0, \quad (\text{from (10)}). \end{aligned}$$

The solution of the planning problem in this, more general, case is based on the following assumption:

$$(13) \quad \frac{d}{d\tau_t} \left[ \frac{B_t}{Y(\tau_t)} \right] < 0.$$

In words, increasing the tax rate today, given current spending and interest payments, reduces the debt/output ratio at the end of the period. A more detailed form of assumption (13) is obtained using the periodical budget constraint

$$B_t = (1+r)B_{t-1} - G_t - R(\tau_t),$$

which implies that

$$(14) \quad \frac{dB_t}{d\tau_t} = -R'(\tau_t)$$

Then,

$$\frac{d}{d\tau_t} \left[ \frac{B_t}{Y(\tau)} \right] = \frac{-R'(\tau_t) - Y'(\tau) \frac{B_t}{Y(\tau_t)}}{Y(\tau_t)}$$

Hence, (13) requires that

$$(15) \quad R'(\tau_t) + Y'(\tau_t) \frac{B_t}{Y(\tau_t)} > 0$$

This condition has two parts. One is  $R'(\tau_t) > 0$  i.e.,  $\tau_t$  has to be in the upward sloping range of the Laffer curve. Hence, a higher  $\tau_t$  should make it possible to reduce  $B_t$ , the numerator in  $B_t/Y(\tau_t)$  ratio. However, this is not enough for complying with (13). The reduction in  $B_t$  should not be offset by the decline in output, represented by the negative term  $Y'(\tau_t)B_t/Y(\tau_t)$ .

The current version of the planning problem is

$$(16) \quad \text{Min} \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} \left[ v(\tau_t) + \frac{\lambda}{2} \left( \frac{B_t}{Y(\tau_t)} - \bar{b} \right)^2 I_t Y(\tau_t) \right],$$

subject to the periodical budget constraints, which are written here as

$$(17) \quad F(\tau_t, B_t, G_t, B_{t-1}) \equiv B_t - (1+r)B_{t-1} + G_t - R(\tau_t) = 0, \quad t = 1, 2, \dots, \infty$$

and the intertemporal budget constraint.

In Section 2, the optimality conditions were obtained by substituting the expression for  $\tau_t$  from the periodical budget constraints into the objective function, leaving  $B_t$  as the decision variable. In (17)  $\tau_t$  is an implicit function of  $B_t$ . The relevant partial derivatives of this implicit function are

$$\frac{d\tau_t}{dB_t} = -\frac{F_{B_t}}{F_{\tau_t}} = -\frac{1}{R'(\tau_t)} < 0$$

$$\frac{d\tau_t}{dB_{t-1}} = \frac{1+r}{R'(\tau_t)} > 0$$

where the signs follow from (15). The current counterparts of the first-order conditions in Section 2 are

$$(18) \quad -\frac{v'(\tau_t)}{R'(\tau_t)} + \frac{v'(\tau_{t+1})}{R'(\tau_{t+1})} + \lambda \left( \frac{B_t}{Y(\tau_t)} - \bar{b} \right) I_t \left[ \frac{Y(\tau_t) + \frac{Y'(\tau_t)}{R'(\tau_t)} B_t}{Y(\tau_t)} \right] - \frac{Y'(\tau_t)}{R'(\tau_t)} \frac{\lambda}{2} \left( \frac{B_t}{Y(\tau_t)} - \bar{b} \right)^2 Y(\tau_t) I_t = 0$$

$t = 1, 2, \dots, \infty.$

Note that when  $\lambda = 0$ , these conditions become:  $-\frac{v'(\tau_t)}{R'(\tau_t)} + \frac{v'(\tau_{t+1})}{R'(\tau_{t+1})} = 0$

Since  $\frac{v'(\tau_t)}{R'(\tau_t)}$  is a monotonic and increasing function of  $\tau$  (from (15) and (12)), the solution

is  $\tau_t = \tau_{t+1}$ , i.e., tax smoothing.

If  $\lambda > 0$ , the interesting case is when  $b_0 > \bar{b}$ . As in equation (5),  $I_t = 0$  holds until period  $\bar{t}$ , and thus the tax rate is smooth till then. The main question is whether the tax rate follows the same type of pattern from period  $\bar{t}$  onwards, as in Section 2.

Following a similar reasoning as Section 2,  $B_t/Y(\tau_t) - \bar{b} > 0$  holds. The terms in square brackets in (18), which can be written as  $\frac{1}{R'(\tau)} \left[ R'(\tau_t) + Y'(\tau_t) \frac{B_t}{Y(\tau_t)} \right]$  is also positive from

(15). Hence,  $\frac{v'(\tau_{\bar{t}})}{R'(\tau_{\bar{t}})} > \frac{v'(\tau_{\bar{t}+1})}{R'(\tau_{\bar{t}+1})}$  holds, which implies  $\tau_{\bar{t}+1} < \tau_{\bar{t}}$ , i.e., the tax rate declines

at time  $\bar{t}$ . So long as  $B_t/Y(\tau_t) - \bar{b} > 0$ , as  $(b - \bar{b}) \rightarrow 0$ , the tax rate converges again to a smoothened rate, at a lower level.

We conclude that output being a negative function of the tax rate should not alter the main conclusion in the text, i.e., tax rates are reduced only after the debt is substantially reduced towards the level implied by the implicit debt/output guideline.

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