# The Adjustment of Prices to Monetary Shocks When Trade is Uncertain and Sequential

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#### ABSTRACT

Trade is both uncertain and sequential. Money surprises are not neutral because prices at the beginning of the trading process cannot depend on its end. Unlike fixed price models, here sellers can change prices during trade. Unlike Lucas (1972), here there is no asymmetry in the information about the money supply. The price quoted by individual sellers may adjust slowly to changes in the targeted money supply, but the distribution of quoted prices adjusts perfectly to these changes and the real price distribution is independent of the anticipated rate of change in the money supply.

### INTRODUCTION

Prices do not follow the predictions of the standard competitive spot market model. Using the Stigler-Kindahl data, Carlton (1986) found that "It is not unusual in some industries for prices to individual buyers to remain unchanged for several years," and "Even for what appear to be homogeneous commodities, the correlation of price changes across buyers is very low" (p. 638). Recently, Lach and Tsiddon (1992) looked at disaggregated price data during a high inflation period in Israel (1978-85) and report similar findings: Individual stores change their prices infrequently and price changes are not synchronized across stores. This is usually interpreted as evidence of a monopolistically competitive environment with non-trivial fixed costs for changing nominal prices.

Here I argue that the above findings do not require fixed menu type costs and market power. I use a monetary version of Eden (1990) which is a competitive version of the Prescott (1975) - Butters (1977) model. From the point of view of an individual seller, purchasing power arrives in batches after each new injection of cash. The number of batches that will arrive is random because the number of cash injections is. The seller is a pricetaker: He knows the prices that he can sell to each batch. He makes a contingent plan as to how to utilize capacity. He may plan, for example, to utilize and sell only part of his capacity to the first batch of buyers,

speculating on the possibility that the second batch will arrive and buy at a higher price. This is modelled as a sequence of Walrasian spot markets. The arrival of a new batch opens a new market. Because the number of batches that arrive is random, the number of markets that will open is random. At the beginning of the trading process, producers allocate capacity across markets. Capacity is utilized only in markets that open, and these markets are cleared.

In equilibrium there is a tradeoff between price and the probability of making a sale, and the expected payoff per unit of capacity is the same in all markets. Since producers are indifferent to the way they allocate capacity across markets, the equilibrium number of sellers in each market is undetermined: Only the total quantity supplied to each market is determined. This indeterminacy allows for equilibria in which not all sellers adjust their prices to observed changes in the money supply. Sellers who do not change prices are compensated for the reduction in real price by the increased probability of making a sale. They may therefore let inflation erodes the real price until it falls to the level that guarantees a sale and only then change the nominal price.

When the realization of the money supply is relatively high, more markets are opened, more capacity is utilized, and more output is produced. Thus, unanticipated variations in the money supply are associated with variations in capacity utilization and real activity, despite the fact that all transactions occur at market-clearing prices. Money is non-neutral because prices at the beginning of the trading process cannot depend on its

end. This is different from fixed price models (Fischer, 1977, McCallum, 1977, Phelps and Taylor, 1977, Taylor, 1979, and Svensson, 1986) because here sellers have no incentive to change prices during trade. Unlike Lucas (1972), here there is no asymmetry in the information about the money supply.

This model has some features in common with those of Lucas (1989), Woodford (1990), and the recent work by Lucas and Woodford (1992). By contrast with Lucas (1989), this model shows that the non-neutrality of money shocks does not depend upon sellers having to sell at money prices that they already regret having committed themselves to at the time that the sales occur. By contrast with Lucas and Woodford, this model shows that the non-neutrality does not depend upon any asymmetry of information between buyers and sellers as to the current state of aggregate demand.

In the Lucas and Woodford model, trade is sequential but the transfer of money is not: trade starts only after buyers know the total amount of money transferred. This asymmetry in information about the money supply may lead to rationing. Here both the money transfer and trade are modelled as perfectly synchronized processes: Each batch of dollars transferred triggers more trade and because the market that opens is cleared, there is no rationing. Lucas and Woodford use an infinite horizon model, a Nash equilibrium concept, and exogenous capacity. The fact that the main empirical implications are the same in these two versions of the sequential trading model is evidence of the robustness of these models. THE MODEL

I consider an overlapping generations model. At the beginning of each period, a known number of ex-ante identical individuals are born. The population does not change over time. Individuals live for two periods. They produce and sell their output for money in the first period. They then use, in the second period, the proceeds of first period sales plus a transfer that they receive from the government to buy goods. Fiat money is the only asset.

The buyer (an old agent) shops in many locations. The amount of money available to the buyer at the beginning of period t (M<sub>t</sub>) equals the proceeds of period t-1 sales. The money transfer occurs sequentially and is proportional to the initial amount of money held. At the first location the buyer receives a transfer of ( $\theta_1\lambda_t - 1$ ) per dollar, where  $\lambda_t - 1$  denotes the anticipated rate of change in the money supply and  $\theta_1$  is the lowest realization of an i.i.d. random variable  $\theta$  which takes the realizations:  $\theta_1 < \theta_2 < ... < \theta_S$ . The realization  $\theta_1$  occurs with probability  $\Pi_1$ . For convenience I set  $\theta_0 = 0$ .

Thus in the first location the buyer has  $(\theta_1 - \theta_0)\lambda_t M_t$  dollars and in equilibrium, he spends all of it. He then goes home. If there are no additional transfers, trade for period t stops and the buyer consumes whatever was bought in the first location. But, with probability  $q_2 = 1 - \Pi_1$ , he gets an additional transfer of  $(\theta_2 - \theta_1)\lambda_t M_t$  dollars. If he gets it he spends it. In general, the buyer spends  $\Delta_i = (\theta_i - \theta_{i-1})\lambda_t M_t$  dollars, immediately after getting it, with probability  $q_i = \sum_{j=1}^{S} \Pi_j$ . The end of period money supply is:  $M_{t+1} = \theta(t) \lambda_t M_t$ .<sup>1</sup>

The seller-producer (a young agent) stays in one location. To simplify, I assume that there is a single price-taker seller in each location and at each round of trade a single buyer may appear in his location. It is assumed that a buyer does not visit a given location more than once. This prohibits trade in contingent contracts and forces agents to complete their transactions before they have full information about the realization of demand. Since all locations are the same, I focus on the point of view of a representative seller at a representative location. From his point of view demand arrives in batches: The first buyer that arrives spends  $\Delta_1$  dollars and disappears. Then we have two possibilities: Either trade for the period ends or a second buyer arrives (with probability  $q_2$ ) and spends  $\Delta_2$  dollars. This process stops after  $\tilde{s}$  batches of dollars arrive, where  $\tilde{s}$  is a random variable that may take the realizations  $1, \ldots, S$ . Thus the seller is uncertain about the number of batches that will arrive and not about the size of each batch. This is illustrated by

<sup>1</sup> We may think of  $\theta$  as "control errors" of the central bank. The targeted money supply is  $\lambda_t M_t$ , and the post-transfer money supply is:  $M_{t+1} = \theta(t)\lambda_t M_t$ . It is assumed that the distribution of  $\theta$  does not depend on the targeted money supply ( $\lambda M$ ) and the rate of change in the target:  $d(\lambda M)_t = \ln(\lambda_t M_t) - \ln(\lambda_{t-1} M_{t-1})$ . Thus, the probability that the post-transfer money supply ( $\theta \lambda M$ ) will be 0.9 when the target ( $\lambda M$ ) is 1 is the same as the probability that the post-transfer money supply will be 90 when the target is 100. Figure 1. (In our special case the number of batches is equal to the number of buyers).

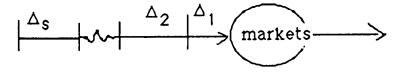


Figure 1

The seller is a price-taker and knows that he can sell to the first buyer at the dollar price  $P_1$ . He can sell to the second buyer, if he arrives, at the dollar price  $P_2 > P_1$ , and so on.<sup>2</sup> He chooses total capacity before the beginning of trade and makes a contingent plan as to how to utilize it. He may plan for example to utilize and sell 10% of his capacity to the first buyer, 20% to the second buyer, if he arrives, and so on. This program maximizes his expected utility (to be specified shortly). It is important to realize that the seller prefers to sell rather than not to sell but if buyers fail to appear sale does not take place. The seller's choice takes this possibility into account and the contingent plan is dynamically consistent.

<sup>&</sup>lt;sup>2</sup> The airline industry may serve as an example. The airline can sell tickets for a certain flight well in advance at a relatively cheap price. It can also sell high price tickets to last minute travelers. But the last minute travelers may not arrive and therefore the airline makes on average the same revenue from each type of ticket.

Capacity can be costlessly moved from one state of nature to another. The seller may also change the allocation of capacity (the contingent plan) during trade, as uncertainty unravels, but because the plan is dynamically consistent he does not have an incentive to do so. The decision to reserve capacity (not to utilize all of it for the first buyer) can be described as speculating on the possibility of a price rise following a further cash injection.

This is modelled as a sequence of Walrasian spot markets, one after each new injection of cash. The first buyer buys in market 1 that opens with certainty. If the second arrives he buys in market 2 and so on. The seller chooses total capacity and allocates it to the S markets. The dollar price in market s at time t is  $P_{st}$ . Dollars that arrive earlier buy in markets with lower indices because in equilibrium:

 $P_{1t} < P_{2t} < \ldots < P_{St}$ . Thus, there are S markets in the representative location. The first market opens with certainty. The second market opens if  $\theta(t) \ge \theta_2$ . In general, market s opens if  $\theta(t) \ge \theta_3$ . The process from the seller's point of view is described by Figure 2.

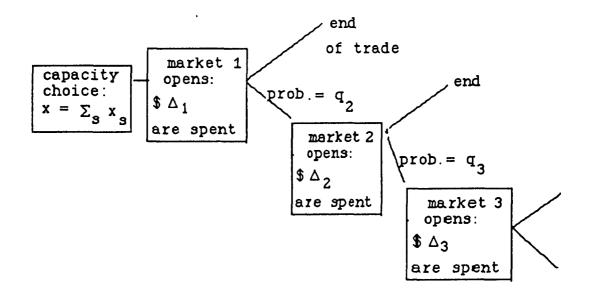
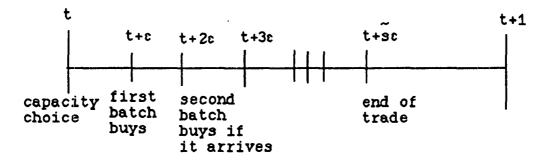


Figure 2

We may also describe the trading process for period t in real time. The first batch of purchasing power arrives at t +  $\varepsilon$  with certainty. The second arrives at t +  $2\varepsilon$  with probability q<sub>2</sub>. The third may arrive at t +  $3\varepsilon$ , only if the second batch arrived and so on. The uncertainty is about the date, t+ $\tilde{s}\varepsilon$  < t+1, at which the process will end and not about the amount that will be spent at each sub-period. This is illustrated by Figure 3.





It is assumed that  $\varepsilon$  is arbitrarily small, and therefore the econometrician cannot distinguish goods by the time of sale. I distinguish among the various goods by the events that will lead to a sale, rather than by the time of delivery.

It takes one unit of labor to produce one unit of capacity. The representative young agent at time t, has a utility function:

(1) 
$$c_{t+1} - v(x_t)$$
;

where lower case letters are used to denote quantities,  $c_{t+1}$  is the expectation at time t of consumption at time t+1 and  $x_t$  is the total amount of work (capacity). The function v() describes the disutility from work. It is assumed that v() is twice differentiable with v'(x) > 0 for all x > 0, v'' > 0 everywhere and v'(0) = 0. It is assumed that there is some limit on labor supply which is normalized to unity (24 hours if we think of a day as our period), and when this limit is reached, the marginal cost of increasing labor is infinity:  $v'(1) = \infty$ .

The representative young agent at time t chooses total capacity,  $x_t$ , and allocates it among the S markets before the beginning of period t trade. Capacity allocated to market s is denoted by  $x_{st}$  ( $\sum_s x_{st} = x_t$ ) and is utilized if market s opens. For simplicity, I assume that it is costless to convert capacity into output, and storage is not possible.<sup>3,4</sup>

It is assumed that the seller forms point estimates of the prices next period for each realization of the current period  $\theta$ : When in the current period j markets were opened, the seller expects the price in next period's market i, to be  $P_{it+1}^{j}$ . A dollar earned this period will entitle its owner to a sequence of transfers next period and as a result to the expenditure of  $(\theta_i - \theta_{i-1})\lambda_{t+1}$  dollars in next period's market i if it opens

- <sup>3</sup> We may think of a restaurant that prepares a certain number of meals that can be served during lunch hour. This is the choice of capacity. It is assumed that serving the meal is costless and if the meal is not served it is thrown into the garbage. In an earlier draft (Eden, 1986) I show that the introduction of costly storage will lead to a lag in the effect of money on output. For a model which also includes a variable factor of production, see Eden (1990).
- <sup>4</sup> An alternative interpretation would be to use output instead of capacity and sales instead of output. Under this interpretation a national income accountant would find that net national product was equal to sales since unsold goods are fully depreciated: A meal prepared and not sold, does not add to NNP.

(i = 1,...,S). The expected purchasing power of a dollar earned in the current period if exactly j markets opens in the current period is therefore:  $\sum_{i=1}^{S} q_i(\theta_i - \theta_{i-1}) \lambda_{t+1} / p_{it+1}^{j}$ . The probability that exactly  $j \ge s$  markets will open given that market s opens is  $(\Pi_j/q_s)$ . The expected purchasing power of a dollar earned in market s (i.e., given that market s opens) is:

(2) 
$$Z_{st} = \sum_{j=s}^{S} (\Pi_j/q_s) \sum_{i=1}^{S} q_i (\theta_i - \theta_{i-1}) \lambda_{t+1}/P_{it+1}^{j}$$

The expected real revenue from a unit of capacity utilized in market s is  $P_{st}Z_{st}$  and the expected real revenue from a unit allocated to market s is:  $q_sP_{st}Z_{st}$ . The representative young agent's problem is to choose the capacities  $x_{st}$  which solve:

(3) 
$$\max \sum_{s} q_{s} P_{st} Z_{st} x_{st} - v (\sum_{s} x_{st}).$$

The first order conditions for an interior solution to (3) require the expected real revenue per unit of capacity to equal marginal cost and thus be the same across markets:

(4) 
$$q_s P_{st} Z_{st} = v'(x_t)$$
 for all s.

Since  $(\theta_s - \theta_{s-1})\lambda_t M_t$  is the amount of money per seller that will be spent in market s if it opens, the market clearing conditions are:

.

(5) 
$$(\theta_s - \theta_{s-1})\lambda_t M_t / P_{st} = x_{st}$$
, for all s.

Equilibrium requires (4) and (5).

Let,  $M_{st+1} = \overline{\theta}_{s}\lambda_{t}M_{t}$ , where  $\overline{\theta}_{s} = 1/\{\sum_{j\geq s} (\Pi_{j}/q_{s})/\theta_{j}\} = 1/\{E((1/\theta)| \ \theta \geq \theta_{s})\}$ . This is approximately the expected post-transfer money supply given that market s opens, because  $\overline{\theta}_{s} \approx E(\theta| \ \theta \geq \theta_{s})$ .<sup>5</sup> I focus on a steady-state equilibrium in which dollar prices are proportional to the expected level of the post-transfer money supply:

$$P_{st} = P_s M_{st+1},$$

where ps is the real price in market s.

The discount factors Z<sub>st</sub> are:

<u>Claim</u>: Under (6),  $Z_{st} = z/M_{st+1}$ , where  $z = \sum_i q_i (\theta_i - \theta_{i-1}) / \overline{\theta_i} p_i$ .

The proof of the Claim as well as all other proofs is in the Appendix. Note that z is a weighted sum of  $1/p_s$ . The weights will be discussed shortly. We can think of z as the expected purchasing power of a

<sup>&</sup>lt;sup>5</sup> Note that because of Jensen's inequality, an increase in the variability of  $\theta$  has the same effect on  $M_{st+1}$  as a reduction of the mean of  $\theta$ . This somewhat puzzling result is discussed in Eden (1976) and Brock and Scheinkman (1980). It explains why increasing the variability of the money supply may be welfare improving when the transfer payments are in a lump sum form.

dollar when the expected post-transfer money supply is unity. When  $M_{st+1}$  is increased the purchasing power of a dollar, Z, is proportionally reduced.

Substituting the Claim and (6) in the equilibrium conditions (4) and (5) leads to the following steady-state conditions:

(7) 
$$q_{s}p_{s}z = v'(x);$$

(8) 
$$(\theta_s - \theta_{s-1}) / \theta_s p_s = x_s.$$

Note that the expected real revenue of a unit of capacity allocated to market s,  $q_s p_s z$ , is independent of the targeted money  $\lambda M$  and of its rate of change:  $d(\lambda M)_t = ln\lambda_t M_t - ln\lambda_{t-1} M_{t-1}$ .

A solution  $(p_1, p_2, ..., p_S, x_1, x_2, ..., x_S)$  to (7) and (8) is a symmetric steady-state equilibrium. Let,  $\mu_j = x_j/x$  denote the fraction of total capacity allocated to market j. I now turn to show,

#### Theorem:

There exists a unique symmetric steady-state equilibrium with the following properties:

(a) high realizations of  $\theta$  are associated with high rates of capacity utilization and output;

(b) capacity and output are independent of the targeted money supply ( $\lambda M$ ) and its rate of change ( $d\lambda M$ );

(c)  $q_3p_3 = p_1$  for all s;

(d) 
$$\mu_{j} = \{q_{j}(\theta_{j} - \theta_{j-1})/\theta_{j}\} / \{\sum_{s} q_{s}(\theta_{s} - \theta_{s-1})/\theta_{s}\};$$

(e)  $z = x \sum_{s} q_{s} \mu_{s};$ 

(f) v'(x) =  $(z/x)\sum_{s} \Pi_{s}(\theta_{1}/\theta_{s});$ 

(g) eliminating the uncertainty about the end of period money supply (replacing the random variable  $\theta$  by a scalar) will increase capacity, average capacity utilization, and welfare.

(a) and (b) are obvious (but nevertheless important). (c) says that after deflating by the appropriate expected post-transfer money supply, the expected revenue per unit of capacity must be the same in all markets. (d) says that the fraction of capacity allocated to market j,  $\mu_j$ , is proportional to the expected percentage of the post-transfer money which will be spent in market j:  $q_j(\theta_j - \theta_{j-1})/\tilde{\theta}_j$ .

Substituting the market clearing conditions (8) in the expression for z, yields  $z = \sum_{s} q_{s}x_{s}$ . Thus, z is the equilibrium level of expected

consumption. Since in equilibrium  $M_{st+1}$  buys the entire total consumption,  $Z_{st} = z/M_{st+1}$  is the expected purchasing power of a dollar.

The equilibrium expected real wage (f), is equal to the average capacity utilization ((z/x) =  $\sum_{s}q_{s}\mu_{s}$ ) times the average fraction of the proceeds from the sales in the first market in the post transfer money supply  $\sum_{s} \Pi_{s}(\theta_{1}/\theta_{s})$ . To see why this is the case, let us think of an hypothetical case in which the seller allocates his entire capacity to market 1. (Since in equilibrium the representative seller is indifferent to the way he allocates capacity across markets, this is a way of deriving the expected real wage). If in the current period only market 1 opens, he will own the entire money supply (per seller) and this will buy on average  $x(\Sigma_s q_s \mu_s)$  units next period. The expected amount of future consumption per unit of x is  $(\Sigma_s q_s \mu_s)$  in this case. If in the current period, two markets are opened the representative seller will own only a fraction of  $( heta_1/ heta_2)$  of the post transfer money supply and will therefore buy on average  $(\theta_1/\theta_2) \times (\sum_s q_s \mu_s)$  units next period or  $(\theta_1/\theta_2) (\sum_s q_s \mu_s)$  per unit of x. Therefore the unconditional expected future consumption per unit of x supplied to the first market is equal, in equilibrium, to  $\Sigma_s \Pi_s(\theta_1/\theta_s) (\Sigma_s q_s \mu_s)$  and this is equal to the marginal cost, v'(x).

From (f) it is clear that the expected real wage is maximized (attaining the level of unity) in the absence of uncertainty about the money supply. This leads to (g).

<u>Remark:</u> It is shown in Eden (1990) that sellers do not have an incentive to change prices (or change the way they allocate their capacity among the remaining markets) during the trading process. This is also true here.

I now characterize the implied behavior of prices and output. I use  $dx_t = lnx_t - lnx_{t-1}$ .

#### Corollary 1:

(a) The distribution of relative prices,  $P_{st} / P_{1t}$ , is independent of the rate of change of the targeted money supply,  $d(\lambda M)_t = \ln \lambda_t M_t - \ln \lambda_{t-1} M_{t-1}$ ; (b) The rate of change of nominal prices is equal to the rate of change of the targeted money supply:  $dP_{st} = d(\lambda M)_t$  for all s; (c) The rate of change of real output is an increasing function of the rate of change of the actual money supply, minus the rate of change of the targeted money supply:  $dy_t = F(dM_{t+1} - d(\lambda M)_t)$ , where F() is strictly increasing.

Parts (a) and (b) of the Corollary follow directly from (6). Part (c), which follows from  $dM_{t+1} - d(\lambda M)_t = \ln\theta_t - \ln\theta_{t-1}$ , says that output grows if the actual rate of change is higher than the rate of change of the target. Here anticipated money is:  $dM_{t+1}^e = d(\lambda M)_t + \ln\theta_{t-1}$ , and unanticipated money is:  $dM_{t+1} - dM_{t+1}^e = \ln\theta_t$ . It therefore follows that only unanticipated money affects real output. This hypothesis has been tested by Barro (1977), Barro and Hercowitz (1980), and Boschen and Grossman (1982), among others. Extension to many goods: The distinction between the physical characteristics of the goods can be made at the cost of adding an index. Assume that there are G goods indexed g. Markets are now indexed by the physical characteristic of the good and the event that will lead to the opening of the market. There are GS markets. In market gs there is trade in good g and it opens when  $\theta \ge \theta_s$ . As before, it costs one unit of labor to produce a unit of capacity, and total capacity, x, is allocated among the GS markets. Capacity allocated to market gs is denoted by  $x_{sg}$  ( $\Sigma_s \Sigma_g x_{sg} = x$ ). Because of the constant returns to scale assumption, in equilibrium the prices of all the G goods that will be sold if  $\theta \ge \theta_s$  are the same and will be denoted by  $P_s$ .

The representative buyer's utility function is  $u(c_1, \ldots, c_G)$ , where ( $c_1, \ldots, c_G$ ) denotes his consumption of the G goods. It is assumed that u()is strictly monotone and strictly quasi concave and all the G goods are normal. Let  $V(W) = \max u(c_1, \ldots, c_G)$ ; s.t.  $\Sigma_g c_g = W$ . It is assumed that V(W) = W. Thus the consumer is risk neutral.

The problem of the representative young agent is:

(3') max 
$$\sum_{s} \sum_{q} q_{s} P_{st} Z_{st} x_{sqt} - v (\sum_{s} \sum_{q} x_{sqt})$$
.

As before, I focus on nominal prices that satisfy (6). The equilibrium condition (7) is now:

(7') 
$$q_s p_s z = v' (x = \sum_s \sum_g x_{sg}).$$

When markets with index s are opened, the representative buyer chooses  $(c_{s1}, \ldots, c_{sG})$  which solve:

max 
$$u(\sum_{j=1}^{s-1} c_{j1} + c_{s1}, \dots, \sum_{j=1}^{s-1} c_{jG} + c_{sG})$$
  
s.t.  $P_s \sum_g c_{sg} = (\theta_s - \theta_{s-1}) \lambda M;$ 

where  $\sum_{j=1}^{s-1} c_{jg}$  denotes the total quantities of good g which were bought in previous rounds. Let  $c_{sg}^{*}$  denote the solution to the above problem. Market clearing requires:

(8') 
$$c_{sg}^* = x_{sg};$$
 for all s and g.

Equilibrium requires (7') and (8').<sup>6</sup> Because income elasticity is the ratio of marginal to average propensity to consume, it follows that:

<u>Corollary 2:</u> Positive money surprises increase the share of goods with income elasticity greater than 1, in total output.

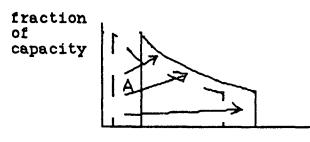
<sup>&</sup>lt;sup>6</sup> Since nominal spending is the same as in the single good case it must be that  $\sum_{g} x_{sg} = x_{s}$ , where  $x_{s}$  is total capacity allocated to market s in the single good case. Thus the total capacity allocated to markets with index s does not depend on the number of goods.

This is different from the prediction of the market clearing with asymmetric information literature, which emphasizes the role of supply elasticities. See Hercowitz (1981), for example.

# ASYMMETRIC EQUILIBRIA

Equilibrium conditions (7) and (8) determine the total capacity allocated to each market  $(x_s)$  but do not determine the number of sellers in each market. This indeterminacy allows for many asymmetric equilibria. I now describe one possibility in detail.

I assume that  $(\theta_j - \theta_{j-1}) = 1$  for all j. This leads to a distribution of real prices which is skewed to the left, as shown by the solid line in Figure 4.<sup>7</sup>



real price

Figure 4

<sup>7</sup>  $\mu_j \bar{\theta}_j / \mu_s \bar{\theta}_s = q_j / q_s = P_s / P_j$  implying  $\mu_j \bar{\theta}_j P_j = \mu_s \bar{\theta}_s P_s$ . Since  $\bar{\theta}_s P_s$  are monotonically increasing in s, the  $\mu_s$  are monotonically decreasing.

The equilibrium distribution of prices is achieved by variations of prices across sellers: A fraction  $\mu_s$  of all sellers participates in market s. In a more realistic model, in which sellers live for many periods, not all sellers will change their nominal prices in response to an increase in the targeted money supply because most sellers are compensated for the reduction in the real price by the increased probability of making a sale.<sup>8</sup> Sellers will change their nominal price only when the real price falls below  $p_1$ , because in this case the probability of making a sale remains unity.

In terms of Figure 4, changes in the targeted money supply shift the distribution of real prices from the solid line to the broken line. To restore equilibrium only the sellers who are in box A change their nominal prices. Thus nominal price changes occur in discrete jumps.

As indicated by the arrows, the change in nominal price is different across sellers. The change in prices must be coordinated across sellers. If there are many sellers, this can be achieved if all sellers in box A follow a mixed strategy. Therefore, the magnitude of price changes made by the same seller may vary across time, as illustrated by Figure 5. This accounts for the observation of many small changes in prices (see, Carlton, 1986, Kashyap, 1991, and Hanoch and Gal-Yam, 1985).

<sup>&</sup>lt;sup>8</sup> Alternatively, we may assume that the selling itself is done by longlived marketing firms which, minimize price changes subject to a zero profit constraint.

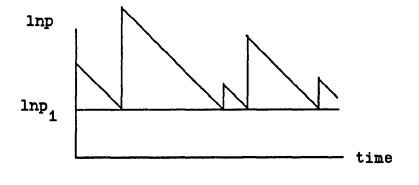


Figure 5

### CONCLUSIONS

Money surprises have real effects. This result is obtained here in the absence of price rigidity and asymmetric information about the money supply.

Measured productivity is procyclical: It goes up when the money supply is up because more output is obtained for the same inputs (see Hall, 1988 and 1990, Rotemberg and Summers, 1990, Rotemberg and Woodford, 1991, Eden, 1990, and Eden and Griliches, 1993). In Lucas (1972) production is always efficient and monetary shocks cause no variations in measured productivity.<sup>9</sup>

Money surprises have a greater effect on the production of goods with high income elasticity. According to the market clearing with asymmetric

 $^9$  I owe this point to the discussion in Lucas and Woodford (1992).

information approach, supply elasticities are important for determining the size of the money surprise effect (see Hercowitz, 1981, for example).

Uncertainty about the money supply causes lower capacity, lower average capacity utilization and, as a result, lower average output.<sup>10</sup> This is not true in the fixed price models: When prices are fixed in advance at the level that will clear the market in the absence of uncertainty, and producers are committed to sell any amount demanded, monetary uncertainty affects the variance of output but not its mean.

When inflation erodes the real price but the real price remains in the equilibrium range, a seller is compensated by the increase in the probability of making a sale and has therefore no incentive to change his nominal price. Therefore, it is possible to get many asymmetric equilibria in which nominal price changes occur in discrete jumps and are not synchronized across sellers.

Changes in anticipated inflation do not affect the distribution of real prices. I argue in a different paper (Eden, 1993) that this is consistent with the findings that the variance of nominal price changes is higher in periods of high inflation.

<sup>&</sup>lt;sup>10</sup> Using a similar framework, I argue in Eden (1986), that distinguishing between money and bonds by imposing a cash in advance constraint may lead to an increase in average consumption per capita (and to a Pareto improvement) because it reduces the uncertainty about demand.

#### APPENDIX

<u>Claim</u>: Under (6),  $Z_{st} = z/M_{st+1}$ , where  $z = \sum_{i} q_{i}(\theta_{i} - \theta_{i-1}) / \overline{\theta}_{i}p_{i}$ .

<u>Proof</u>: When exactly j markets open this period,  $M_{t+1} = \theta_j \lambda_t M_t$ . Using (6) this leads to:

(A1) 
$$P_{it+1}^{j} = P_{i}\overline{\theta}_{i}\lambda_{t+1}(\theta_{j}\lambda_{t}M_{t}).$$

Substituting (A1) in (2) and rearranging leads to:

(A2) 
$$z_{st} = \{ \sum_{j=s}^{S} \Pi_{j} / (q_{s}\theta_{j}\lambda_{t}M_{t}) \} \{ \sum_{i=1}^{S} q_{i}(\theta_{i} - \theta_{i-1}) / p_{i}\overline{\theta}_{i} \}.$$

The Claim follows by substituting the definition of  $M_{st+1}$  in (A2).

#### Theorem:

There exists a unique symmetric steady-state equilibrium with the following properties:

(a) high realizations of  $\theta$  are associated with high rates of capacity utilization and output;

(b) capacity and output are independent of the targeted money supply ( $\lambda M$ ); and its rate of change ( $d\dot{\lambda}M$ );

(c)  $q_s p_s = p_1$  for all s;

(d) 
$$\mu_{j} = \{q_{j}(\theta_{j} - \theta_{j-1})/\theta_{j}\} / \{\Sigma_{s} q_{s}(\theta_{s} - \theta_{s-1})/\theta_{s}\};$$

(e)  $z = x \sum_{s} q_{s} \mu_{s};$ 

(f) v'(x) =  $\sum_{s} \Pi_{s} \left( \theta_{1} / \theta_{s} \right) \left( \sum_{s} q_{s} \mu_{s} \right)$ ;

(g) eliminating the uncertainty about the end of period money supply (replacing the random variable  $\theta$  by a scalar) will increase capacity, average capacity utilization, and welfare.

<u>Proof:</u> Since a solution to (7) and (8) does not depend on  $\lambda M$ , the supplies  $(x_1, x_2, \ldots, x_S)$  are independent of  $\lambda M$ . This implies that capacity and consumption are independent of  $\lambda M$ . Thus we have shown (b).

Using the first order conditions (7) leads to:

(A3) 
$$P_{ij}/P_{s} = q_{s}/q_{j}$$
,

which implies (c).

The market clearing conditions (8) lead to:

(A4) 
$$P_{j}/P_{s} = \{(\theta_{j}-\theta_{j-1})/(\theta_{s}-\theta_{s-1})\}(\overline{\theta}_{s}/\overline{\theta}_{j})(\mu_{s}/\mu_{j})$$

(A3) and (A4) imply:  $q_s/q_j = \{(\theta_j - \theta_{j-1})/(\theta_s - \theta_{s-1})\} (\overline{\theta}_s/\overline{\theta}_j)(\mu_s/\mu_j)$ ,

which leads to:

(A5) 
$$\mu_{s} = \mu_{i}(q_{s}/q_{i}) \{ (\theta_{s} - \theta_{s-1}) / (\theta_{i} - \theta_{i-1}) \} (\theta_{i}/\theta_{s}) = \mu_{i}a_{si}.$$

Substituting (A5) in  $\sum_{\rm S}\,\mu_{\rm S}$  = 1, leads to:

;

(A6) 
$$\mu_j = 1/\{1 + \Sigma_{s\neq j} \mid a_{sj}\} = \{q_j(\theta_j - \theta_{j-1})/\tilde{\theta}_j\} / \Sigma_s q_s(\theta_s - \theta_{s-1})/\tilde{\theta}_s.$$

Thus we have shown (d).

To show (e) I substitute the market clearing conditions (8) in the expression for z. This leads to:

$$(A7) z = x \sum_{s} q_{s} \mu_{s} .$$

Using (A7) and the market clearing conditions (8) leads to:

(A8) 
$$p_1 z = p_1 x \Sigma_3 q_3 \mu_3 = (\theta_1 - \theta_0) / \tilde{\theta}_1 \Sigma_3 q_3 \mu_3$$
.

Substituting (A8) in (7) for s = 1, leads to:

(A9) 
$$w = [(\theta_1 - \theta_0)/\overline{\theta}_1] \Sigma_s q_s \mu_s = v'(x),$$

where w is the equilibrium expected real wage. Thus we have shown (f).

Since (A6) implies that the left hand side of (A9) is a strictly positive expression involving only the probabilities and the  $\theta$ 's, the assumptions about v imply a unique positive solution for x. We can now use (A7) and (A8) to derive unique and positive solutions for z and p<sub>1</sub>. Given  $p_1$  we can use (A3) to solve for all the  $p_s$  and (A5) to solve for all the  $x_s$ . Thus there exists a unique symmetric steady state equilibrium.

To show (g), note that the expected real wage, w, is maximized in the absence of monetary shocks (i.e., when  $\mu_1 = 1$  and  $\mu_s = 0$  for all s > 1). Now, (A9) implies that equilibrium capacity, x, is strictly increasing in w. The welfare of the representative consumer, max<sub>x</sub> wx - v(x), is also strictly increasing in w. Therefore, both are maximized in the absence of monetary shocks when w = 1. Consumption is maximized by eliminating the monetary shocks because capacity is maximized and everything is

consumed.

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