OPTIMIZATION OF BROADBAND INFRASTRUCTURE DEPLOYMENT IN ISRAEL*

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Abstract

This paper studies a resource-allocation problem faced by governments when public subsidies are used to expand broadband infrastructure in areas where private deployment is not commercially viable. The regulator must select, from many potentially overlapping deployment offers, the set of offers that maximizes coverage subject to a budget constraint, while avoiding redundant public spending on the same households or areas. We compare two optimization approaches for broadband infrastructure investment under such constraints. The first is a procurement auction based on a greedy base-2 algorithm, which guarantees a worst-case performance ratio of $1 - e^{-1} \approx 0.632$ relative to the optimum, with a runtime of $O(n^4)$. The second is a method implemented by the Israeli Ministry of Communications in recent procurement auctions, which runs in $O(n^2)$ time but has no known performance bound. Using the case study of fiber-optic deployment in Israel, we show that the Ministry's algorithm achieves broader coverage at lower cost, thus more effectively meeting auction objectives. Benchmarking both methods against the optimal solution on small instances, we find that each approaches optimality, with a slight budget-efficiency advantage for the Ministry's algorithm.

Keywords: Approximation algorithms; Broadband deployment; Combinatorial auctions; Knapsack problem; Resource allocation; Submodular maximization.

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1 INTRODUCTION

In recent years, there has been growing global recognition that expanding public access to broadband Internet contributes to economic growth and the reduction of inequality.[4] However, Bezeq and HOT, the two incumbent companies in the Israeli communications market, concluded that without additional incentives, deploying broadband infrastructure in the country's geographic and social periphery was not economically viable for them. This situation posed a significant challenge for the Israeli government, which had designated connecting all citizens to high-speed Internet as a strategic goal of national importance. Consequently, the Israeli government initiated a dedicated tax intended to subsidize private entities to deploy infrastructure in these areas.[5]

Thus, in 2020, a law came into effect allocating a total budget of approximately NIS 800 million, spread over ten years, for the purpose of connecting peripheral areas to the national fiber-optic network. The Ministry of Communications (MoC) was tasked with managing a series of annual auctions designed to maximize the number of households in the periphery connected to the network within the approved budget.

The auctions were considered a major success. Within two years, after only two auction rounds, 99.5 percent of the pre-defined peripheral areas were selected for connection to the national network at a total cost of approximately NIS 160 million, instead of the NIS 800 million originally allocated by law.

The central challenge facing the MoC when designing the auction was information asymmetry: the costs of connecting different areas and the potential synergies between them were known only to the firms operating in the private market and not to the Ministry itself. To address this problem, the Ministry chose to design an auction aimed at extracting information from the firms in a manner that would assist in selecting the optimal set of areas to be connected within the given budget.

The auction was designed to maximize the benefits arising from connecting adjacent areas. The procurement auction rules allowed participating companies to submit several combined offers, proposing prices to connect clusters of neighboring areas. The auction rules

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[4] Research indicates that a 10 percent increase in broadband penetration in a country increases economic growth by 0.9–1.5 percent [4]. Other studies have shown that broadband access improves public health through telemedicine [1] and helps bridge technological gaps, particularly in low-density rural areas characterized by high capital costs [5, 3].

[5] Communications (Telecommunications and Broadcasting) (Amendment No. 74) Law, 5781-2020 [6].

determined how the winning offers would be selected from all submitted offers.

The chosen mechanism was a "pay-your-bid" auction, in which the winning company is required to connect the areas specified in its offer in exchange for a payment equal to the offer price. Furthermore, each company could win multiple offers within the same auction.[6]

The efficiency of the auction adopted by the MoC naturally depends on how the winning offers are selected from the total pool of submissions. Formally, the selection problem is defined as follows: Given a set of m different geographical areas $\{a_1, \dots, a_m\}$ and a total budget B allocated by the MoC for the auction. Assume that connecting area a_i to the national network generates a value $v(a_i)$ for the state, proportional to the number of households in that area. Given a collection of submitted offers $\{q_1, \dots, q_n\}$, each offer q_j includes a subset of areas from the set of areas and a requested price for connecting the areas in the offer to the national network, to be paid to the proposer from the auction budget B. The question is: what is the collection of offers that maximizes the total number of households in the areas selected for connection within the budget constraint B?

It is important to note that the set of winning offers may contain "overlapping offers". Overlap means that the same area may appear in several different winning offers. In such cases, the value calculation for the set of offers must account for the overlap and avoid double counting the value of connecting an area that appears in multiple offers.

In general, a problem where one must choose an optimal combination of a set of discrete objects, each with a given value and cost, under a budget constraint, is known in the literature as the *knapsack problem*. A "standard" knapsack problem is a special case of the problem discussed in this paper, where each offer contains only one area, and thus there are no overlaps between different offers. The standard knapsack problem belongs to the class of *NP-complete* problems, which are problems of high computational complexity. For this class, no algorithm is currently known that can find the optimal solution in a polynomial running time, relative to the input size.[7] However, given a solution to a problem in this class, one can verify in polynomial time whether the proposed solution meets the problem's conditions. Furthermore, it is known that if a polynomial algorithm is found for the optimal solution of one problem in this class, all other problems in the class could also be solved optimally. Therefore, solving NP-complete problems optimally is considered exceptionally difficult;

[6] For a full description of the auction rules, see (Hebrew): https://www.gov.il/he/pages/13102021

[7] Polynomial running time means that the time required to run the algorithm grows at a rate that is a polynomial function (as opposed to exponential) of the number of offers. Polynomial time is considered practical for solutions, whereas exponential time is not.

known optimal algorithms have an exponential running time relative to input size, making the solution of large problems impractical even with current computing power.

While approximate solutions for the standard knapsack problem exist in reasonable time using polynomial algorithms [18], the standard version does not involve overlaps. However, in the MoC auction, overlaps must be considered. For this reason, we refer to the problem described above as the *knapsack problem with overlaps*. The potential for overlaps makes this problem even more difficult than the standard knapsack problem. The practical implication is that the knapsack problem with overlaps cannot be solved optimally for a large number of offers, leaving no choice but to use algorithms that approximate the optimal solution.

Figure 1 | Areas to deploy

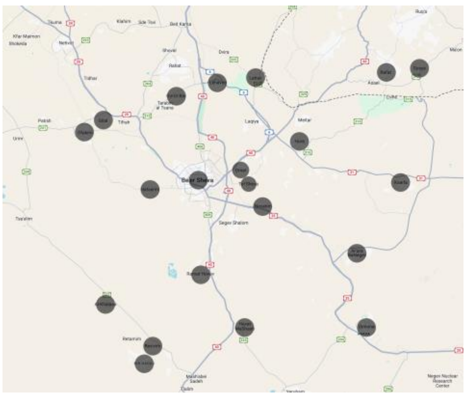
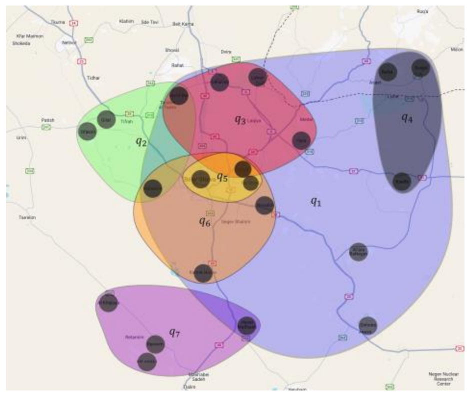


Figure 2 | Seven offers $\{q_1, \ldots, q_7\}$



To illustrate the complexity of the knapsack problem with overlaps, Figure 1 presents an example of 21 potential areas for deployment in the vicinity of the city of Beersheba, where each area contains a given number of households. For these areas, there are seven deployment offers, denoted as $q_1, \ldots, q_7$, which are described in Figure 2. Each of the seven offers shown in Figure 2 includes a cluster of areas. The question is: under a given budget constraint, how should one select the set of offers that maximizes the number of connected households, taking into account the overlaps between different offers without double-counting households that appear in more than one offer?

A greedy algorithm constructs a solution sequentially, adding at each step the offer that provides the highest marginal contribution according to a specified criterion (e.g., marginal coverage or marginal coverage per cost). The greedy base-2 algorithm, which is a generalization of this approach, offers the best approximation ratio known in the literature for the knapsack with overlaps problem. The algorithm operates in two stages: in the first stage, it enumerates all possible solutions consisting of only one or two offers, aiming to capture edge cases in which a small number of high-value offers satisfies the budget constraint. In the second stage, the algorithm expands each solution containing a pair of offers according to the criterion of "maximizing marginal coverage per cost" until the budget is exhausted. That is, the algorithm continues to add the offer that provides the largest incremental coverage per cost, in descending order, until the budget is fully utilized. Finally, the algorithm selects the solution with the maximum coverage from all the solutions generated in the first two stages.

Kulik, Schwartz, and Shachnai [8] show that the greedy base-2 algorithm produces a solution that guarantees an approximation ratio of at least $1 - e^{-1} \approx 0.632$ of the optimal solution for any monotone submodular objective function under a budget constraint, with a polynomial running time of $O(n^4)$, where n represents the number of offers. The notation $O(n^4)$ means that the running time grows on the order of the fourth power of the number of offers. The *approximation ratio* of an algorithm is a metric indicating the ratio of the optimal solution value that the algorithm is guaranteed to achieve for any possible input. This ratio is used for worst-case analysis and provides a standard measure for comparing approximation algorithms for NP-complete problems. Although the greedy base-2 algorithm has a polynomial running time, its relatively high complexity, $O(n^4)$, limits its practical application when the number of offers is large, as is the case in the MoC auctions.

In this paper, we compare the performance of the greedy base-2 algorithm to the algorithm used by the Israeli Ministry of Communications, developed with the assistance of the authors.[8] The MoC algorithm employs a method that selects the set of winning offers based on the principle of "maximizing marginal coverage per marginal cost." Specifically, the algorithm begins with the offer that achieves the highest coverage per cost and then sequentially adds the offer that achieves the maximum additional coverage per additional cost. Crucially, it accounts for the fact that adding new offers to the solution may render previously included offers redundant, thereby saving budget by removing them from the solution. Similar to the greedy base-2 algorithm, this iterative process continues until the budget is exhausted.

Unlike the greedy base-2 algorithm, the MoC algorithm explicitly accounts for potential overlaps and can improve solution efficiency by identifying and removing redundant offers that might remain in a greedy base-2 solution. However, the MoC algorithm does not examine as many initial combinations as the greedy base-2 algorithm and may therefore miss certain combinations that the latter would capture. Unlike the established approximation ratio of the

[8] Hadar Binsky, Zvika Neeman, and Roy Shalem served as economic advisors to the Tender Committee of the Israeli Ministry of Communications.

greedy base-2 algorithm, the approximation ratio of the MoC algorithm is currently unknown. Its running time is $O(n^2)$, making it significantly faster and more practical for large-scale applications than the greedy base-2 algorithm.

In the following sections, we present two simple examples. In the first example, the MoC algorithm produces a better solution than the greedy base-2 algorithm, while in the second example, the greedy base-2 algorithm performs better. Furthermore, we run both algorithms on a selection problem based on data similar to that received in the MoC auctions, showing that in this scenario, the solution produced by the MoC algorithm is superior.

To deepen the performance analysis, we conducted several additional comparisons. First, we compared the MoC algorithm to a “simple” greedy algorithm, which selects the offer with the highest coverage-to-cost ratio at each step until the budget is exhausted, to evaluate the specific benefit of removing overlapping offers. Second, we compared these algorithms to the optimal solution. Due to computational constraints and the exponential growth of the solution space, the optimal solution was calculated for small samples (28 offers) using an exhaustive search of all 2^n possible combinations under the budget constraint.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the law and the requirements of the Ministry of Communications. Section 4 describes the theoretical model. Section 5 presents and compares the algorithms, including the simple greedy algorithm and the optimal-solution benchmarks on a small sample, which allows this comparison. Finally, Section 6 provides concluding remarks.

2 LITERATURE REVIEW

In the literature known to us, the most similar problem to the one faced by the Ministry of Communications is described by Bernheim and Whinston [2], who analyzed bus route licensing in the UK during the 1980s. Bidders in that context were asked to propose “menus” of multiple bus routes and a corresponding price, similar to the offers in the MoC auction which include clusters of areas and a price. Bernheim and Whinston demonstrate that under full information (where all bidders are aware of each other’s costs), a “pay-your-bid” auction can lead to an equilibrium with an optimal allocation. However, they also show the existence of other, inefficient equilibria, and it remains unclear which equilibrium, if any, would be realized in practice. Furthermore, for simplicity, Bernheim and Whinston assumed that the selection of winning offers from the total pool is performed optimally, even though, as explained above, optimal selection is practically impossible when the number of offers is large. Thus, their analysis largely bypasses the primary implementation challenge in environments with many offers, which is the focus of this paper.

In the field of telecommunications, combinatorial auctions have long been used to allow for offers that combine different frequency bands [10]. A prominent example in the Israeli market is the MoC's 5G frequency auctions held in 2020 and 2023, where companies competed for spectrum packages. These auctions allowed bidders to submit offers for various frequency combinations based on their strategic needs.[9] Another example comes from the aviation industry: the International Air Transport Association (IATA), in coordination with airport authorities around the world, manages the allocation of "slots" (takeoff and landing time windows) for airlines, accounting for runway availability and the preferences of airlines for specific takeoff-landing pairs.[10] These mechanisms highlight the inherent complexity of designing auctions where participants can submit customized, bundled offers.

These cases underscore the growing need to integrate economic considerations with computational constraints in auction design, particularly when synergies exist between items or when allocations are interdependent. Combining economic principles, such as marginal value analysis, with advanced computational solutions has become critical. This trend emphasizes the broader contribution of combinatorial auctions to resource allocation across various economic sectors.

The bidding language is important in the present setting because it determines how bidders can express synergies, overlaps, and substitutabilities between different deployment areas. The MoC auction utilized the **OR** *bidding language*, where a bidder can submit a collection of offers, and any subset of these can be selected. This language provides significant flexibility and allows for the expression of complex preferences for item combinations, such as clusters of adjacent areas. Consequently, it is the preferred language for auctions characterized by item synergies. However, the number of possible combinations of subsets grows exponentially with the number of offers, complicating the allocation problem. In contrast, other languages, such as the **XOR** bidding language, allow only one offer from a bidder's set to be selected. While the XOR language is computationally simpler and better suited for situations involving high substitutability between items, it fails to capture complex dependencies or synergies between disparate items. As noted by Nisan [15] and Sandholm [16], the choice of bidding language directly impacts the participants' ability to express preferences accurately and plays a key role in achieving efficient auction outcomes.

As explained in the introduction, selecting winning offers for the peripheral fiber-optic deployment requires solving a knapsack with overlaps problem. As detailed in Section 5, this problem can be represented as *maximizing a monotone submodular function under a budget*

[9] See auction results: https://www.gov.il/en/pages/12082020_2

[10] Further information on slot coordination can be found on the IATA website: https://www.iata.org/en/programs/ops-infra/slots/slot-guidelines/

constraint.

Nemhauser et al. [14] first demonstrated that for the maximum coverage problem with identical costs, a simple greedy algorithm achieves an approximation ratio of at least $1 - e^{-1} \approx 0.632$ of the optimal solution with a running time of $O(n^2)$. Khuller et al. [7] extended this case with heterogeneous costs (Budgeted Maximum Coverage problem) and proposed a greedy base-k algorithm. This algorithm enumerates all possible subsets of size 1 to k that do not exceed the budget, and expands each subset greedily based on the marginal coverage per- cost criterion. This achieves an approximation ratio of at least $1 - e^{-1}$ with a running time of $O(n^{k+2})$, where $k \geq 3$.

Sviridenko [17] generalized this result to any monotonic submodular function maximization under a budget constraint, showing that a greedy base-3 algorithm maintains the $1 - e^{-1}$ ratio with a running time of $O(n^5)$. Subsequently, Leskovec et al. [9] proposed a faster algorithm with a running time of $O(n^2)$, although its approximation ratio decreases to $\frac{1}{2}(1 - e^{-1})$. Work by Kulik et al. [8] showed that the $1 - e^{-1}$ ratio could be preserved while reducing the running time to $O(n^4)$ by using a greedy base-2 algorithm that starts from every possible pair and expands it greedily.

As stated, we compare the MoC algorithm's performance to the greedy base-2 algorithm because the latter represents the best approximation ratio proposed in the literature for the knapsack with overlaps problem. However, we emphasize that the practical application of the base2-greedy algorithm on a large number of offers remains challenging due to its high running time complexity.

3 REGULATION AND LEGISLATION OF BROADBAND INFRASTRUCTURE DEPLOYMENT IN ISRAEL

Amendment No. 74 to the Communications (Telecommunications and Broadcasting [6]) Law, which entered into force in 2020, established an incentive fund and an auction management mechanism with a budget of approximately NIS 800 million over ten years. Its primary objective was to subsidize and support the deployment of advanced networks in areas where such deployment is not economically viable for private firms.

The objective analyzed in this paper is households passed, namely the number of households for which fiber infrastructure becomes available as a result of the selected offers. This is a natural and observable objective for the winner-determination problem, but it is not equivalent to realized usage. From a policy perspective, households passed is a necessary but not sufficient condition for broader adoption of fiber services, since take-up also depends on retail pricing, service quality, consumer demand, and the availability of wholesale or bitstream access for additional service providers. The analysis below should therefore be interpreted as an evaluation of the allocation rule relative to the Ministry's stated deployment objective, rather than as a complete welfare assessment of downstream market outcomes.

On October 13, 2021, the Tender Committee published the first auction for the expansion of licenses for advanced network deployment and the receipt of financial grants to encourage the deployment of fixed advanced networks in areas lacking economic viability (Tender No. RM2021/1). According to the auction terms, each participant was allowed to submit up to 1,000 offers. Each offer could include a single area or a cluster of incentive areas, provided it aligned with the bidder's existing authorization, with the total requested grant per offer not exceeding NIS 10 million. Encouraging bidders to include clusters of areas in a single offer stemmed from the goal of creating synergies between adjacent areas and maximizing the overall value of the deployment.

The auction model adopted by the Ministry of Communications (MoC) is a "pay-your-bid" type. In this framework, each winning offer entitles the proposer to a grant equal to the offered amount in exchange for connecting the specified areas to the national network. This approach encourages participants to submit offers that reflect their expected deployment costs, as the offered price constitutes the final funding the proposer will receive for the project. However, since this approach may create an incentive for "cost padding" (inflating offers beyond actual costs), the Tender Committee established a maximum threshold for the average cost per household. This threshold remains undisclosed to the participants and serves as an additional criterion for evaluating the reasonableness of the financial offers submitted. Furthermore, the bidding stage was transparent to all participants. This feature makes the selection process critical, as proposers must balance the advantage of a lower price (increasing the probability of winning) against the advantage of a higher price (covering actual deployment costs plus a profit margin). The selection process consisted of two parts: the first involved creating a pool of winning offers using an algorithm (which we will elaborate on in Section 5), and the second involved an auction to obtain an exemption from deployment for winners of overlapping areas.

The eligibility conditions in the tenders required prior experience in carrying out infrastructure projects, such as water, sewage, energy, or communications works. This criterion is relevant for rollout capability, but it does not necessarily ensure experience in the provision of retail communications services. This distinction may matter for the transition from physical deployment to active use of the network, since firms with stronger commercial and operational experience in serving end users may be better positioned to convert network availability into effective subscriber adoption.

The scoring rule in the financial stage did not include an explicit quality component beyond the threshold conditions and the cap on the average cost per household. As a result, bidders submitting multiple geographically differentiated offers had to form their own expectations, for each proposed cluster, regarding deployment costs, expected take-up, future revenues per connection, and project risk. In particular, for each cluster, a bidder had to assess the net present value (NPV) of the deployment project. If the NPV was negative absent support, the bid had to be set at a level such that, once the requested incentive grant was taken into account, the project's NPV would become positive. Thus, each bid reflected the bidder's assessment of the economic viability of deployment in that cluster conditional on the subsidy requested. This feature helps explain why a flexible bidding language may be valuable: it allows bidders to encode localized cost and demand assessments that are difficult for the regulator to observe directly.

The fact that the total subsidy allocated in the first two rounds was substantially below the total budget authorized by law may reflect strong cost efficiency, but it may also reflect forecasting errors by some bidders. In particular, when bidders must submit many offers under substantial uncertainty regarding costs, demand, and take-up, there is a risk that some bids are based on overly optimistic assessments of project viability. For that reason, low expenditure relative to the overall fund should not be interpreted mechanically as evidence of full policy success. Ex post implementation and service outcomes remain important for evaluating whether the auction selected projects that were not only inexpensive but also feasible in practice.

An indication of the difficulty involved in correctly assessing project viability may be seen in the relatively small share of areas won by experienced communications providers such as HOT, Partner, and Cellcom. Firms with substantial experience in communications services may have applied stricter profitability thresholds, particularly in areas where the path from rollout to actual take-up was uncertain. We note this point cautiously, since the paper does not include bidder-level evidence that would allow a direct identification of this mechanism.

In the first auction round, 10 participants submitted over 1,800 offers collectively. In March 2022, the Tender Committee announced the winners, who committed to deploying infrastructure across 469 incentive areas, covering approximately 287,000 households.[11] The budget allocated to the winners from the incentive fund amounted to approximately NIS 82 million, reflecting an average allocation of NIS 285 per household.

The following year, the Tender Committee published a follow-up auction (Tender No. RM2022/2) to further expand licenses or permits for advanced network deployment. The

[11] See first auction results (Hebrew): https://www.gov.il/en/pages/10032022

incentive fund allocated for this follow-up auction was NIS 92,111,889.[12] This round saw 9 participants submitting 299 offers. At the end of the second round, winners were selected to deploy infrastructure in 271 areas covering approximately 130,000 households. The cost of this auction was approximately NIS 78 million, reflecting an average allocation of NIS 600 per household.[13]

By the end of the first two rounds, deployment covered 99.5percent percent of households in the designated areas. Consequently, the MoC announced the completion of the deployment auction eight years ahead of the original forecast. The MoC published a deployment map detailing the location of the areas, the number of households, and the entities responsible for deployment. This map is available to the public on the Ministry's website.[14] However, these maps only specify the final results of the deployment; they do not include the number of winning offers or the specific composition of the submitted offers, both of which were classified as confidential.

An additional qualification concerns implementation after the award stage. Public announcements by the Tender Committee indicate that, in some cases, winning bidders did not fully meet their deployment commitments and their awards were subsequently cancelled, including bids covering approximately 40,000 households across the two tenders [11, 12, 13]. At the same time, official publications concerning the tender results and the subsequent tender rounds indicate substantial progress in reducing the remaining uncovered scope. The first tender covered approximately 287,000 households across 470 areas, and the second tender covered approximately 130,000 households across 270 areas, representing the last 5 percent of households without a deployment obligation. Moreover, the current third and final tender concerns only 39 remaining areas comprising approximately 28,000 households. The current final tender appears to focus mainly on a small number of particularly challenging and remote areas that remained after the substantial deployment achievements of the first two tenders.[15]

This limited residual scope is consistent with the view that non-compliance in the earlier tenders was not widespread enough to undermine the overall deployment process. If non-compliance had been broad, one would expect the Ministry to face a substantially larger number of uncovered areas and households in the final tender. Thus, while ex post

[12] See auction rules (Hebrew): https://www.gov.il/BlobFolder/rfp/26102022/he/RM2-2022-24012023.pdf

13 See second auction results: https://www.gov.il/en/pages/01022023

[14] See the deployment map: https://public.tableau.com/views/_17255232362800/sheet0?:embed=y&:showVizHome=no

[15] See current third auction (Hebrew): https://www.gov.il/he/pages/31122025

implementation should be distinguished from the winner-determination problem studied here, the successive reduction in the scale of the tenders provides an additional indication of substantial deployment progress. This observation does not alter the algorithmic comparison in the paper, but it does underscore the importance of monitoring, enforcement, and implementation capacity in procurement design.

4 MODEL

Let $A = \{a_1, \dots, a_m\}$ be a set of m geographical areas where broadband services can be deployed. Connecting area a_i generates a social value $v(a_i)$, which is proportional to the number of households residing in that area. Let $S = \{q_1, \dots, q_n\}$ be a set of offers. Each offer q_j describes a plan for deploying broadband services across a subset of areas $q_j \subseteq A$ at a requested price $c(q_j)$. (To minimize the notation, the letter q may denote either a specific offer or the set of areas included in that offer, depending on the context). Let B denote the budget.

The social value obtained from deployment in the set of areas included in a given offer $q \in S$ is described by the function $V: 2^A \rightarrow \mathbb{R}_+$, defined by the sum:

$$V(q) = \sum_{a \in q} v(a).$$

The social value obtained from the set of offers $Q \subseteq S$ is given by the sum of values obtained from all areas included in the collection of offers Q:

$$V(Q) = V\left(\bigcup_{q \in Q} q\right).$$

It is important to note that the value from each area included in the set of offers Q is counted only once.

The social value function V satisfies the following properties. First, the social value from connecting an empty set of areas is zero ($V(\emptyset) = 0$). Second, since increasing the set of areas designated for deployment increases the social value, the function V is monotonically non-decreasing. That is, for any two sets of offers $Q \subseteq Q' \subseteq S$:

$$V(Q) \leq V(Q')$$

Given a set of offers $Q \subseteq S$ and an additional offer $q \in S \backslash Q$, we define the *marginal coverage* of q relative to Q as the incremental social value obtained by adding q to the set Q:

$$V(Q \cup q) - V(Q)$$

Because the value function V counts each area only once (even if it appears in more than one offer), the marginal coverage of offer q reflects only the value of the *new* areas that q adds beyond what is already covered by Q.

The fact that different offers for a given set of areas can overlap means that the function V is *submodular*. That is, for any two sets of offers $Q \subseteq Q' \subseteq S$ and for any offer $q \in S \backslash Q'$:

$$V(Q' \cup q) - V(Q') \leq V(Q \cup q) - V(Q)$$

This inequality reflects the diminishing marginal coverage of any offer as the existing set of offers expands, due to overlaps with areas already covered.

The objective is to find a set of offers Q^* that deploys broadband infrastructure for the largest number of households within the budget constraint B. Formally, we seek to find a set of offers Q^* that solves the following optimization problem:

$$\max_{Q \subseteq S} \quad V(Q) \qquad \mathrm{s.t.} \qquad \sum_{q \in Q} c(q) \leq B. \qquad (1)$$

5 ALGORITHMS

As explained in the introduction, we compare two algorithms for solving the winner–determination problem (1).

5.1 The Greedy base-k Algorithm

The greedy base-k algorithm is based on a greedy approach and is divided into two stages. In the first stage, the algorithm examines all potential solutions consisting of offers of size 1 to k and selects the one that connects the largest number of households within the budget constraint. In the second stage, the algorithm starts from every possible solution set of size k and expands it incrementally. In each expansion step, the algorithm evaluates all unselected offers and adds the one that provides the highest marginal coverage to price ratio, i.e., the incremental increase in the number of connected households resulting from adding the offer, divided by its cost. This process continues until the budget is exhausted. Finally, the algorithm selects the best solution found in both stages.

Algorithm 1 | Greedy base-$\boldsymbol{k}$ algorithm

Input: Collection of offers S; value function $V(\cdot)$; Budget B.

Output: $Q_{best} \subseteq S$.

$Q_{best} \leftarrow \varnothing$;

// Enumerate all solutions of size at most k.

$$Q_{best} \leftarrow \mathrm{argmax}_{Q \subseteq S} V(Q) \quad \text{such that} \quad |Q| \leq k, \sum_{q \in Q} c\,(q) \leq B;$$

// Iteratively add offers until the budget is exhausted.

For *each* $Q \subseteq S$ *with* $|Q| = k$ ***do***

While *budget allows* ***do***

$$q \leftarrow \mathrm{argmax}_{q \in S \setminus Q} \left\{ \frac{V(Q \cup q) - V(Q)}{c(q)} \right\};$$

$Q \leftarrow Q \cup q$;

end

if $V(Q) > V(Q_{best})$ ***then***

$Q_{best} \leftarrow Q$.

end

end

Theorem 1 (Kulik et al., 2021): The solution obtained by the greedy base-2 algorithm achieves an approximation of at least $1 - e^{-1} \approx 0.632$ of the optimal solution Q^*. The algorithm's running time is $O(n^4)$, where n represents the number of offers.

The polynomial running time of $O(n^4)$ ensures that for a sufficiently large n, the greedy base-2 algorithm is faster than a simple exhaustive search of all possible solutions, which has an exponential running time in the number of offers. However, the greedy base-2 algorithm does not guarantee an optimal solution, and when the set of offers S is very large, the algorithm's execution time may still be significant.

5.2 The Ministry of Communications (MoC) Algorithm

In the MoC auction, 1,000 different areas were defined. In the first round, over 1,800 offers were submitted, while the second round saw 299 offers. Each offer could include a single area or a cluster of areas, ranging in size from one area to several dozen areas. The large number of offers and areas highlights the diversity and computational complexity involved. The MoC implemented a two-stage mechanism to determine the winning offers.

Stage 1: Selection based on Marginal Coverage to Marginal Cost ratio

The algorithm starts with an empty set of selected offers and an available budget. In each step, the algorithm calculates the marginal coverage and marginal cost for each offer. The marginal cost of an offer q, denoted as $c(q, Q)$, is defined as the cost of offer q minus the costs of any offers already in Q that become redundant when q is added:

$$C(q, Q) \equiv c(q) - \sum_{q' \in Q : V(Q \cup q) = V\left((Q \setminus q') \cup q\right)} c(q')$$

In cases where the redundant offers are expensive, the marginal cost may be negative, as a new offer could reduce the total budgetary expenditure. The algorithm calculates the marginal coverage to marginal cost ratio for each offer and adds the offer that maximizes this ratio in each iteration. After adding an offer, the algorithm removes redundant offers and updates the budget accordingly. This process continues until the budget is fully utilized.

Algorithm 2 | Ministry of Communications (MoC) Algorithm

Input: Collection of offers S; value function $V(\cdot)$; Budget B.

Output: Selected set of offers $Q^* \subseteq S$.

$Q^* \leftarrow \varnothing$;

While *budget allows* ***do***

// Add the offer that maximizes marginal coverage per marginal cost and remove redundant offers.[16]

$$q \leftarrow \text{argmax}_{q \in S \setminus Q^*} \left\{ \frac{V(Q^* \cup q) - V(Q^*)}{C(q, Q^*)} \right\};$$

$$Q^* \leftarrow \{Q^* \cup q\} \setminus q';$$

Update remaining budget;

end

At each step, the algorithm examines at most n remaining offers. For each, it calculates the marginal coverage by checking at most n offers already in Q^* for redundancy. Since the combined number of selected and unselected offers sums to the total number of offers n, the running time for each step is $O(n)$. Given that one offer is selected in each step, and the total number of selected offers is bounded by n, the overall running time of the algorithm is $O(n^2)$.

The MoC also defined an additional constraint: a maximum threshold for the average cost per household. Any offer exceeding this threshold was disqualified. This constraint aimed to prevent excessively expensive investments and to preserve the budget for future auctions, where funds might be utilized more efficiently. This threshold remained undisclosed to the bidders during and after the auction.

[16] If multiple offers maximize the ratio, the one with the highest marginal coverage is selected. If contributions are equal, one is chosen randomly. In case of negative marginal costs, the offer with the highest budgetary saving is chosen.

Stage 2: Exemption Auction from Deployment

The second part of the mechanism involves an auction for exemption from deployment in overlapping areas. In this auction, all areas included in more than one winning offer are identified. For each such area, relevant participants submit payment offers to receive an exemption from their deployment obligation in the overlapping area. Every bidder, except the one who submitted the lowest payment offer, is required to pay their proposed amount to receive the exemption, thereby avoiding the need to deploy fiber infrastructure in that specific area. The bidder with the lowest offer does not pay and maintains the obligation to deploy in that area.

This auction ensures that at least one provider is selected to deploy infrastructure in every overlapping area. By granting exemptions to all participants but one, the Ministry achieves deployment across the same number of areas at a lower net cost.

5.3 Examples

The following theoretical examples illustrate the application of the two algorithms described above. For simplicity, the household value in each area is set to one. In the first example, the MoC algorithm outperforms the greedy base-2 algorithm, whereas in the second example, the greedy base-2 algorithm achieves a better result than the MoC algorithm.

Example 1

Consider a set of six geographical areas, $A = \{a_1, \dots, a_6\}$, targeted for infrastructure deployment. A collection of six offers, $S = \{q_1, \dots, q_6\}$, is submitted under a total budget constraint of $B = 9$. The specific areas included in each offer and their respective costs are detailed as follows:

Table 1 | Composition and costs of offers in Example 1

Offer	Areas	Cost
q_1	$\{a_1, a_2\}$	3
q_2	$\{a_3, a_4\}$	3
q_3	$\{a_5, a_6\}$	3
q_4	$\{a_1\}$	1
q_5	$\{a_3\}$	1
q_6	$\{a_5\}$	1

Figure 3: The six areas targeted for deployment in Example 1.

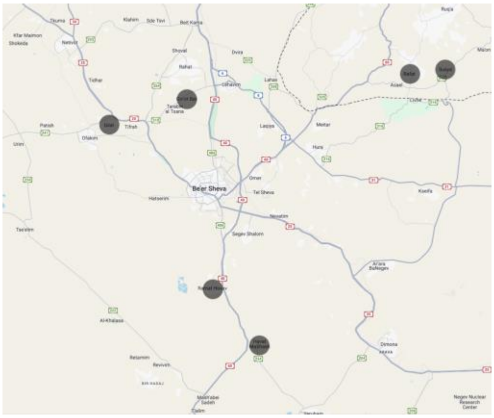
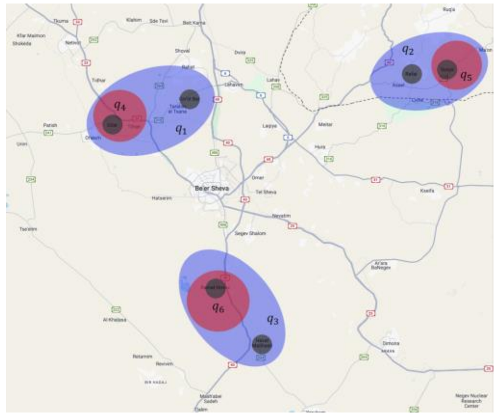


Figure 4: The six offers submitted in Example 1.



Implementation Results:

Table 2 | Implementation Results for Example 1

Algorithm	Selected Offers	Coverage	Budget	Avg. Cost
MoC Algorithm	$\{q_1, q_2, q_3\}$	6	9	1.5
Greedy base-2	$\{q_1, q_2, q_6\}$	5	7	1.4

This example demonstrates that the MoC algorithm provides coverage for a larger number of households within the given budget. The MoC algorithm's advantage stems from its ability to identify that adding offer q_3 renders offer q_6 redundant, thus improving the solution while remaining within budget constraints.

Example 2

Consider a set of seven geographical areas, $A = \{a_1, \dots, a_7\}$, targeted for infrastructure deployment. A collection of three offers, $S = \{q_1, q_2, q_3\}$, is submitted under a total budget constraint of $B = 5$. The specific areas included in each offer and their respective costs are detailed as follows:

Table 3: Composition and costs of offers in Example 2

Offer	Areas	Cost
q_1	$\{a_1, a_2, a_3\}$	1
q_2	$\{a_2, a_4, a_5\}$	2
q_3	$\{a_3, a_6, a_7\}$	3

Figure 5 | The seven areas targeted for deployment in Example 2.

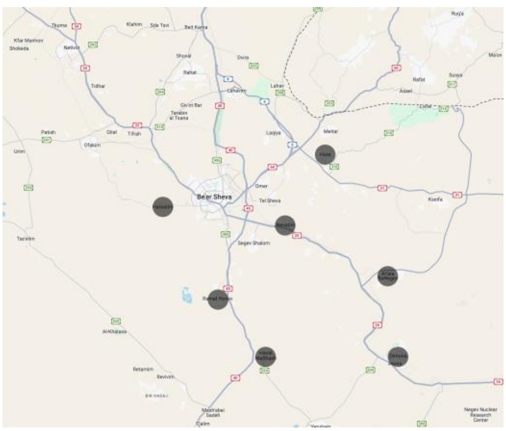
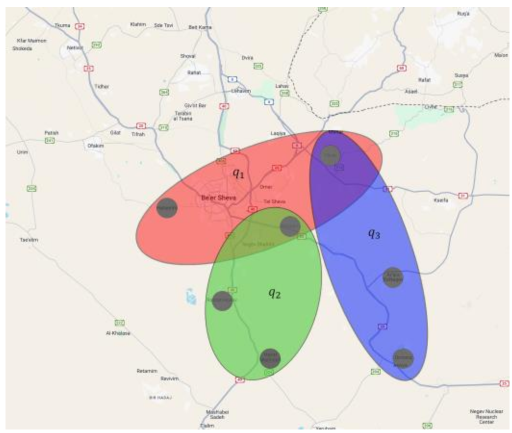


Figure 6 | The three offers submitted in Example 2.



Implementation Results:

Table 4 | Implementation Results for Example 2

Algorithm	Selected Offers	Coverage	Budget	Avg. Cost
MoC Algorithm	$\{q_1, q_2\}$	5	3	0.6
Greedy base-2	$\{q_2, q_3\}$	6	5	0.83

This example illustrates that the greedy base-2 algorithm covers a larger number of households within the given budget. This occurs because the MoC algorithm begins from an empty set, and the first offer added is q_1, which limits the algorithm's ability to explore interactions between different offer mixes. In contrast, the greedy base-2 algorithm selects the pair $\{q_2, q_3\}$, thereby more effectively utilizing the possibilities to improve total coverage within the budget constraint.

5.4 Case Study: The 2022 Ministry of Communications Auction

In this section, we compare the performance of the various algorithms using data based on the offers received during the second round of the MoC's 2022 auction. While the initial round involved approximately 1,800 offers, in the second round, 299 offers were submitted, aimed at serving infrastructure deployment for 142,098 households across 378 distinct geographical areas. The budget allocated for the second round was NIS 92,111,889. It should be noted that both the specific composition of the submitted offers and the maximum average cost-per-household threshold established by the Ministry remain confidential.

Due to the computational complexity of the greedy base-2 algorithm, we compared the performance of the two algorithms on three samples of 100 offers out of the 299 submitted (each offer was chosen proportionally to the number of offers of that bidder). Because we reduced the number of offers by about two-thirds, we also proportionally reduced the budget to NIS 30,806,652. The average results are presented in the following Table 5.

Table 5 | MoC Algorithm vs. Greedy base-2 (Three 100-Offer Sample)

Metric	MoC Algorithm	Greedy base-2
Winning offers	51	55
Covered areas	91	93
Covered households	**59,099**	**58,780**
Total cost	30,126,450	30,753,952
Avg. cost per household	510	523
Overlapping areas	2	5
Overlapping households	1,377	5,539

These results show that the MoC algorithm achieves 0.54 percent higher household coverage compared to the greedy base-2 algorithm, and does so at a 2.08 percent lower cost. Additionally, the MoC algorithm has fewer overlapping areas and fewer households in those areas. This analysis indicates that the advantage of the MoC algorithm in minimizing additional costs from potential overlaps compensates for the fact that it examines fewer possibilities than the greedy base-2 algorithm. Thus, it can be expected that when the set of offers contains many overlapping offers, the MoC algorithm will be more efficient, achieving more coverage while minimizing redundancy.

It is important to emphasize that this comparison is intended to be indicative rather than a complete empirical assessment of the relative performance of the algorithms. Since the analysis is based on reduced samples of offers, part of the small observed difference may depend on the specific composition of the sampled offers. A more systematic comparison would require repeating the analysis over many subsamples or conducting a broader simulation exercise.

In addition to the comparison with the greedy base-2 algorithm, we examined the MoC algorithm against a simple greedy approach. This approach is based on a greedy selection of the offer with the highest coverage-to-cost ratio at each iteration until the budget is exhausted. While the simple greedy algorithm has a fast running time of $O(n\log n)$, it ignores overlaps between offers and does not guarantee an approximation ratio relative to the optimal solution. This comparison was performed on the full set of offers from the second round of the auction (299 offers) and demonstrates the advantage achieved when overlapping offers can be rendered redundant through the MoC algorithm. The results are presented in Table 6.

Table 6 | MoC Algorithm vs. Simple Greedy (Full 299 Offers)

Metric	MoC Algorithm	Simple Greedy
Winning offers	119	136
Covered areas	271	258
Covered households	**129,457**	**127,347**
Total cost (NIS)	77,696,630	80,042,025
Avg. cost per household	600	629
Overlapping areas	5	11
Overlapping households	1,876	18,782

The MoC algorithm covered 1.65 percent more households while using 3 percent less budget. Furthermore, it reduced the number of overlapping households by 90 percent. These results underscore that the treatment of overlaps within the MoC algorithm improves both budgetary efficiency and coverage compared to simple greedy approaches.

Another interesting comparison is between the performance of the algorithms presented above and the *Optimal Solution*. As previously noted, examining all 2^n combinations to find the optimal solution is impractical for a large number of offers; otherwise, it would have been the preferred choice for the Ministry. Therefore, we performed this comparison on three small samples of 28 offers each, allowing for an exhaustive search of every possible mix of offers with a budget proportionally reduced to NIS 8,625,863.

It should be noted that this small sample is limited in its ability to represent the original problem: first, the synergy between a bidder's various offers is reduced, as the sample does not faithfully represent the bidder's overall strategy. Second, the sampling was not adjusted for offer costs, leading to an over-representation of offers that are expensive relative to the reduced budget. Despite these limitations, the sample provides insights into the proximity of the algorithms' performance to the optimal solution. The average results are presented in Table 7.

Table 7 | Comparison against the Optimal Solution (Three 28-Offer Sample)

Metric	Optimal	MoC	Greedy base-2	Simple Greedy
Winning offers	18	16	20	20
Covered areas	37	33	38	38
Covered households	**22,995**	**22,982**	**22,292**	**20,939**
Total cost (NIS)	8,380,936	8,355,286	8,577,181	8,577,181
Avg. cost per household	364	364	385	410
Overlapping areas	2	2	6	6
Overlapping households	1,472	1,472	3,972	1,353

The results highlight the proximity of the algorithms to the optimal solution. The MoC algorithm covered 99.94 percent of the households achievable by the optimal solution while utilizing 99.7 percent of the optimal budget.

In contrast, the greedy base-2 algorithm covered 96.9 percent of the optimal solution's households while using 102.3 percent of the optimal budget, with a higher number of overlaps in both households and areas. Since the number of overlaps in this small sample was naturally low, the advantage of examining multiple starting pairs did not yield a significant benefit. The Simple greedy algorithm performed even lower, covering only 91 percent of the households with a budget 2.3 percent higher than optimal. These results emphasize that in this sample, the MoC algorithm provides a near-optimal solution with high budgetary efficiency and minimal overlaps.

Finally, it is worth noting that one may also consider a hybrid algorithm combining the greedy base-2 algorithm with the algorithm used by the Ministry of Communications. Such an algorithm would begin by enumerating all possible pairs of offers, and would then iteratively select additional offers according to a marginal coverage-to-marginal cost criterion, while removing redundant offers in the presence of overlaps, until the budget is exhausted. Preliminary sample-based tests suggest that the hybrid algorithm yields results similar to those of the Ministry's algorithm, and in some cases slightly better. At the same time, because the hybrid algorithm examines a larger set of combinations in which some offers cannot be redundant, the number of overlapping households under the hybrid algorithm may be higher than under the Ministry's algorithm. More generally, when the set of offers involves greater overlap and combinatorial complexity, the hybrid algorithm may offer a more substantial advantage, improving coverage while limiting the additional costs associated with overlaps.

6 CONCLUSION

This paper addresses the optimization of broadband service allocation under a budget constraint, with the goal of achieving maximum coverage in peripheral areas. The use of a combinatorial auction allows each offer to include a variety of geographical areas to exploit the synergies arising from geographical proximity. Consequently, the paper tackles the knapsack with overlaps problem, or alternatively, the maximization of a submodular function under budget constraint. Both problems are defined as NP-complete, highlighting the theoretical and practical complexity of finding exact solutions for large-scale instances within a reasonable timeframe and necessitating the use of heuristic and approximation approaches.

In this study, we compared the greedy base-2 algorithm, which achieves the best known approximation ratio, guaranteeing a solution within a factor of $1 - e^{-1} \approx 0.632$ of the optimal solution with a running time of $O(n^4)$, with the algorithm implemented by the Israeli Ministry of Communications (MoC), which features a faster running time of $O(n^2)$ but with an unknown approximation ratio. Additionally, we conducted a direct comparison against the optimal solution, a task feasible only on a small sample due to high computational complexity. The empirical comparisons based on the 2022 auction data suggest that the MoC algorithm may improve budgetary efficiency by identifying redundant overlaps, thereby ensuring broader coverage while minimizing budgetary redundancy at a lower cost. However, the sample-based comparison should be interpreted as indicative rather than as a complete empirical assessment of the algorithms' relative performance.

A central question not addressed in this paper is the strategic impact of the algorithmic choice on bidder behavior. Different auction mechanisms influence how bidders formulate their offers, particularly in complex combinatorial auctions where the interdependencies between offers are critical. For example, algorithms incorporating greedy considerations might lead bidders to focus their offers on small clusters with high benefit-to-cost ratios, while other algorithms might encourage broader and more diverse submissions.

Even in areas in which deployment obligations are fulfilled, an important unresolved question is the extent to which households actually switch to fiber service and whether the resulting market structure supports effective competition. Ex post evaluation should therefore examine realized take-up, the availability of user-friendly interfaces and wholesale access arrangements for additional service providers, and the extent to which subsidized rollout created competitive local markets rather than localized monopolies. These downstream outcomes lie beyond the scope of the present algorithmic analysis, but they are central for assessing the broader welfare consequences of the policy.

Future research could examine the strategic influence of auction mechanisms on bidder decision-making processes, aiming to design mechanisms that balance computational efficiency with strategic simplicity. Furthermore, deeper integration between different algorithms could be explored to create solutions that maintain existing complexity while improving resource allocation efficiency.

Beyond the specific context of broadband infrastructure, this research contributes to the broader discussion of resource allocation optimization. The approaches and algorithms discussed here can be adapted and applied to various fields, such as healthcare, transportation planning, and environmental management, offering a general framework for solving optimization problems and combinatorial auctions.

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