# **Research Department**



## Bank of Israel

# A Constrained Dynamic Model for Macroeconomic Projection in Israel\*

Yosi Djivre and Yossi Yakhin

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\* Research Department, Bank of Israel. http://www.boi.org.il

Joseph Djivre – Phone: 972-2-655-2607; Email: joseph.jdjivre@boi.org.il Yossi Yakhin – Phone: 972-2-655-2616; Email: yossi.yakhin@boi.org.il

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## A Constrained Dynamic Model for Macroeconomic Projection in Israel

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#### Abstract

In this paper we developed an empirical small open economy model for macroeconomic projection of the Israeli economy. The model is estimated for the period between 1997Q3 and 2008Q3. Our primary purpose is inflation forecasting and the evaluation of the effect of monetary policy on prices. The endogenous variables in the model are: growth of business sector product, the nominal effective depreciation of the Shekel, the CPI inflation rate, and the Bank of Israel policy interest rate. Our empirical strategy constitutes a compromise between "letting the data speak" and imposing structural constraints on the coefficients of the model – as suggested by standard New-Keynesian theory. To that end we estimated four models: a simple unrestricted VAR (Vector Auto-Regression), a model with selective exclusion restrictions on the exogenous variables in each equation, a model with structural constraints (in addition to the exclusion restrictions), and finally the structural model is augmented by rational expectations. The performance of the model improved, both in term of forecasting ability and in terms of its dynamic properties (as reflected by the Impulse Response Functions), with the imposition of structural constraints. However, following the imposition of rational expectations its forecasting performance deteriorated, leaving the model with structural constraints as the version generating the best results. Finally, analyzing the dynamic responses of the model to structural shocks, we conclude that the imposition of additional long-run restrictions is required.

## מגבלות מבניות במודל דינמי לחיזוי מקרו-כלכלי בישראל

## יוסי ג'יברה ויוסי יכין

## תקציר

פיתחנו מודל אמפירי של משק קטן ופתוח לחיזוי מקרו-כלכלי של התפתחויות במשק הישראלי. המודל נאמד לתקופה 1997/III עד 2008/III מטרתנו העיקרית היא יצירת כלי אמפירי לחיזוי האינפלציה ולניתוח השפעת המדיניות המוניטרית עליה. המשתנים האנדוגניים במודל הם שיעור הצמיחה של התוצר העסקי, שיעור הפיחות הנומינלי האפקטיבי של השקל, שיעור האינפלציה וריבית בנק ישראל. גישתנו האמפירית מאזנת בין מתן חופש לנתונים לומר את דברם ובין אמידה מבנית הכופה על הקשרים שבין המשתנים מגבלות בהתאם לתיאוריה הניאו-קיינסיאנית. לשם כך אמדנו ארבעה מודלים: מודל VAR (Vector Auto-Regression) ללא מגבלות, מודל עם בחירה סלקטיבית של המשתנים האקסוגניים בכל משוואה, מודל מבני עם מגבלות על הקשרים שבין המשתנים (בנוסף למגבלות על המשתנים האקסוגניים) והמודל המבני בתוספת ציפיות רציונליות. ביצועי המודל השתפרו עם הוספת המגבלות המבניות - הן מבחינת כושר החיזוי והן מבחינת התכונות הדינמיות, כפי שמשתקף מניתוח ה-Impulse Response Functions; לעומת זאת הוספת הביצועים הרציונליות פגעה בכושר החיזוי של המודל. משמע שהמודל המבני ללא ציפיות רציונליות סיפק את הביצועים הטובים ביותר. לבסוף, מניתוח התגובה הדינמית של המודל לזעזועים מבניים, מצאנו שדרושות לו גם מגבלות טווח ארוד.

## 1. Introduction

This paper presents a small open economy model for macroeconomic projection of the Israeli economy. The primary purpose of the model is inflation forecasting and the evaluation of the effect of monetary policy on prices. Our empirical strategy was to start from an unrestricted Vector Auto-Regression (VAR) model and then augment it with structural restrictions in order to bring it closer to our prior beliefs which, in turn, are driven by standard theory.

VAR models are a popular workhorse for analyzing monetary policy and for generating macroeconomic forecasts in both academic and policy circles. To keep our specification compact we estimated a four-equation system in first differences with the exception of the nominal interest rate which is in levels, while other considerations such as rest-of-the-world developments and special features of the Israeli economy (geopolitical factors and the inflation target) were captured by exogenous variables. Our four endogenous variables were: the growth rate of business sector GDP (DLYB), the depreciation rate of the effective exchange rate (DLEFX), CPI inflation (DLCPI), and the interest rate set by the Bank of Israel, BoI hereafter, (IBOI).2 This specification is very similar to that adopted in the VAR literature according to which the specification typically includes a measure of real activity, of domestic prices' CPI and/or GDP deflator, and of monetary policy; open economy models also include the nominal or the real exchange rate to the analysis. The models are usually estimated in levels in either monthly or quarterly frequency [Cushman and Zha (1997), Kim and Roubini (2000) and Mountford (2005)], but also in first differences, similarly to our model [Gerlach and Smets (1995), Peersman and Smets (2001) and Bjornland (2005)]. In the first difference models the nominal interest rate, which is usually considered to be the monetary policy variable, is still expressed in levels. In addition to the aforementioned variables, monetary aggregates, total bank reserves and non-borrowed reserves are also included in the endogenous variables, with shocks to these variables being sometimes considered as monetary policy shocks [Christiano et al. (1998) and Leeper et al. (1996)]. The exogenous variables typically include a commodity price index or foreign prices, in the case of open economies, so as to disentangle monetary policy shocks from endogenous policy reactions to signs of inflation pressures reflected in such price indices. Absent such variables, information about higher future prices to which the central bank

Sims (1980a), Bernanke and Blinder (1992), Christiano et-al (1996), Bernanke and Mihov (1998), among others, use VARs for the analysis of monetary policy in the US. Small open economy VARs are applied by Camarero et-al (2002) for the Spanish economy, Bjornland (2005) for Norway, and Dungey and Pagan (2008) for Australia, to name a few.

Since we use variables in log first differences, we express them as deviations from steady state in order to impose some long-run properties that are lost when differencing the data.

reacts turns out as a component of the monetary policy structural shock leading to the price puzzle, i.e. an increase in the price level following an unexpected monetary policy contraction [Sims (1992)]. In the case of small open economies the exogenous variables also include foreign interest rates and indices of foreign economic activity when the latter affects domestic activity [Cushman and Zha (1997) and Peersman and Smets (2001)]. In this respect our model conforms to the specifications in recent literature in that it includes measures of imported inflation, foreign activity and foreign interest rates.

The identification constraints imposed in the estimation of these models vary from short run zero-constraints on the contemporaneous coefficients of the endogenous variables to long run constrains on the effect of various structural shocks on some of the endogenous variables [Peersman and Smets (2001)]. These long run constraints usually boil down to the inability of nominal, monetary policy or aggregate demand shocks to affect permanently real variables. According to the VAR tradition the identification restrictions are not usually applied on the structural coefficients of the lagged endogenous variables, as it is usually the case in the estimation of simultaneous equation models, and when they are, it is generally in the context of long run restrictions. Contrary to this tradition we imposed such restrictions in our estimation in an effort to give some structure to our model, linking, for instance, the nominal exchange rate foreign prices and domestic inflation so as to obtain a real exchange rate effect on economic activity. In a similar way we did also impose restrictions on the nominal interest rate and on inflation so as to obtain the real interest effect on economic activity. Moreover our estimation results were consistent with the interest rate differential between domestic and foreign interest rates affecting the nominal exchange rate depreciation. With respect to the short term zerorestrictions some regularities stand out in surveyed VAR literature which can be summarized as follows: The monetary policy variable, which is usually a short-term nominal interest rate, is some times affected on impact by economic activity and inflation [Bjornland (2005), Gerlach and Smets (1995)], and some times it is not, on the ground that these variables are not econometrically observable by the policy maker especially in the case of monthly models [Kim and Roubini (2000), Christiano et al (1998), Sims and Zha (2006) and Cushman and Zha (1997)]. Regardless of the endogeneity or exogeneity of the monetary policy variable it is almost always assumed that unexpected changes in monetary policy affect economic activity with a lag even though they may sometimes affect prices contemporaneously (Gerlach and Smets (1995)). But even in these cases inflation does not affect contemporaneously economic

activity. In the open economy models the nominal exchange rate or the real exchange rate is also introduced as an endogenous variable and it is almost always assumed that this variable is contemporaneously affected by shocks to all the other endogenous variables. However shocks to the exchange rate do not affect prices on impact except in the model of Peersman and Smets (2001) with long-run restrictions.

Even though our model has been constructed for forecast purposes, relying only on its reduced form constrains considerably its ability to evaluate different scenarios, especially monetary policy scenarios. The reason is that reduced form specifications are silent about the contemporaneous effect of policy on the rest of the economy, and if one believes that policy does have a contemporaneous impact<sup>3</sup> one must specify a Structural VAR (SVAR) in order to capture this effect. We opted therefore for the estimation of a structural model.

We used data in quarterly frequency for the period between 1997q3 and 2008q3 and estimated different specifications of our system by FIML. These specifications varied in the structure we imposed on the model.

The basic structure we imposed on the model is consistent with output growth being contemporaneously exogenous, with the depreciation rate being affected by the current interest rate, with inflation being affected by both contemporaneous output growth and the depreciation rate, and with the interest rate being affected by current output growth and inflation. As we proceeded in imposing more structure on the model some of these assumptions were relaxed.

The first version of the model is a standard VAR with all exogenous variables entering all four equations. We refer to this version as the "unrestricted model", hereafter UR. In the second version we imposed exclusion restrictions on the exogenous variables introducing them selectively in the various structural equations. We refer to this version as the "exogenous variables constrained model", hereafter EXOGR. In the third version of the model, hereafter the "fully constrained model", ENDOGR, we imposed additional restrictions on structural coefficients to capture variations in the real interest rate, in the real exchange rate and in the interest rate spread between domestic and foreign interest rates.

We used these specifications in order to evaluate the importance of structural restrictions in terms of dynamic simulation, out of sample performance, and impulse response functions (IRF).

Most probably policy interest rate affects the exchange rate even in high frequency data.

<sup>&</sup>lt;sup>4</sup> All restrictions are presented explicitly in Section 3.

As an extension, we also experimented with a version that incorporates rational expectations to the model.

In all estimated versions of the model we also imposed long-run restrictions which ensure that the model converges to a steady state equilibrium that is consistent with economic theory and stylized facts for the Israeli economy. Specifically, in the long-run the model is constrained to converge to the inflation target (2 percent), the business sector GDP growth rate to 4 percent per annum, and we also assumed that the relative PPP and interest rate parity conditions hold.

With respect to both the definition of the endogenous variables and the exogenous variables and the identification restrictions the fully constrained version of our model comes closest to the first difference specification of Peersman and Smets (2001) with long run restrictions. Likewise to their paper we have assumed that the nominal interest rate does not affect economic activity on impact it does however affect inflation mainly through the effect of the interest rate on the exchange rate depreciation, the contemporaneous exchange rate depreciation effect on inflation being a stylized characteristic of the Israeli economy for decades. In our case however not only do changes in the nominal exchange rate affect inflation on impact but they also allowed to affect economic activity through a real exchange rate effect.

In spite of the different identification restrictions most of the models share common traits with respect to their reaction to monetary policy shocks. More precisely, a monetary policy contraction leads to higher interest rates, lower economic activity and falling prices. The fall in prices is usually more protracted than the fall in economic activity [Sims and Zha (2006)]. This pattern of price evolution is also a characteristic of open economies [Cushman and Zha (1997) for Canada, Kim and Roubini (2000) for the non-US G-7 economies, Bjornland (2005) for Norway, Buckle et al. (2002) for New Zealand, Anzuini and Levy (2007) for the Czech Republic, Hungary and Poland and Peersman and Smets (2001) for the Euro area] and it is accompanied by an exchange rate appreciation, reflected also in a real exchange appreciation. However the estimation results differ with respect to the evolution of the exchange rate following the monetary policy shock. More precisely, according to theory, the creation of a positive interest rate differential in favor of domestic interest rates should be followed by expectations for depreciation of the domestic currency, implying an overshooting of the appreciating exchange rate on impact. This is not however the result obtained in all models;

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<sup>&</sup>lt;sup>5</sup> We did not succeed to obtain FIML estimators when imposing long run restrictions on the effect of shocks on economic activity and on the real exchange rate and this remains one of the objectives of further research.

Eichenbaum and Evans (1995), Anzuini and Levy (2007), Buckle at al (2002) and Peersman and Smets (2001)<sup>6</sup> obtain a protracted exchange rate appreciation contrary to Cushman and Zha (1997), Kim and Roubini (2000) and Bjornland (2005) whose results conform with the UIP hypothesis. Our estimation results give also rise to a nominal and real exchange rate appreciation after an unexpected monetary policy contraction and a higher real interest rate. They conform however to the UIP theory and give rise to a protracted period of exchange rate depreciation following the initial appreciation. Moreover, similarly to the surveyed literature, the inflation rate remains below its steady state level implying a protracted fall in prices. However these results are only marginally, as a result in spite of the qualitative similarity with the results obtained in similar research abroad the lack of statistical significance of the impulse responses requires some caution on our side with respect to the drawing of conclusions.

The results of the dynamic simulation of the model over the sample period indicate that the UR model outperforms the other models in terms of RMSE. This result is not particularly surprising in view of the fact that unconstrained estimation results are characterized by a better goodness of fit which is also reflected in a more accurate within-sample dynamic simulation. However, the introduction of structural restrictions improved the performance of out-of-sample projections as the ENDOGR model generated the most accurate projections, both in terms of RMSE and in terms of the variance of the forecast error and the UR model fared worse than the other two specifications. Moreover, the constrained models gave rise to coefficient signs and impulse response functions closer to our a-priori beliefs based on standard economic theory. In particular, the most disturbing results of the UR model were that a rise of the interest rate implied a depreciation of the domestic currency, and that higher inflation implied lower interest rates on impact, though this effect was insignificant and lagged inflation did lead to higher interest rates. These results are, of course, inconsistent with economic theory and were reversed in both the EXOGR and the ENDOGR models. It should be noted that the rational expectations model resulted in opposite signs to most forward looking variables, did not improve out-ofsample projections, and did not exhibit better dynamic characteristics than the ENDOGR model. In sum, we conclude that structural constraints bring the empirical results closer to theory and improve the out-of sample forecasts, while incorporating rational expectations in the model does not improve its performance.

<sup>&</sup>lt;sup>6</sup> In Peersman and Smets (2001) this is true only for the model with first differences and long run restrictions.

The constrained versions of our model also have some drawbacks. In particular, the coincident and lagged effects of business sector GDP growth on inflation were found to be insignificant. Moreover according to economic theory it is the output-gap which should affect the inflation rate and not the growth rate, even though there may exist speed limit effects which could justify the introduction of the latter in the inflation equation, but in addition to the output-gap. As a result, we experimented with a simplified specification of the gap in the inflation equation. The overall effect of the gap across its lags was positive but statistically insignificant and did not affect substantially the IRFs of the ENDOGR model.

Moreover while the GDP growth rate and the effective nominal exchange rate depreciation converge to their steady state value, the real exchange rate (in levels) and the output-gap do not.<sup>7</sup> As a result in its present specification our model should be considered as being suitable for providing forecasts for short time horizons not longer than two years.

We proceed as follows: In section 2 we present the variables of the estimated system and discuss the choice of the sample period. In section 3 we present the specification of the models and discuss their steady state properties and their constraints. In section 4 we discuss the estimation results and in section 5 we examine the robustness of the estimated coefficients and analyze the impulse response functions, the in-sample fit and the out of sample forecasts. In section 6 we present the rational expectations extension of the model and in section 7 we conclude.

#### 2. The Data

## The Endogenous Variables

The vector of endogenous variables includes the variables: *DLYB*, *DLEFX*, *DLCPI*, and *IBOI*. These variables stand respectively for the growth rate of the business sector GDP, the depreciation rate of the NIS against a basket of four foreign currencies with weights of 6% for the Japanese Yen, 13% for the British Pound, 49% the US dollar and 32% for the Euro, the CPI inflation rate and the BoI effective interest rate. Unit root tests performed for these variables led to the conclusion that the first three are stationary while the last one, the BoI interest rate, is not (see Appendix 1). The non-stationarity of the interest rate may reflect the disinflation process in Israel which was accompanied by a gradual fall of the nominal interest rate over time. 8 It should

After we imposed additional restrictions in this direction the solver did not attain convergence.

<sup>&</sup>lt;sup>8</sup> It may also reflect type II error as both inflation and the real interest rates reject the unit root hypothesis.

be noted that unit root tests for the interest rate net of the inflation target, which captures this falling trend, provide mixed results: the ADF test rejected the hypothesis of a unit root at the 5 percent significance level but the PP test could not reject it at any reasonable significance level. We nevertheless decided to use the interest rate in levels when estimating our models, assuming it is stationary.

Although many papers in the literature estimate VAR models in levels, despite non-stationarity of the endogenous variables, we nevertheless decided to specify the model in first differences as in Gali (1992), Gerlach and Smets (1995), Peersman and Smets (2001) and Bjornland (2005).

## The Exogenous Variables

The vector of exogenous variables includes the variables *TARGET*, *DSECURITY*, *EFGR*, *DLEFCPI*, *IEF* and dummies. These variables stand respectively for: the average inflation target for the next four quarters, the first difference of tourist entrances to Israel relative to the US – aimed to capture the security and political environment in Israel, <sup>10</sup> the foreign GDP growth rate, calculated as a weighted average of growth rates in the aforementioned four economies with the same weights as in the effective exchange rate depreciation, and similarly weighted foreign CPI inflation and foreign inter-bank interest rates respectively.

The variables *EFGR* and *DLEFCPI* and *IEF* entered the structural equations of the model both contemporaneously and with a lag of one quarter while the remaining two variables entered only contemporaneously. Seasonal dummies include dummy variables for the second and third quarters, *DUMQ2* and *DUMQ3* respectively. We also introduced a dummy variable for the fourth quarter of 1998, *DUM98Q4*, to account for the LTCM crisis, in all equations.

## The Sample

We used data in quarterly frequency. Our sample starts in the third quarter of 1997 and ends in the third quarter of 2008. We chose the third quarter of 1997 as the sample's first observation since it is common to refer to that point in time as a turning point in the monetary policy and the *de-facto* adoption of a flexible exchange rate regime with the cessation of the Bank of Israel intervention in the foreign exchange market.

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<sup>9</sup> Both tests allow for a constant and a linear time trend as the interest rate still display a downward path even after subtracting the inflation target. The ADF test includes one lag of the first difference of the adjusted variable. For more details on these tests see Apendix 1.

<sup>&</sup>lt;sup>10</sup> See Barnea and Djivre (2004).

## 3. Model's Specification and the Steady State.

## 3.1 The General VAR Specification

All three versions of the model can be written in the following general form:

$$\Gamma_0 y_t = \sum_{i=1}^p \Gamma_i y_{t-i} + \Lambda x_t + F \varepsilon_t \qquad \varepsilon_t \sim N(0, I)$$
 (1)

where  $y_t$  is an  $n \times 1$  vector of endogenous variables;  $x_t$  is an  $m \times 1$  vector of exogenous variables;  $\varepsilon_t$ is an  $n\times 1$  vector of structural innovations; p is the lag order; and  $\Gamma_0,...,\Gamma_p$ , F and  $\Lambda$  are coefficient matrices. In particular the matrix  $\Gamma_0$  captures the contemporaneous relationship among the endogenous variables. We assume that  $\Gamma_0$  and F are invertible. F and the  $\Gamma_i$  matrices are also nxn matrices and  $\Lambda$  is an nxm matrix.

We estimated the model by FIML procedure, to that end the system in (1) had to be transformed into a reduced form representation, obtained by pre-multiplying the model by  $\Gamma_0^{-1}$ :

$$y_{t} = \sum_{i=1}^{p} A_{i} y_{t-i} + \sum_{i=0}^{k} B_{i} x_{t-i} + e_{t} \qquad e_{t}^{iid} \sim N(0, \Omega)$$
 (2)

We first estimated the unrestricted model, and after performing the relevant tests we chose a lag length of two quarters.<sup>11</sup>

In estimating the various versions of our model we imposed four types of restrictions. The first consists of long-run restrictions regarding the s.s. values of the variables of the model – endogenous and exogenous; the second type refers to the choice of the exogenous variables entering each of the equations; the third regards the structural specification of the contemporaneous relationships among the endogenous variables, and the last imposes within equation restrictions on the coefficients of the regressors. 12

## 3.2. The Steady State

Table 1 below presents the s.s. values of the model's endogenous variables that were imposed in the estimation. Alongside these values the table reports the values to which an unconstrained VAR model converged to when no long-run restrictions were imposed. Figure 1 also presents the variation in the s.s. values of the same unconstrained VAR as the end period of the sample size expands from 2005q4 to 2008q3. After some fluctuations in the s.s. values in the shorter samples, these values eventually converged to values in the neighborhood of those we assigned

For details see Appendix 2. From the standpoint of formulating the estimation problem restrictions type 2, 3, and 4 are in fact identical.

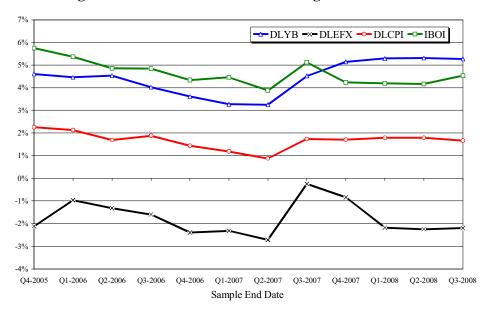
to all three estimated versions of the model with the exception of the depreciation of the nominal exchange rate.

The fluctuation of the s.s. values of the endogenous variables with changes the sample size supports the imposition of long-run restrictions since without them the model loses an anchor for convergence especially in out of sample forecasts.

Table 1: The Steady State Values in the Unconstrained Model (1997q3-2008q3, Annual Rates)

Variable	S.S Value	Imposed S.S Value
DLYB	5.3%	4.0%
DLEFX	-2.2%	0.0%
DLCPI	1.7%	2.0%
IBOI	4.5%	4.0%

Figure 1: The S.S. Values of the Endogenous Variables



We justify our choice of the s.s. equilibrium values as follows: The inflation targeting regime implies that inflation should equal its target. Relative purchasing power parity (PPP) suggests that the depreciation rate of the NIS against the basket of currencies we used must equal the inflation differential between Israel and the countries whose currencies make up the basket. The uncovered interest parity (UIP) suggests that the depreciation of the nominal exchange rate

should also equal the differential between nominal domestic and foreign currency interest rates in terms of the currency basket. Therefore, given an inflation target of 2 percent, and assuming foreign inflation of 2 percent on average, and a weighted foreign interest rate of 4 percent, we pinned down steady state domestic inflation at 2 percent, the effective exchange rate depreciation at zero, in line with the relative PPP in the long-run, and the domestic nominal interest rate at 4 percent in line with UIP. These values are consistent with a 2 percent real interest rate in the long run.

In addition, we imposed a long run growth rate of 4 percent. Our choice was driven by the experience of the Israeli economy. The business sector GDP average growth rate between 1965 and 2007 was 5 percent per annum, but between 1973 and 2007, after excluding the post 1967 years, which were characterized by exceptionally high growth, it was 4.1 per cent per annum.

To impose the aforementioned restrictions we simply expressed all variables in deviations from their s.s. value and omitted the constant term from the estimated regression equations. The s.s. equilibrium restrictions on the values of endogenous variables, however, have to be imposed alongside the s.s. values of the exogenous variables – these are presented in Table 2.

Table 2: The S.S. Values of the Exogenous Variables\*

Variable	SS Value
TARGET	2.0%
DSECURITY	0.0%
EFGR	3.2%
DLEFCPI	2.0%
IEF	4.0%
DUM98Q4	0.0%
DUMQ2	0.25
DUMQ3	0.25

<sup>\*</sup> Rates are annualized

## 3.3. Selection of the Exogenous Variables

While in the unrestricted version of the VAR model we assumed that all exogenous variables enter all equations, in the case of the other two versions, EXOGR and ENDOGR we assumed instead a particular relationship between the exogenous and the endogenous variables of the model.

We introduced foreign inflation, *DLEFCPI*, in the growth and in the inflation equations in order to reflect the effect of changes in the real exchange rate depreciation on GDP growth and of the prices of imported goods on the inflation rate, respectively.

Positive values of the variable *DSECURITY* are consistent with an improvement in the security conditions which affect both economic activity and the depreciation of the effective nominal exchange rate, *DLEFX*, the latter through their effect on the perceived risk of the Israeli economy to foreign investors. The effect of the remaining exogenous variables on the endogenous variables is self-evident and does not require further elaboration.

Table 3: Specification of Exogenous Variables in the Structural Model

	DLYB	DLE	DLCPI	IBOI
TARGET			X	X
<b>DSECURITY</b>	X	X		
EFGR	X			
<i>EFGR(-1)</i>	X			
DLEFCPI	X		X	
DLEFCPI (-1)	X		X	
$I\!E\!F$		X		
<i>IEF</i> (-1)		X		
DUM98Q4	X	X	X	X
DUMQ2			X	
DUMQ3			X	

Table 3 presents the specification of the exogenous variables in both the EXOGR and the ENDOGR models.<sup>13</sup> The latter is characterized also by within equation restrictions imposed on the regression coefficients of the various regressors on the basis of priors based on economic theory which are specified below.

## 3.4. Restrictions on the Contemporaneous Relationship among the Endogenous Variables

Our baseline structural specification assumed contemporaneous relationships as follows: Output growth is contemporaneously exogenous. The depreciation of the exchange rate is affected only by the interest rate; inflation is affected by output growth and the depreciation of the exchange

<sup>&</sup>lt;sup>13</sup> In the UR model all the exogenous variables enter each of the equations.

rate; and finally, the BoI interest rate is affected by output growth and inflation. Under this specification the matrix  $\Gamma_0$  has the following form:

$$\Gamma_{0} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -\beta_{ei} \\
-\beta_{\pi g} & -\beta_{\pi e} & 1 & 0 \\
-\beta_{ig} & 0 & -\beta_{i\pi} & 1
\end{bmatrix}$$
(3)

where  $\beta_{yx}$  represents the contemporaneous effect of variable x on y, with g indicating business sector GDP growth, e exchange rate depreciation,  $\pi$  CPI inflation, and i the BoI interest rate. The structure of the matrix as in (3) holds for the UR and the EXOGR models and it was slightly modified in the ENDOGR model (see section 3.5). Note also that this specification implies that our benchmark model is over-identified.

## 3.5. Within Equation Restrictions

Within equation restrictions are imposed only on the coefficients of the fully-constrained model, **ENDOGR**. These restrictions are driven by economic theory. Specifically, we assumed that the business sector GDP is affected by the current and lagged real exchange and ex-post interest rates. The former is defined as the ratio between foreign CPI in domestic currency and domestic CPI while the latter equals  $i_{t-1}-\pi_t$ . The assumption that the ex-post real interest rate affects economic activity is consistent with the credit channel theory of monetary policy transmission whereby external funds are costlier to firms than internal funds so that a higher ex-post real interest rate erodes the internal funds available for investment, raising its marginal funding cost and affecting thereby negatively economic activity. Moreover if the level of GDP is affected by the real interest and exchange rates, then the GDP growth rate should be affected by the first difference of these variables. Another assumption is that the effective nominal exchange rate depreciation depends on the spread between domestic and foreign nominal interest rates. These assumptions affect the specification of the growth equation and the effective nominal exchange rate depreciation equation as follows:

In the equation of the business sector GDP growth rate the coefficient of the coincident ex-post real interest rate should be of the same magnitude but have the opposite sign with the coefficient of the real ex-post interest rate lagged by one quarter. Moreover in view of the aforementioned definition of the ex-post real interest rate, our assumption also implies that the coefficient of the nominal interest rate lagged by one quarter should be of the same magnitude

but have the opposite sign with the coefficient of current inflation rate. The introduction of the real exchange rate depreciation as a regressor in this equation implies that the coefficient of the nominal exchange rate depreciation should be equal to the coefficient of foreign inflation and have the same magnitude but the opposite sign with that of the coincident domestic inflation rate.

Note that, in line with our assumptions, the coincident inflation rate constitutes a component of both the ex-post real interest rate and the coincident real exchange rate depreciation. This observation requires an additional restriction, namely that the estimated coefficient of the inflation rate should equal the negative of the sum of the coefficient of the nominal interest rate lagged by one quarter and of the coefficient of the coincident nominal exchange rate depreciation.

In the effective nominal exchange rate depreciation equation our assumption implies that the domestic and foreign nominal interest rates should have coefficients of equal magnitude but of opposite signs.

The restrictions above imply the following structure for  $\Gamma_0$ :

$$\Gamma_{0} = \begin{bmatrix}
1 & -\beta_{ge} = \lambda_{g\pi^{*}} & -\beta_{g\pi} = \beta_{ge} + \beta_{gi_{-1}} & 0 \\
0 & 1 & 0 & -\beta_{ei} = \lambda_{ei^{*}} \\
-\beta_{\pi g} & -\beta_{\pi e} & 1 & 0 \\
-\beta_{ig} & 0 & -\beta_{i\pi} & 1
\end{bmatrix}$$
(4)

In the matrix  $\Gamma_0$  above the coefficients  $\lambda_{g\pi^*}$  and  $\lambda_{ei^*}$  stand respectively for the coefficient of the foreign inflation rate in the growth equation and for the coefficient of the current foreign interest rate in the nominal exchange rate depreciation equation.

## 4. The Estimation Results

In a small open economy the transmission mechanism of the monetary policy is generally based on two channels, the interest rate channel and the exchange rate channel. On the inflation front higher interest rates give rise to a nominal exchange rate appreciation which feeds back directly into the CPI inflation rate through its effect on the price of tradable goods. As a result of frictions, the exchange rate appreciation and the higher nominal interest rate are reflected in higher real interest rates and a real exchange rate appreciation both of which slow down economic activity. The weakened economic activity mitigates further inflation at a later stage. This description of the characteristics of the transmission mechanism of monetary policy in a

small open economy implies that: (1) the coefficient of domestic interest rate in the effective nominal exchange rate depreciation equation should be negative, (2) the coefficient of the nominal interest rate in the growth equation should be also negative, and (3) that the coefficient of the exchange rate depreciation in the growth equation should be positive. Moreover the mitigating effect of the slowdown in economic activity on domestic inflation implies that the coefficient of the variable measuring economic activity in the inflation equation should be positive.<sup>14</sup>

The estimation results of the UR model are inconsistent with the aforementioned transmission mechanism. The imposition of exclusion restrictions on the exogenous variables and restrictions on regressor coefficients within the equations delivered better results. More precisely:

In the UR model the estimation results did not provide any evidence of the existence of the main channel of transmission of monetary policy in small and open economies as described above. This is true for both the effect of the nominal interest rate on the nominal exchange rate depreciation, whose coefficient was positive and the effect of the exchange rate depreciation on economic activity which on average was nil.

Higher GDP growth rates were found to have mitigating direct and indirect effects, through the appreciation of the exchange rate, on inflation. The direct effect was statistically insignificant. In the indirect effect, via the change in the exchange rate, higher growth gave rise to a statistically significant nominal exchange rate appreciation and thereby to lower inflation, as the exchange rate depreciation coefficient in the inflation equation is positive and statistically significant.

The effect of economic activity and of contemporaneous inflation on the nominal interest rate were statistically insignificant, and in the case of inflation only longer inflation lags displayed a positive effect on the BoI nominal interest rate.

The coefficient of the first lag of nominal interest rate in the growth equation was negative and significant while the second lag was positive significant and of a similar magnitude to that of the coefficient of the first lag, supporting our constrained specification according to which it is the *change* in the interest rate that affects the GDP growth rate.

<sup>&</sup>lt;sup>14</sup> The estimation results for the three VAR models, the unconstrained model, the exogenously-constrained model and the fully constrained mode are reported in appendices 3-5.

In the exogenous variables constrained model, EXOGR, the introduction of exclusion restrictions on the exogenous variables improved the estimation results, especially those with respect to the exchange rate channel of the monetary policy transmission mechanism and with respect to the effect of inflation on the nominal interest rate.

In the interest rate equation the effect of contemporaneous inflation became positive and significant similarly to the effect of its lagged values and the positive effect of economic growth on the nominal interest rate improved.<sup>15</sup> Moreover the estimation results indicated a very high interest rate smoothing factor.

The coefficient of current domestic interest rate in the nominal exchange rate depreciation equation has become negative although insignificant, and the overall effect of exchange rate depreciation on growth has become positive due to a significant coefficient of the depreciation rate lagged by two quarters.

The signs of the coefficients of the exogenous variables were found to be in the expected direction, an improvement in security conditions enhancing economic activity and supporting an exchange rate appreciation, higher foreign currency interest rates leading to an exchange rate depreciation, and higher foreign inflation promoting economic activity by enhancing real exchange rate depreciation alongside higher inflationary pressures. The coefficient of the inflation target in the interest rate equation was found to be negative and statistically significant at the 10 percent significance level, implying that a higher inflation target is consistent, other things equal, with a lower nominal interest rate.

Like in the unrestricted model, in this version of the model the coefficient of the nominal interest rate with one lag in the growth equation was negative while the corresponding coefficient for the second lag was positive, statistically significant and of a similar absolute magnitude to that of the first lag. Again, this supports our conclusion that it is the change in the interest rate that affects growth.

The estimation results of the fully constrained model, ENDOGR, did not indicate any obvious improvement relative to the estimation of the EXOGR model and were qualitatively similar both with respect to the exchange rate and interest rate channels of the transmission mechanism of the monetary policy and with respect to the determination of the nominal interest

<sup>&</sup>lt;sup>15</sup> The coefficient of growth lagged by one quarter is positive and significant at a confidence level of 7.2% in a one tailed test.

rate. The significance of the coefficient of the coincident interest rate in the exchange rate depreciation equation became though significant at 10 percent significance level.

The value added of this estimation lies in the fact that it did not reject the restrictions imposed on the regression coefficients endowing the model with a structure consistent with economic theory.

Examining the effect of the GDP growth rate on the inflation it appears that the coincident effect is negative but the overall effect is nil, as the sum of growth coefficients across its lags is not substantially different from zero. This result is consistent with expressing the inflation rate as a negative function of the first difference in growth.

We claim that this interpretation of the estimation results may be traced to the fact that the change in the GDP growth rate is negatively correlated with the output gap along the cycle which in its turn is positively correlated with the inflation rate. More precisely, periods in which the economy is characterized by a positive output-gap the growth rate slows down, it equals potential growth rate as the economy hits the peak in activity and becomes smaller than potential growth when the positive output-gap starts decreasing. Along this path the first difference of the growth rate is negative but reaches zero at the inflection point of the cycle when the output gap becomes negative. Similarly, negative gaps are associated with positive first difference in growth. This evolution implies that the first difference of the GDP growth rate and the output-gap are negatively correlated. This hypothesis is supported by empirical evidence of the Israeli economy. In the data the correlation between the output gap obtained on the basis of an HP filter and the first difference of the business-sector GDP growth rate was found to be equal to -0.41 for the period between 1995q1 and 2009q4. If we exclude from this period the sub-period until 2000q3 during which the output gap hovered around zero, then the correlation coefficient reaches -0.69.16 This interpretation may also justify the negative sign of the contemporaneous growth rate in the inflation equation. It does not provide though an explanation for the negative coefficient of the growth rate in the equation of the depreciation of the exchange rate.

The drawback of the above interpretation is that this proxy for the output gap cannot be used for impulse response analysis. The reason is that in this case shocks to the growth rate imply higher economic activity and hence a higher output gap, while the aforementioned negative

<sup>&</sup>lt;sup>16</sup> These correlations are based on a two-quarter (simple) moving average of both the output gap and the change in growth rate. The regression weighs the lags differently, as reflected by their coefficients.

correlation between the gap and the first difference of growth implies that higher growth would have been interpreted by the model as a smaller gap.

## 5. Testing the Performance of the Models

In this section we performed several exercises in order to assess the models' structural characteristics, their fit to the data and their suitability as forecasting instruments. The structural characteristics of the models were examined in the context of impulse response functions (IRF) and of robustness tests of the regression coefficients. The fit to the data was examined by performing within and out of the sample dynamic simulations and comparing their outcomes to the data. The out of the sample simulations for the period 2007q1-2009q3 served us also to assess the suitability of the estimated models as forecasting instruments together with an evaluation of the plausibility of the forecast results of the various models for the years 2010 and 2011, for which though it is still impossible to make a counterfactual comparison.

#### 5.1. Robustness of the Estimated Coefficients

To assess the robustness of the regression coefficients we used the results from a rolling estimation along an expanding sample. The first sample of this estimation covered the period between 1997q3 and 2006q4 and the last one the period between 1997q3 and 2008q3. We thus obtained a total of 8 estimates. We performed this exercise for the two constrained models, the EXOGR and ENDOGR models. A graphic representation of the results appears in appendix 6 in which we also present confidence intervals of two standard deviations around the coefficient estimate.

The estimation results indicate that the obtained coefficients are in general stable, both those which were found to be different from zero and those which were not. In cases in which the coefficients changed, it may be still said that the corresponding changes in the size of the coefficient was in most cases smaller than one standard deviation prior to the change.

The coefficients of the regressors in the exchange rate depreciation equation, in the EXOGR model, exhibited instability and more precisely a monotonic trend, sometimes increasing and some times decreasing and this is true for all coefficients with the exception of the coefficients of the BoI interest rate which were relatively stable.

In both models some instability was also observed in the coefficients of the inflation equation. It should be mentioned however that in these cases the changes in coefficients did not

affect substantially the overall effect of the regressors as they canceled out, to a large extent, across their lags. This is true for the coefficients of economic growth, the coefficients of the exchange rate depreciation, and for the coefficients of the nominal interest rate.

## 5.2. IRF of the Constrained Models

We limited the Impulse Response exercise to the EXOGR, and the ENDOGR models (see corresponding diagrams in Appendix 7). We did not analyze the IRFs of the unrestricted model as it failed to generate signs for the coefficients of some key variables which are consistent with economic theory, making thereby the analysis of the dynamics of that model redundant.

It is apparent from the IRFs that there exist two monetary policy transmission channels. The first transmission channel is that of the interest rate and the second, which depends on the former, is that of the exchange rate. An increase in the central bank interest rate leads to a nominal exchange rate appreciation which lowers inflation. This appreciation is characterized by an overshooting of the effective nominal exchange rate as a result of which the initial exchange rate appreciation is followed by depreciation.

The lower inflation rate and the higher nominal interest rate are reflected in a higher real interest rate while the exchange rate appreciation leads to a real exchange rate appreciation in view of the fact that the downward adjustment of inflation brought about by the nominal exchange rate appreciation is only partial. The IRFs indicate that the higher ex-post real interest rate adversely affects economic activity on impact while the real exchange rate appreciation does so mainly after two quarters, the real interest rate effect on economic activity being however more substantial than that of the exchange rate appreciation. On the other hand the nominal exchange rate appreciation is much more effective in reducing inflation pressures than the second round effect of slower growth, which is negative on impact, contrary to expectations, but statistically insignificant and of a small magnitude. In other words the estimation results point to a decoupling between the real activity and inflation so that the monetary policy effects on activity and inflation are limited to the first round effects.

The model's dynamic specification allows also for a higher nominal interest rate to directly mitigate inflation pressures. Moreover, the impulse reaction results indicate a substantial nominal interest rate smoothing.

The difference between the two models with respect to the monetary policy transmission mechanism is found in the reaction of the business sector GDP growth to a policy shock. In the

EXOGR model there is no contemporaneous effect of the interest rate on the growth rate while in the ENDOGR model there is, through the effect of the contemporaneous inflation on the expost real interest rate whose coefficient was set equal to zero in the EXOGR model. As a result the rise in the interest rate in the former affects growth with a one quarter lag while in the latter the slow down in growth is immediate.

In addition to the characterization of the transmission mechanism of the monetary policy the results of the IRF exercise indicate that in both models the response of the nominal interest rate to inflation shocks either direct or indirect (following an exchange rate depreciation) is hump shaped reaching its peak two quarters after the shock. This is a result of the fact that the coefficient of the first lag of the interest rate in its own equation is greater than unity while the second lag is negative. Similarly to the appreciation of the exchange rate following shocks to the nominal interest rate, shocks to the inflation rate and to the exchange rate do also affect in the short run the real exchange rate. Moreover, an inflation shock leads to a contemporaneous rise in growth as a result of the erosion of the ex-post interest rate, in the ENDOGR model, but not in the EXOGR model in which we did not allow for such a contemporaneous effect of the inflation rate.

Some additional results of the impulse response exercise are the following: Higher growth reduces inflation through its effect on the exchange rate which appreciates one quarter after the shock. This fall with a lag of one quarter is statistically significant for both models. The inflation mitigating effect of a positive shock to the GDP growth rate is further enhanced by the direct contemporaneous effect of higher growth in the inflation equation which is negative, contrary to economic theory, and statistically insignificant but less so in the ENDOGR model. It is possible that shocks to growth may reflect sectorial shocks to export-demand which are not accounted for in the GDP growth of the G4 countries entering the domestic growth equation in both models.

Two important comments have to be made at this point. The IRF of a shock to the business sector GDP-growth rate converges eventually to its unperturbed level. However as it transpires from the corresponding diagram, this convergence occurs through positive growth rates. This implies that following a positive shock to growth, which according to the specification of the equation is a demand shock, the economy experiences a permanent increase in GDP and as a result it diverges permanently from potential output. A similar comment is relevant for the real exchange rate depreciation. This is an undesirable property of the model since we should in general expect that after demand shocks economic activity returns to its long-run level. This

implies that our model, in spite of its theoretical underpinnings, will be unsuitable for providing long-horizon forecasts because for long periods the aforementioned deviation will surely affect the accuracy of the forecasts. Typically, long-run restriction a-la Blanchard and Quah (1989) are imposed on the model in order to cope with this type of criticism – see for example Gali (1992). However, we cannot apply the Blanchard-Quah methodology in our framework since it assumes that the reduced-form coefficients are unrestricted, while all the structure of the model is imposed on the contemporaneous relationships among the endogenous variables. This assumption simplifies significantly the estimation procedure as it implies a simple OLS regression for the reduced-form coefficients. However, this is not the case in our model due to exclusion restrictions on the exogenous variables and some inter-temporal restrictions among the endogenous variables.<sup>17</sup>

The second comment relates to the fact that the inflation rate is generally affected by the output-gap and not by the rate of GDP growth, even though there could also exist speed limit effects with a positive coefficient, contrary to the negative sign we obtained. In order to cope with this criticism we introduced to the inflation equation a measure of the output gap calculated as the deviation of log GDP from a linear trend. <sup>18,19</sup> The estimation results we obtained by applying this methodology to the ENDOGR specification of the model delivered IRF which were almost identical to those of the benchmark model. The overall effect of the output gap across its different lags on inflation was positive, as a result of the positive sign of the coefficient of the first lag of the output gap in the inflation equation, but statistically insignificant. The IRF of the output-gap to shocks to economic growth demonstrated our earlier

<sup>&</sup>lt;sup>17</sup> For this reason we tried to impose an additional restriction when estimating our model, namely that the sum of deviations of DLYB from its unperturbed level following shocks is equal to zero. However, the solvers we used in the ML estimation process in MATLAB did not ensure convergence, implying the need to search in the direction of other solvers. We reached this conclusion after experimenting with constrained optimization problems for which we knew ex-ante the answer to and nevertheless the solver did not succeed to provide the correct solution or did not converge to a solution at all. It should be noted that up to this point all restrictions were substituted into the likelihood function and hence we actually solved unconstrained optimization problems, while restricting the growth (and/or exchange rate depreciation) IRFs to sum to zero requires a solution via the specification of a Lagrangian.

<sup>&</sup>lt;sup>18</sup> The "gap" variable was entered the model as an additional endogenous variable and was linked by an identity to the growth variable DLYB. More precisely, in order to avoid singularity of the variance-covariance matrix of the structural shocks we allowed for a tiny standard deviation (of 0.0001 – one hundredth of a percent) to the shock of the gap "identity".

<sup>&</sup>lt;sup>19</sup> We also experimented with introducing the level of unemployment to the inflation equation in order capture the position of the economy relative to its natural rate. Unemployment was found to reduce inflation with a lag of two quarters. However, since unemployment is an endogenous variable and we do not have an equation to describe it, the resulting impulse response functions of the system are misleading as they hold unemployment fixed and do not allow changes in the real economy to fully feed back into inflation. We thus decided to let growth alone to represent the real side of the economy in the system.

observation and indicated a permanent rise of the gap following a positive shock to the growth rate.

## 5.3. Dynamic Simulations within the Sample

We performed a dynamic simulation of the model for our sample period 1997q3 to 2008q3 using the actual values for the exogenous variables. The reason for using the realizations of the exogenous variables was that in this way projection errors stem from the choice of the model alone, rather than from forecast errors for the exogenous variables. That is, by using the realizations of the exogenous variables we isolate the effect of model selection on forecast errors (Appendix 8 presents diagrams of the dynamic simulation results).

We calculated the RMSE for this exercise and its results are reported in Table 4 below. We report in bold characters the forecast with the lowest RMSE and in italics the second lowest.

Table 4 indicates that constraining the models increases their in-sample RMSE, the UR model having generated the lowest RMSE in all variables, the EXOGR model the second in line with the exception of the interest rate. This outcome is not very surprising because the results of the dynamic simulations are closely related to the goodness of fit of the model, which is generally the best for unconstrained models.

In contrast to the data reported in table 4, the figures of the dynamic simulation in Appendix 8 include also a full year out of the sample. An interesting observation arising from the diagrams is the clustering of the results of the different models within the sample and their relative dispersion, out of the sample, in 2009. More precisely the models generated higher interest rates for 2009 and lower economic growth relative to the actual data. This pattern is also observed in some forecasting exercises as discussed below.

**Table 4: RMSE of Dynamic Simulation 1997q3 – 2008q3** (percentage points, 45 observations)

Variables	DLYB	DLEFX	DLCPI	IBOI
Model				
UR	3.75	8.99	2.40	1.46
ExogR	4.34	10.51	2.89	1.78
EndogR	4.42	10.99	2.91	1.51

## 5.4. Out of Sample Forecasts

We performed two types of out of sample forecasts. In the first we estimated the model on the basis of expanding samples with varying end-dates, the latest ending date being 2008q3, and then generated out-of sample forecasts starting with the quarter following the last quarter of the sample.<sup>20</sup> This allowed us to conduct a counterfactual exercise by comparing the models forecasts to the realizations of the data. In the second exercise we generated forecasts for the years 2010 and 2011 for which, at the time of the writing of this paper, we cannot make any counterfactual comparisons. We can, however, evaluate the plausibility of the forecast results.

#### 5.4.1. Counterfactuals

The objective of this section is to provide a measure for the accuracy and the reliability of the out-of sample forecasts. As mentioned, the forecasts were based on eight consecutive estimations of the models for expanding samples all starting at 1997q3 and with end dates from 2006q4 to 2008q3. The forecasts were performed for three different time horizons, one, two and four quarters ahead using the realizations of the exogenous variables for the reasons we expounded in the previous section. We compared these forecasts to the actual data and calculated their RMSE.

However the forecasts' RMSE cannot serve as a unique criterion for the models' suitability for forecasting purposes since the deviation of the endogenous variables from their observed values were obtained for an *endogenous* path of the nominal interest rate. From the policy maker's view the relevant deviation should most probably be that obtained when it is assumed that the evolution of the interest rate is identical to its realization. For this reason we also conducted a similar exercise in which the forecasts were made conditional on the realized path of the interest rate.

The results of these exercises are reported in Tables 5 and 6 below. The RMSEs reported in Table 5 indicate that the out of the sample performance of the ENDOGR model exhibited the lowest RMSE across all forecast horizons and all variables, with the exception of the four quarters ahead forecast of inflation, for which the EXOGR model delivered only a moderately better result. Even though the UR model displayed the most accurate in-sample fit, its out of sample performance was the poorest. These results justified our imposing some structure on the

We perform the forecasts using the realizations of the exogenous variables.

models especially in view of the small sample size which does not allow for the asymptotic properties of the estimators to become effective.

Table 5: RMSE of Out of Sample Forecasts for samples end-dates 2006q4-2008q3 (percentage points, 8 observations)

	1 Quarter					2 Qu	arters		4 Quarters			
	DLYB	DLEFX	DLCPI	IBOI	DLYB	DLEFX	DLCPI	IBOI	DLYB	DLEFX	DLCPI	IBOI
UR	8.2	15.2	4.9	0.9	8.3	12.5	4.1	1.4	9.7	13.4	2.8	2.7
ExogR	6.2	17.4	4.1	0.7	7.4	13.3	2.5	1.3	9.3	11.3	1.1	2.3
EndogR	4.1	14.1	3.7	0.7	4.8	11.3	2.2	1.3	6.8	8.5	1.2	2.3

It is also interesting to note that the RMSE increases across models with the forecast horizon for the GDP growth rate and the nominal interest rate and falls for inflation and for the effective nominal exchange rate depreciation. For the latter however only with respect to the two restricted versions of the model.

The results of the exercise with the actual interest rate are reported in Table 6. The ENDOGR model generated the most accurate forecasts in terms of the RMSE criterion in this case as well. It is worth noting that the accuracy of the forecast increased with the imposition of restrictions. The only exception is the forecasts for the exchange rate depreciation where the UR did better than the EXOGR model.

Table 6: RMSE of Out of Sample Forecasts with Actual Interest Rate for samples end-dates 2006q4-2008q3 (percentage points, 8 observations)

1 Quarter 2 Quarters 4 Quarters DLYB DLEFX DLCPI IBOI DLYB DLEFX DLCPI IBOI DLYB DLEFX DLCPI IBOI UR 17.2 7.2 8.2 5.7 0.011.3 4.5 0.08.9 10.4 3.3 0.0**ExogR** 6.2 18.0 4.2 0.0 7.1 14.9 2.8 0.0 8.5 13.2 2.0 0.0 4.0 0.0 0.0 **EndogR** 15.1 3.8 4.6 12.0 2.5 6.3 9.4 1.9 0.0

From comparing the figures in Table 6 to those in Table 5 it appears that forcing the RMSE of the nominal interest rate to zero has improved the RMSE of growth but has also led to higher RMSE for the exchange rate depreciation and inflation. The reason lies apparently in the fact that the unconditional forecasts generated higher interest rate paths than those realized,

especially during the crises period starting 2008q4, together with negative growth rates that were lower than their realization, as it can be seen in the diagrams in Appendix 9.<sup>21</sup> As a result, forcing the interest rate to its actual (lower) level pushed growth forecasts upward and therefore improved their accuracy.

To evaluate the reliability of the forecasts of each model we also calculated the forecast standard error, a too high standard error being considered as undesirable since it renders the corresponding model worthless as a forecasting tool. In Table 7 we report this standard error for the last estimation of the models based on the sample between 1997.q3 and 2008q3 where forecasts were generated for the period 2008q4 to 2009q3. Similarly to the RMSE, the standard deviations were calculated for one, two and four quarters ahead forecasts of the cumulative growth rate, of the cumulative devaluation of the exchange rate, and of the cumulative inflation rate. The forecasts of the nominal interest rate however refer to their quarterly level. Similarly to the results of the RMSE it clearly transpires from Table 7 that the ENDOGR model delivered the lowest forecast standard deviations. This, together with the accuracy of the forecasts as measured by the RMSEs, indicates that this model delivered not only the most accurate forecasts but also the most reliable ones.

Table 7: Standard Deviations of Forecast Errors, 2008q4-2009q3 (percentage points)

	1 Quarter				2 Quarters				4 Quarters				
	DLYB	DLEFX	DLCPI	IBOI	DLYB	DLEFX	DLCPI	IBOI	DLYB	DLEFX	DLCPI	IBOI	
UR	7.9	21.1	5.7	1.3	10.2	23.9	7.3	2.6	8.1	20.9	6.0	3.7	
ExogR	7.3	13.2	3.6	0.8	10.7	15.6	4.4	1.5	7.0	19.3	5.1	2.3	
EndogR	5.9	12.3	3.4	0.7	7.2	14.0	3.9	1.3	6.8	15.4	3.9	2.1	

We finally examined the extent to which the actual values of the endogenous variables fell within or out of the one standard deviation confidence level of the forecast. Forecasts start from 2008q1 and are based on estimation of the models for the sample period 1997q3-2007q4. The diagrams referring to this exercise are reported in Appendix 9. In this exercise we provided four quarters ahead forecasts for the quarterly nominal interest rate of the Bank of Israel and for the cumulative rates of growth, of depreciation of the exchange rate and of the inflation rate.

<sup>&</sup>lt;sup>21</sup> The diagrams in Appendix 9 present out of sample forecasts for the period 2008q1-2009q4 together with their one S.D. confidence intervals.

The unambiguous conclusions we may draw from this exercise are that for forecasts longer than a year the actual values for the business sector rate of growth fall out of the one standard deviation confidence interval of the forecast implying that all specifications of the model may be unsuitable for long run projections in line with our earlier comments concerning the long run effects that shocks on the growth rate have on the level of economic activity. Nevertheless the ENDOGR model seems to fare a little better than the UR and the EXOG models. The same is true for the interest rate forecasts in the case of the two constrained models. The UR delivers apparently better results in this case but this is probably due to the fact that its confidence interval is wider. Finally for both of the constrained models the realized cumulative rate of the four quarter inflation and exchange rate depreciation fell, to a large extent, within the one standard deviation confidence interval of the forecast.

## 5.4.2. Forecasting into the future: 2010-2011

This section presents forecasts for the years 2010 and 2011 of the different models. Estimation is based on the sample between 1997q3 and 2008q3. The path of the exogenous variables in the forecasts is presented in Table 8 below. For 2010 the assumption regarding foreign variables is taken from private sector forecasts. These forecasts suggest moderate growth in the industrialized countries, EFGR, while nominal interest rates, IEF, were assumed to remain low, and foreign inflation, DLEFCPI, is projected to be moderate, i.e. lower than its s.s value. For 2011 we assumed that these variables start converging to their s.s. values, full convergence being achieved in 2012. In addition, we assumed that the security situation, DSECURITY, will remain unchanged during the forecast horizon. Forecast results for the various models are reported in Table 9 below.

The forecasts of the fully constrained model are in our opinion the most plausible especially for the business sector GDP growth rate, the depreciation of the exchange rate and the inflation rate. The other two models deliver a too high depreciation rate especially for 2011 while output growth for 2010 looks too low while that for 2011 is too high for the UR model and too low for the EXOGR model. We present diagrams of the forecasts together with their confidence intervals for the ENDOGR model in Appendix 10.

**Table 8: The Evolution of the Exogenous Variables 2010q1-2011q4** (Annualized Rates in Percentage Points)

	DSECURITY	EFGR	DLEFCPI	IEF
2010q1	0	2.6	1.2	0.25
2010q2	0	2.7	2.7	0.25
2010q3	0	2.5	0.7	0.43
2010q4	0	2.5	0.1	0.56
2011q1	0	2.7	0.4	1.0
2011q2	0	2.8	0.7	1.6
2011q3	0	2.9	1.0	2.2
2011q4	0	3.0	1.4	2.8

The ENDOGR model delivered business sector GDP growth somewhat lower than that of potential output and this implies a slight increase in the output gap during the forecast period. However, the real interest rate delivered by the model remains at historically low levels and this raises questions about the sustainability of the monetary policy and hence of the recovery in view of the fact that in 2011 projected growth is somewhat moderated as the real interest rate increases, even though it still remains below its s.s. value.

**Table 9: Forecasts for 2010-2011** (Annualized Rates in Percentage Points)

Forecast		DLYB		DLEFX			DLCPI			IBOI		
Period	UR	ExogR	EndogR	UR	ExogR	EndogR	UR	ExogR	EndogR	UR	ExogR	EndogR
2010q1	3.0	3.9	4.1	20.8	7.9	2.7	1.3	3.2	2.4	1.3	1.5	1.4
2010q2	-0.7	1.1	4.2	10.3	3.1	1.4	3.8	6.0	5.5	2.2	2.2	2.0
2010q3	0.2	0.3	2.5	19.2	7.2	1.6	1.9	4.3	3.3	2.6	2.9	2.7
2010q4	0.4	-0.6	2.2	21.1	10.1	3.3	-1.1	1.6	0.3	2.9	3.4	3.0
2010	0.7	1.2	3.3	17.8	7.1	2.3	1.5	3.8	2.9	2.9	3.4	3.0
2011q1	3.8	-0.2	2.1	22.6	15.8	7.5	-1.9	1.2	-0.3	2.9	3.5	2.8
2011q2	5.2	0.0	3.3	10.5	17.4	7.7	-0.1	4.8	3.0	3.3	3.8	2.7
2011q3	5.2	-0.3	2.2	16.1	14.6	5.3	0.3	4.0	2.4	3.1	4.3	2.9
2011q4	6.9	2.3	3.1	12.3	13.2	4.4	-0.7	2.4	1.0	3.3	4.9	3.1
2011	5.3	0.5	2.7	15.4	15.2	6.2	-0.6	3.1	1.5	3.3	4.9	3.1

Furthermore, in view of the (unexpected<sup>22</sup>) narrowing of the spread between domestic and foreign interest rates, especially during 2011, we should expect the nominal effective exchange rate to depreciate especially in 2011. However a depreciation rate of 15 percent with much lower inflation, as suggested by the UR and EXOGR models, does not conform with the experience of the Israeli economy. We therefore consider the forecasts of the ENDOGR model as being more plausible.

## 6. An Extension: The Constrained Model with Model Consistent Expectations

## **6.1. The Specification**

In this section we report the results on an extension of the ENDOGR specification by introducing model consistent rational expectations to the model. We refer to this version as the RE model. In this specification expectations are formed with respect to inflation and business-sector GDP growth. The expected one quarter ahead inflation was introduced in the growth equation affecting the expected real interest rate and as a result the coefficient of the coincident nominal interest rate was not set to zero. In the ENDOGR model the relevant real interest rate at time t was the ex-post real interest rate, the relevant nominal interest rate being the one set at t-1, so that the coefficient of the nominal interest rate at t was set in that case to zero and the current inflation rate affected the determination of both the ex-post real interest rate and the real exchange rate depreciation. In the present specification however the current inflation rate affects only the latter and as a result the coefficient of current inflation has to be of the same magnitude but of the opposite sign to that of the nominal exchange rate depreciation. The one quarter ahead expected inflation rate was also introduced in the inflation equation as determining the current inflation rate and in the nominal interest rate equation, affecting the setting of the nominal interest rate by the central bank.<sup>23</sup>

The expected growth rate was introduced in the growth rate equation. The introduction of the expected rate of growth into the growth equation is based on the New Keynesian version of output-gap equations according to which current economic activity, in terms of the output-gap, depends on next period's expected level of activity. In order to reach our specification we derived the first difference of the level equation, which implies not only the introduction of the

<sup>22</sup> Since the model does not contain forward looking expectations.

In the version we present here the interest rate reacts to expected inflation in the following quarter. We also experimented with a specification that takes into account expected inflation in the following 4 quarters, but results hardly change. For simplicity of presentation we remain with the simplified specification.

expected rate of growth at time t for t+1 but also the expected rate of growth at t-1 for time t. A similar specification was also imposed with respect to expected inflation as a component of the lagged (by one quarter) ex-ante real interest rate. After adjusting the structural form of equation (1) to allow for forward looking variables we obtained the following general expression:

$$\Gamma_{0}y_{t} = \Gamma_{0}^{e}E_{t}(y_{t+1}) + \Gamma_{1}^{e}E_{t-1}(y_{t}) + \sum_{i=1}^{p}\Gamma_{i}y_{t-i} + \Lambda x_{t} + F\varepsilon_{t} \qquad \varepsilon_{t}^{iid} \sim N(0, I)$$
(5)

Here we added two components to the baseline specification:  $\Gamma_0^e E_t(y_{t+1})$  and  $\Gamma_1^e E_{t-1}(y_t)$  in line with our exposition above. Our restrictions imply the following structure for matrices  $\Gamma_0^e$  and  $\Gamma_1^e$ :

$$\Gamma_0^e = \begin{bmatrix} \gamma_{gg} & 0 & -\gamma_{gr} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \gamma_{\pi\pi} & 0\\ 0 & 0 & \gamma_{i\pi} & 0 \end{bmatrix}$$
 (6)

Because of the use in the growth equation of ex-ante real interest rate and the relevance of the contemporaneous inflation for the real exchange rate depreciation only, the specification of the matrix  $\Gamma_0$  changed also and it is now given by:

$$\Gamma_{0} = \begin{bmatrix}
1 & -\beta_{ge} & -\beta_{g\pi} = \beta_{ge} = \lambda_{g\pi^{*}} & -\beta_{gi} = -\gamma_{gr} \\
0 & 1 & 0 & -\beta_{ei} = \lambda_{ei^{*}} \\
-\beta_{\pi g} & -\beta_{\pi e} & 1 & 0 \\
-\beta_{ig} & 0 & -\beta_{i\pi} & 1
\end{bmatrix}$$
(8)

For FIML estimation, the system in expression (5) had to be expressed in reduced form as in equation (2). To that end we had to formalize the process defining the evolution of the exogenous variables over time, so as to solve for expected growth rate and inflation. We assumed for simplicity that the exogenous variables follow an autoregressive process of order 1:

$$x_{t} = Rx_{t-1} + u_{t} \qquad u_{t} \stackrel{iid}{\sim} N(0, \Sigma)$$
 (9)

In solving for the model-consistent rational expectations we used Sims' (2002) algorithm.

#### 6.2. The Estimation Results

The estimation results and the impulse response functions are presented in Appendix 11. The estimation results do not conform with economic theory as all coefficients of the forward looking variables, with the exception of the ex-ante real interest rate, had the opposite sign to that consistent with economic theory. Most disturbing was the coefficient of expected inflation in the interest rate equation – this coefficient turned out negative and significant. As a result the response of the interest rate to an exogenous rise in inflation, although in the right direction because of the positive effect of contemporaneous inflation, is more moderate relative to the impulse response in the ENDOGR model. Also, the coefficients of expected inflation in the inflation equation and of expected business sector GDP growth in the growth equation were both negative, contrary to theory, but insignificant.

The IRFs of the RE model were however qualitative similar to those of the ENDOGR model. The difference between the RE version of the model and the ENDOGR specification is to be found mainly in the IRF of the nominal interest rate. The response of the interest rate to an exogenous rise in inflation has become much more moderate relative to the ENDOGR model and statistically insignificant. The introduction of rational expectations to the model reduced the persistence of the interest rate. A shock to the nominal interest rate dies out after only one quarter in the RE model while in the ENDOGR version it takes approximately six quarters for the shock to disappear, although the effect becomes insignificant after 3. Likewise the real interest rate became much less persistent in the RE model.

From the results of the forecast exercises, reported in Table 10, it transpires that the ENDOGR model delivered more accurate forecasts in terms of the RMSE criterion than its RE version except for the interest rate.

Table 10: RMSE of Out of Sample Forecasts: ENDOGR vs RE Samples End-Dates 2006q4-2008q3 (percentage points, 8 observations)

(percentage points, 8 observations)

	1 Quarter				2 Qua	arters		4 Quarters				
	DLYB	DLEFX	DLCPI	IBOI	DLYB	DLEFX	DLCPI	IBOI	DLYB	DLEFX	DLCPI	IBOI
EndogR	4.1	14.1	3.7	0.7	4.8	11.3	2.2	1.3	6.8	8.5	1.2	2.3
RE	5.6	18.7	4.2	0.6	5.7	14.6	3.0	0.8	6.6	15.2	1.8	1.1

Given the counterintuitive estimation results and since there is no apparent advantage in forecasting performance we reached the conclusion that it is difficult to justify the use of this model as a forecast instrument.

## 7. Conclusions

In this paper we developed an empirical model for the Israeli economy for the purpose of macroeconomic forecasting. The model includes four endogenous variables: business sector GDP growth, depreciation of the nominal effective exchange rate, CPI inflation and the nominal interest rate set by the BoI. Our approach was to start from an unrestricted VAR model where we "let the data speak" and gradually impose more structure upon the estimated model until we reached a structural model with rational expectations. Imposing structural constraints on the unrestricted VAR improved the performance of the model both in terms of coefficient signs and IRFs that are more in line with standard economic theory and in terms of the accuracy of out of sample forecasts. Nevertheless, too much structure proved to be counter productive as the imposition of rational expectations worsened the results of a structural model without forward looking variables along both fronts – coefficient sings and out of sample forecasts. We conclude that, among the models we have considered in this paper, the structural model without rational expectations is the most adequate for forecasting.

Still, at least one drawback of the model needs to be addressed. As mentioned in the text, the IRFs of growth and real depreciation imply that transitory shocks have permanent effect on the level of output and the level of the real exchange rate. A potential remedy to this problem is imposing that the response of growth over time sum up to zero, and similarly the response of the depreciation rate. Unfortunately we were unable to carry out estimations while imposing these restrictions due to technical difficulties. We therefore leave this extension to future work.

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#### **Appendix 1: Unit Root Tests for the Endogenous Variables**

Table A1.1 presents the results of Phillips-Perron (PP) and Augmented Dickey-Fuller (ADF) tests for each of the variables in levels and first differences. Both tests allow for the inclusion of a deterministic time trend and/or a constant, and both account for potential bias due to serially correlated residuals: the PP test corrects the test statistic for serial correlation while the ADF test adds lagged first differences to its underlying regression in order to remove serial correlation from the residual. Both tests indicate that the levels of all variables are non-stationary, while the first differences are stationary.

The indication that the business sector GDP, the exchange rate, and consumer prices contain a unit root comes at no surprise; however, economic theory suggests that the interest rate should be a stationary variable. Recall that the interest rate displays a downward time trend; this trend is driven by the disinflationary process experienced by the Israeli economy. However, in the second half of the sample, at least since 2004, the interest rate has stabilized at a relatively low level generating a break in its trend.

Table A1.1: Unit Root Tests for the Endogenous Variables

Panel A: Phillips-Perron Test

Level	Prob.	First Dif. Prob.
LYB*	0.7248	DLYB** 0.0002
$LEFX^*$	0.9566	DLEFX*** 0.0000
$LCPI^*$	0.2309	DLCPI** 0.0001
IBOI*	0.3991	DIBOI** 0.0003

Panel B: Augmented Dickey-Fuller Test

Level	Prob.	Lags***	First Dif.	Prob.	Lags***
LYB*	0.2876	2	DLYB <sup>**</sup>	0.0003	0
$LEFX^*$	0.9636	0	DLEFX**	0.0000	0
$LCPI^*$	0.2571	0	DLCPI**	0.0001	0
$\mathrm{IBOI}^*$	0.1709	1	DIBOI**	0.0001	1

<sup>\*</sup> The variable was regressed against a constant and a deterministic trend.

Perron (1989) demonstrates that standard tests cannot reject the null hypothesis of a unit-root against the alternative of a trend stationary process when the true data generating mechanism is

<sup>\*\*</sup> The variable was regressed against a constant.

<sup>\*\*\*</sup> In addition to a constant and deterministic time trend, where applicable, regressions included lagged first differences. The number of lags was chosen by Schwarz Criterion.

trend stationary but its trend contains a one-time break. In order to capture the change in time trend we introduced the average inflation target for the next four quarters (*TARGET*) into the interest rate equation. Figure A1.1 presents the two variables; the common time trend is clearly visible in the figure. Although we will continue with the level of the interest rate as one of our left hand side variables, it should be noted that unit root tests for the interest rate net of inflation target provide mixed results: the ADF test rejects the hypothesis of a unit root at the 5 percent significance level but the PP test cannot reject it at any reasonable significance level.<sup>24</sup>

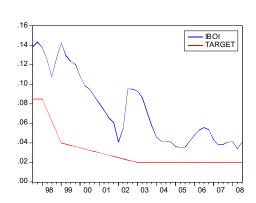


Figure A1.1: The BoI Interest Rate and the Inflation Target

Furthermore, an additional argument supports the view that the nominal interest rate is stationary. Unit root test for the ex-post real interest rate (not shown) support it as stationary; stationarity of both the real rate and inflation suggests that the nominal rate must be stationary as well. The inability of the tests presented in Table A1.1 to reject the unit root hypothesis may therefore suggest type II error.<sup>25</sup>

<sup>25</sup> Gali (1992) makes a similar argument for stationarity of the nominal interest rate.

<sup>&</sup>lt;sup>24</sup> Both tests allow for a constant and a linear time trend as the interest rate still display a downward path even after subtracting the inflation target. The ADF test includes one lag of the first difference of the adjusted variable.

#### Appendix 2: Choosing the Model's Lag Order

In order to evaluate the appropriate lag order we relied on four different statistics: The Akaike Information Criterion (AIC), the Schwarz Criterion (SC), the Hannan-Quinn Criterion (HQC), and a Modified Likelihood Ratio test (MLR). The MLR tests were conducted separately for each lag; that is, in a model with p lags we tested the significance of the pth lag relative to a model with p-1 lags. Lag tests are performed on the unrestricted model only.

The AIC and HQC support a high lag order of at least four lags, while SC and MLR test support only one lag. As a compromise we opted to estimation with two lags. The choice of two quarters lag also balances considerations between our short sample, on the one hand, and the need to allow the monetary transmission mechanism to show up in the data, on the other.

Table A2.1: Evaluation of Lag Order

Lag Order	1	2	3	4
AIC	-16.85	-17.04	-17.01	-17.84
SC	-13.80	-13.34	-12.67	-12.86
HQC	-15.71	-15.66	-15.39	-15.98
MLR Prob.	0.0000	0.2297	0.7257	0.1575

The MLR test corrects the small sample bias of the standard Likelihood Ratio test. For discussion of the modification see Sims (1980b) p. 17.

# Appendix 3: Estimation Results, The Unrestricted Model (UR)

FULL INFORMATION MAXIMUM LIKELIHOOD ESTIMATION

Date: 05-Jan-2010 Time: 09:52 Sample (adjusted): Q3-1997 Q3-2008

Included observations: 45 after adjustments Computation time: 0 seconds

Standard error	s in parenthe		in square b	rackets
SS Values:	DLYB 0.0400	DLEFX 0.0000	DLCPI 0.0200	IBOI 0.0400
DLYB			-0.064276	0.006633 ( 0.027268) [ 0.243249]
DLEFX			0.235940 (0.023400) [10.082884]	
DLCPI				-0.061294 ( 0.066554) [-0.920965]
IBOI		7.213602 ( 4.441253) [ 1.624227]		
DLYB (-1)	0.042010 ( 0.127052) [ 0.330652]	-1.055816 ( 0.352940) [-2.991490]		0.017942 ( 0.022904) [ 0.783348]
DLYB (-2)	0.058020 ( 0.107755) [ 0.538442]	-0.217072 ( 0.287429) [-0.755218]		-0.001548 ( 0.020639) [-0.075020]
DLEFX(-1)	-0.121607 ( 0.074221) [-1.638444]		-0.005898 ( 0.027927) [-0.211193]	0.038791 ( 0.013348) [ 2.906217]
DLEFX(-2)	0.121039 ( 0.060916) [ 1.986973]	-0.220464 ( 0.163109) [-1.351637]	0.040063 ( 0.023813) [ 1.682386]	-0.007601 ( 0.011135) [-0.682658]
DLCPI(-1)	0.027613 (0.270893) [0.101934]	-1.031526 ( 0.770265) [-1.339183]	0.354162 ( 0.099881) [ 3.545827]	0.075576 (0.049514) [1.526332]
DLCPI(-2)		-0.794313 ( 0.807454) [-0.983726]	0.222153 ( 0.092060) [ 2.413143]	0.120734 ( 0.047181) [ 2.558980]
IBOI (-1)	-2.517891 ( 0.904332) [-2.784254]	-7.877355 (4.514428) [-1.744929]	-1.031590 ( 0.361648) [-2.852473]	0.800475 ( 0.200384) [ 3.994701]
IBOI (-2)	2.740455 ( 0.878793) [ 3.118430]	0.667314 ( 2.335465) [ 0.285731]	0.831347 ( 0.355958) [ 2.335521]	0.069066 ( 0.187276) [ 0.368791]

```
3.407858
TARGET
               -0.968602
                                    0.180828
                                               -0.040001
 (SS = 0.0200) (0.547084) (1.517557) (0.214511) (0.110992)
               [-1.770481] [ 2.245621] [ 0.842981] [-0.360394]
DSECURITY
                0.113921
                          -0.338463
                                      0.083655
                                                0.005253
 (SS = 0.0000) ( 0.035312) ( 0.096939) ( 0.015648) ( 0.006972)
               [ 3.226103] [-3.491519] [ 5.345991] [ 0.753545]
EFGR
                0.291868
                           0.817521
                                      0.283220
                                                0.015069
 (SS = 0.0320) ( 0.494942) ( 1.313361) ( 0.182515) ( 0.092386)
               [ 0.589701] [ 0.622465] [ 1.551766] [ 0.163105]
EFGR m1
                0.718946
                         -0.310034
                                     -0.175083
                                               -0.073258
(SS = 0.0320) (0.513983) (1.378125) (0.192118) (0.093650)
               [ 1.398773] [-0.224968] [-0.911330] [-0.782260]
DLEFCPI
                1.037806
                         -1.053215
                                     1.620921
 (SS = 0.0200) (0.577606) (1.532722) (0.219095) (0.142289)
               [ 1.796737] [-0.687153] [ 7.398263] [ 0.604811]
DLEFCPI m1
               -0.184968 -2.086955
                                     0.572412
                                                0.138877
(SS = 0.0200) (0.630203) (1.753637) (0.232356) (0.111674)
               [-0.293506] [-1.190072] [ 2.463511] [ 1.243600]
TEE
                8.653026
                         4.432016 -0.410099
                                               0.419085
 (SS = 0.0400) (2.422218) (6.698006) (1.027484) (0.495060)
               [ 3.572356] [ 0.661692] [-0.399129] [ 0.846533]
                         -8.364939
                                     1.382469
IEF m1
               -8.538043
                                               -0.339141
(SS = 0.0400) (2.213168) (6.078704) (0.981100) (0.459867)
               [-3.857837] [-1.376106] [ 1.409101] [-0.737475]
DUM98Q4
               -0.066509
                          0.468889
                                     0.012832
 (SS = 0.0000) (0.039153) (0.151588) (0.020893) (0.012883)
               [-1.698682] [ 3.093172] [ 0.614204] [ 2.773487]
DUMQ2
                         -0.120271
                                     0.040576
                0.000698
                                                0.006218
 (SS = 0.2500) (0.022022) (0.062422) (0.008284) (0.004086)
               [ 0.031678] [-1.926741] [ 4.898247] [ 1.522000]
DUMO3
                         0.020782
               -0.009173
                                     0.017349
                                                0.005395
               ( 0.022379) ( 0.061423) ( 0.008256) ( 0.004331)
(SS = 0.2500)
               [-0.409879] [ 0.338340] [ 2.101346] [ 1.245513]
______
STD of Eq. 0.031326 0.083079 0.011444 0.005472
Log-likelihood
                                  452.377393
Akaike information criterion
                                   -16.327884
Schwarz criterion
                                   -12.915299
Hannan-Quinn criterion
                                   -15.055707
______
```

# **Appendix 4: Estimation Results, The Exogenous Variables Constrained Model (EXOGR)**

FULL INFORMATION MAXIMUM LIKELIHOOD ESTIMATION

Date: 05-Jan-2010 Time: 10:27 Sample (adjusted): Q3-1997 Q3-2008

Included observations: 45 after adjustments

Computation time: 6 seconds

Computation ti Standard error	s in parenthe			rackets
SS Values:	DLYB 0.0400	DLEFX 0.0000	DLCPI 0.0200	IBOI 0.0400
DLYB	=======	======	-0.013686 ( 0.056113)	0.014012
DLEFX			0.171010 ( 0.025174) [ 6.793129]	
DLCPI				0.060546 ( 0.029264) [ 2.068910]
IBOI		-2.262646 (3.001902) [-0.753738]		
DLYB (-1)	0.215110 ( 0.132734) [ 1.620612]	-0.999522 ( 0.350707) [-2.850016]	( 0.057491)	0.026601 ( 0.018176) [ 1.463539]
DLYB (-2)	0.174325 ( 0.106229) [ 1.641027]		-0.050355 ( 0.049694) [-1.013303]	
DLEFX(-1)	-0.053675 ( 0.070176) [-0.764856]		-0.023134 ( 0.037074) [-0.623994]	0.023767 ( 0.007772) [ 3.058225]
DLEFX(-2)	0.132705 ( 0.064069) [ 2.071286]	-0.219511 ( 0.162441) [-1.351334]	-0.014493 ( 0.029096) [-0.498122]	-0.000854 ( 0.008747) [-0.097620]
DLCPI(-1)	-0.283599 ( 0.252004) [-1.125376]	0.187599 ( 0.539081) [ 0.347998]	0.388213 ( 0.136386) [ 2.846421]	0.096699 ( 0.027738) [ 3.486188]
DLCPI(-2)	0.585841 ( 0.228976) [ 2.558524]	0.174460 ( 0.570885) [ 0.305596]		0.058381 ( 0.035572) [ 1.641216]
IBOI (-1)	-2.404908 ( 1.003554) [-2.396391]	-0.418432 (3.629200) [-0.115296]	-0.907361 ( 0.476247) [-1.905233]	0.969723 ( 0.167550) [ 5.787653]
IBOI (-2)	2.290873 ( 0.907810) [ 2.523517]	2.555805 ( 2.294927) [ 1.113676]	0.596020 ( 0.461604) [ 1.291192]	-0.052360 ( 0.157879) [-0.331645]

```
TARGET
                                   0.512503 -0.129606
(SS = 0.0200)
                                 ( 0.271747) ( 0.088175)
                                 [ 1.885958] [-1.469880]
                       -0.122229
DSECURITY
              0.116158
(SS = 0.0000) (0.036285) (0.086810)
              [ 3.201240] [-1.408014]
EFGR
               0.670889
(SS = 0.0320) (0.529432)
              [ 1.267188]
              1.020798
EFGR m1
(SS = 0.0320) (0.561476)
             [ 1.818060]
DLEFCPI
              0.972878
                                  1.006253
(SS = 0.0200) (0.453930)
                                (0.242590)
              [ 2.143235]
                                 [ 4.147956]
DLEFCPI m1
              0.350825
                                  0.212113
(SS = 0.0200) ( 0.617743)
                                 ( 0.266686)
              [ 0.567913]
                                 [ 0.795367]
IEF
                       15.141341
                        ( 6.028517)
(SS = 0.0400)
                       [ 2.511620]
IEF m1
                       -16.580247
(SS = 0.0400)
                       (6.004209)
                       [-2.761438]
              DUM98Q4
(SS = 0.0000) (0.043636) (0.140972) (0.022841) (0.007479)
              [-2.418799] [ 4.844627] [ 3.597753] [ 1.797682]
DUMQ2
                                   0.036724
                                 ( 0.010212)
(SS = 0.2500)
                                 [ 3.596171]
DUMO3
                                   0.019006
(SS = 0.2500)
                                 (0.011223)
                                 [ 1.693549]
______
STD of Eq. 0.037100 0.098010 0.016140 -0.005532
Log-likelihood
                                417.942389
Akaike information criterion
                                -15.952995
Schwarz criterion
                                -13.584260
Hannan-Quinn criterion
                                -15.069954
______
```

# **Appendix 5: Estimation Results, The Fully-Constrained Model (ENDOGR)**

FULL INFORMATION MAXIMUM LIKELIHOOD ESTIMATION

Date: 06-Jan-2010 Time: 08:39 Sample (adjusted): Q3-1997 Q3-2008

Included observations: 45 after adjustments

Computation time: 3 seconds

Standard errors in parenthesis, Z-stats in square brackets

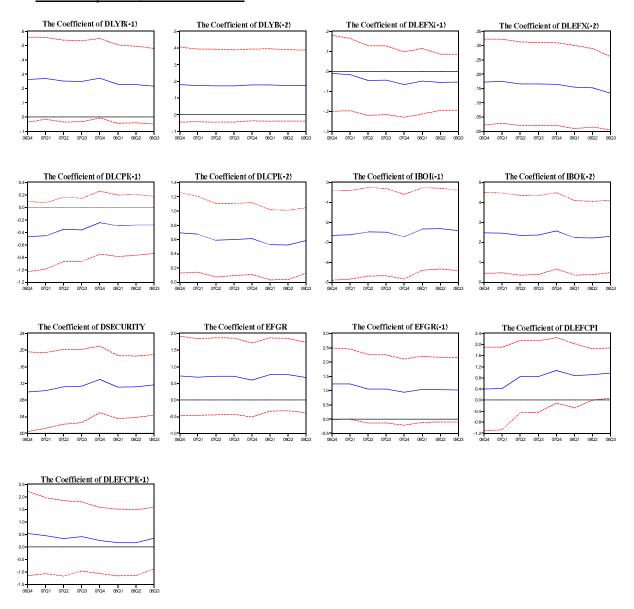
Standard errors	in parenthe	sis, Z-stats	in square b	rackets
SS Values:	DLYB 0.0400	DLEFX 0.0000	DLCPI 0.0200	IBOI 0.0400
DLYB			-0.079483 ( 0.064492)	0.019795 ( 0.020240) [ 0.978024]
DLEFX	-0.022747 ( 0.075761) [-0.300249]		0.176164 ( 0.026824) [ 6.567313]	
DLCPI	0.498843 ( 0.193868) [ 2.573100]			0.073980 ( 0.030825) [ 2.400044]
IBOI		-5.020648 ( 3.461009) [-1.450631]		
DLYB (-1)		( 0.312837)	0.080934 ( 0.059965) [ 1.349678]	
DLYB (-2)			-0.027800 ( 0.051434) [-0.540498]	
DLEFX(-1)		0.124147 ( 0.163613) [ 0.758784]	-0.029180 ( 0.037781) [-0.772349]	
DLEFX(-2)	( 0.062444)	-0.224402 ( 0.176167) [-1.273802]	-0.004406 ( 0.029906) [-0.147323]	-0.002295 (0.008939) [-0.256758]
DLCPI(-1)		0.222068 ( 0.585496) [ 0.379281]	0.393665 ( 0.138960) [ 2.832951]	
DLCPI(-2)	0.332028 ( 0.192905) [ 1.721197]	-0.188974 ( 0.526577) [-0.358873]	0.248841 ( 0.130295) [ 1.909825]	0.053598 ( 0.036316) [ 1.475894]
IBOI (-1)	-0.476095 ( 0.155118) [-3.069241]	5.803831 ( 3.718415) [ 1.560835]	-1.101716 ( 0.496436) [-2.219253]	1.015401 ( 0.174051) [ 5.833914]
IBOI (-2)	0.476095 ( 0.155118) [ 3.069241]	-0.965699 (1.339216) [-0.721093]	0.789604 ( 0.480876) [ 1.642013]	-0.091516 ( 0.163416) [-0.560018]

```
0.444434 -0.137066
( 0.277628) ( 0.088915)
TARGET
 (SS = 0.0200)
                                    [ 1.600825] [-1.541551]
DSECURITY
               0.097977
                        -0.125791
 (SS = 0.0000) (0.039662) (0.097273)
              [ 2.470309] [-1.293177]
EFGR
               0.473628
 (SS = 0.0320) ( 0.515033)
              [ 0.919608]
               0.913796
EFGR m1
(SS = 0.0320) (0.604860)
              [ 1.510757]
DLEFCPI
               -0.022747
                                    1.000012
 (SS = 0.0200) (0.075761)
                                   (0.246667)
              [-0.300249]
                                   [ 4.054098]
DLEFCPI m1
               -0.067517
                                    0.150390
(SS = 0.0200) \qquad (0.063758)
                                   (0.272865)
              [-1.058965]
                                   [ 0.551150]
IEF
                          5.020648
(SS = 0.0400)
                         (3.461009)
                         [ 1.450631]
IEF m1
                         -5.803831
(SS = 0.0400)
                         (3.718415)
                         [-1.560835]
DUM98Q4
               -0.142118
                        0.762089 0.074150 0.012024
 (SS = 0.0000) (0.057932) (0.160913) (0.023587) (0.007557)
              [-2.453197] [ 4.736030] [ 3.143632] [ 1.590968]
DUMQ2
                                     0.037045
                                    ( 0.010393)
 (SS = 0.2500)
                                    [ 3.564294]
DUMO3
                                     0.019211
(SS = 0.2500)
                                    (0.011437)
                                    [ 1.679697]
______
STD of Eq. 0.042219 0.109780 0.016410 -0.005581
Log-likelihood
                                 409.636741
Akaike information criterion
                                  -15.806077
Schwarz criterion
                                  -13.638082
Hannan-Quinn criterion
                                  -14.997871
______
```

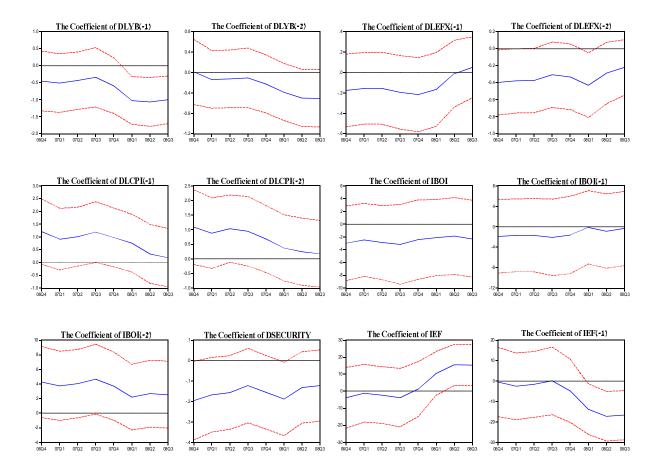
# Appendix 6: Robustness of the Estimated Coefficients +/- 2\*S.D

# A. The Exogenous Variable Constrained Model, EXOGR

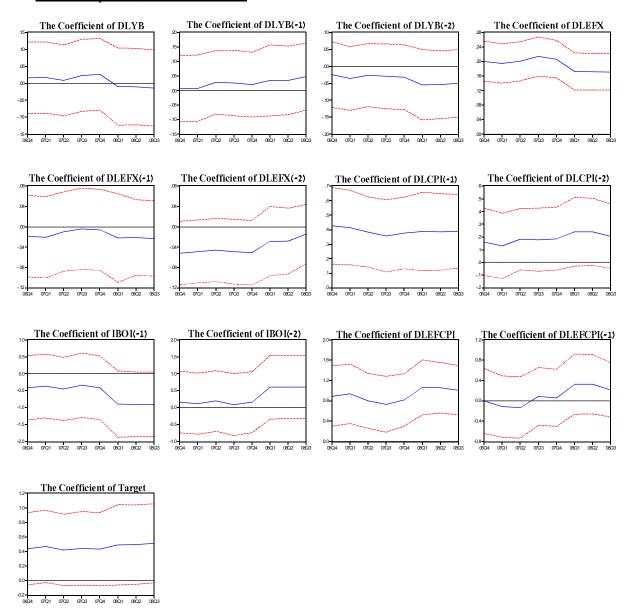
## **DLYB** Equation, EXOGR Model



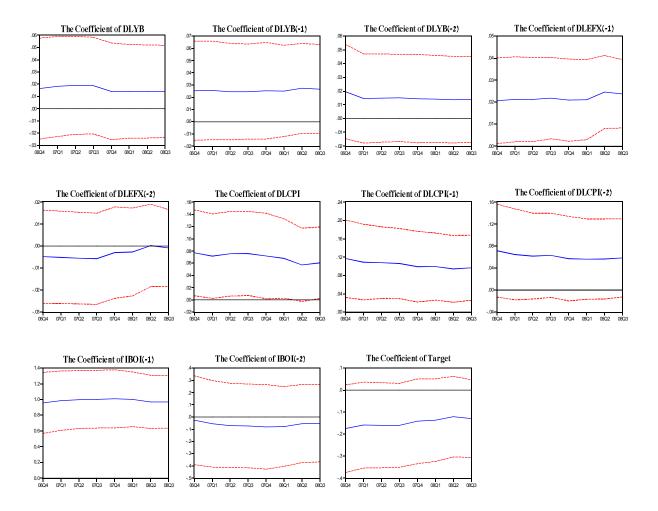
# DLEFX Equation, EXOGR Model



# DLCPI Equation, EXOGR Model

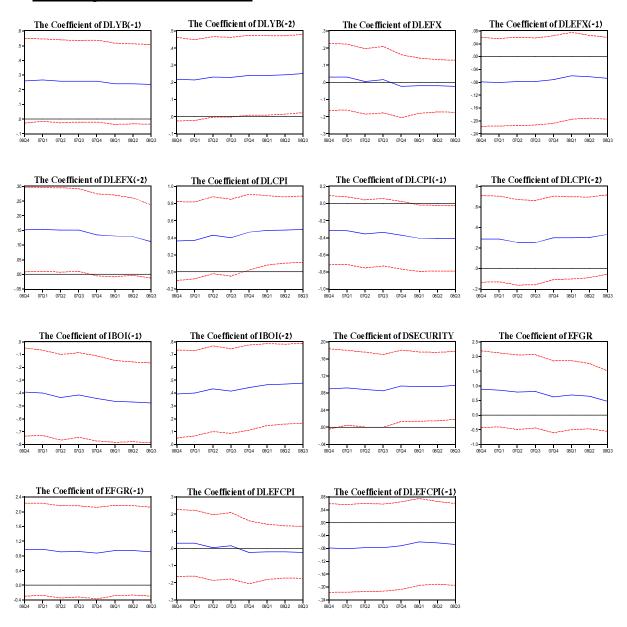


# The IBOI Equation, EXOGR Model

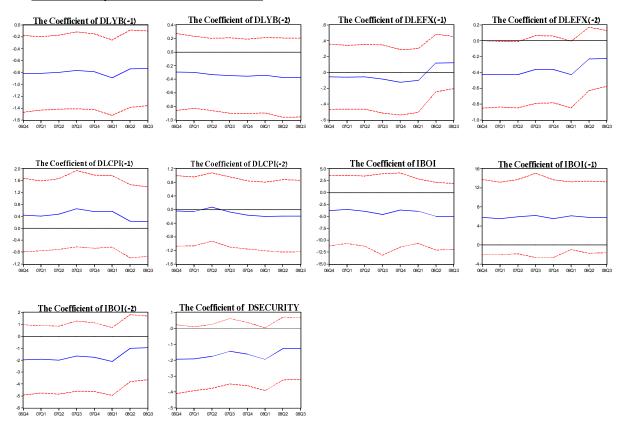


#### **B.** The Fully Constrained Model, ENDOGR

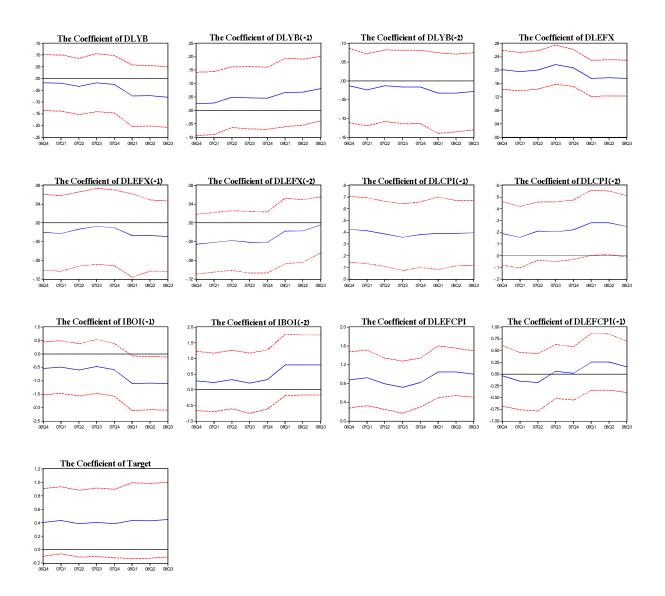
## DLYB Equation, ENDOGR Model



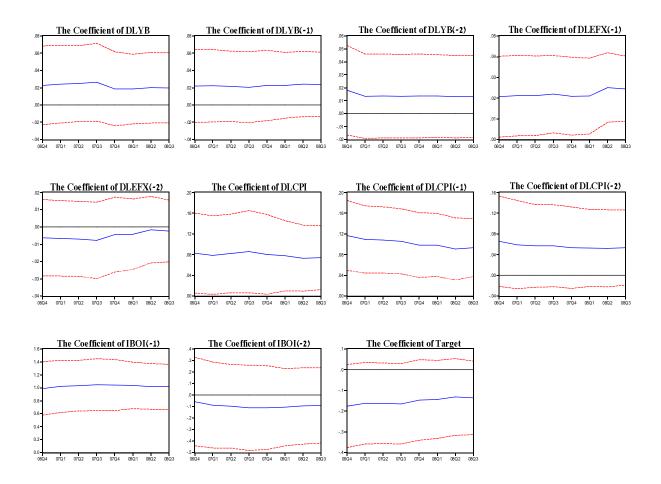
# DLEFX Equation, ENDOGR Model



# DLCPI Equation, ENDOGR Model



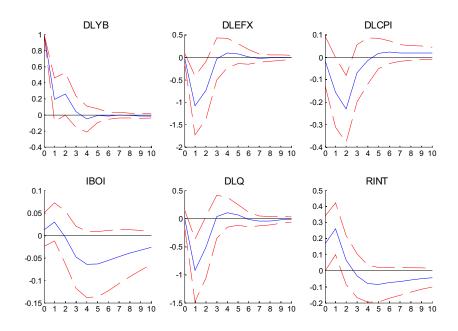
# IBOI Equation, ENDOGR Model



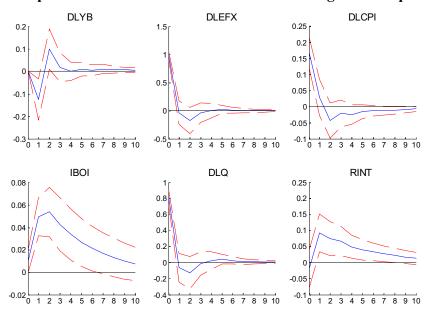
#### **Appendix 7: The Impulse Response Functions (In Percentage Points)**

## A. The Exogenous Variables Constrained Model (EXOGR)

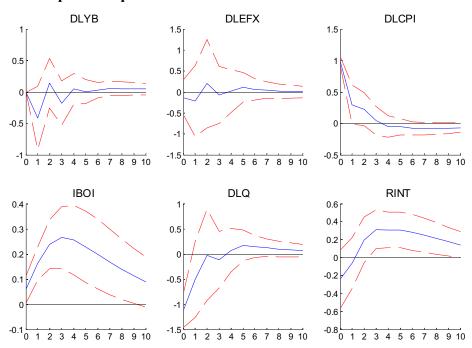
## Impulse Response to 1% Shocks to the Business Sector GDP Growth Rate:



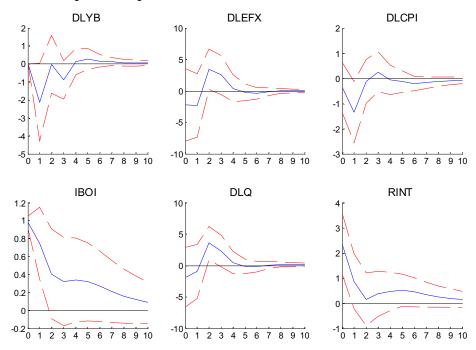
#### Impulse Response to 1% Shocks to the Nominal Exchange Rate Depreciation:



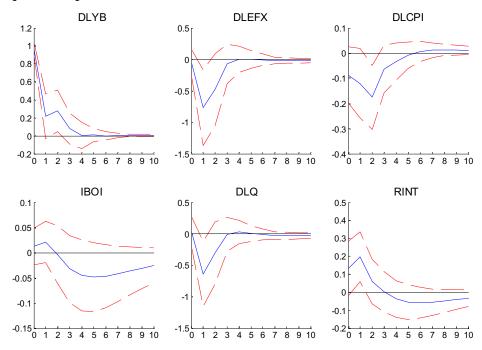
## Impulse Response to 1% Shocks to the CPI Inflation Rate:



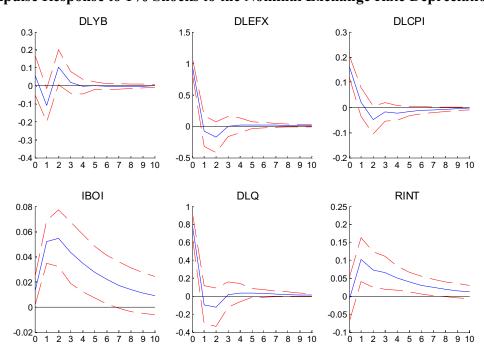
# Impulse Response to 1% Shocks to BOI Interest Rate:



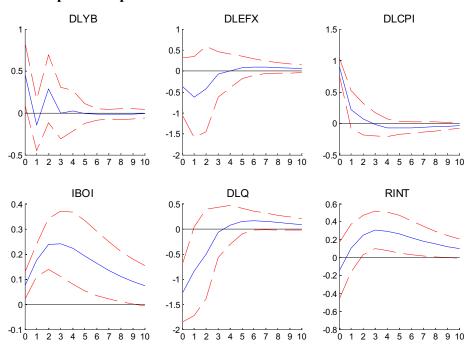
# B. The Fully Constrained Model (ENDOGR) Impulse Response to 1% Shocks to the Business Sector GDP Growth Rate:



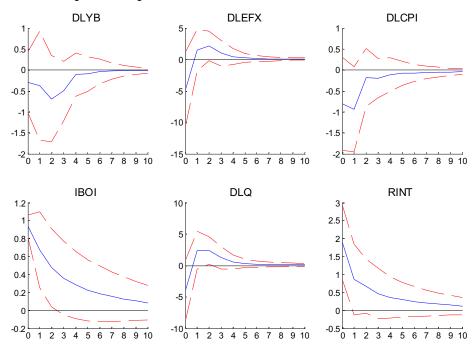
#### Impulse Response to 1% Shocks to the Nominal Exchange Rate Depreciation:



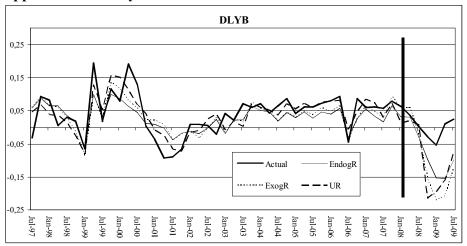
## Impulse Response to 1% Shocks to the CPI Inflation Rate:

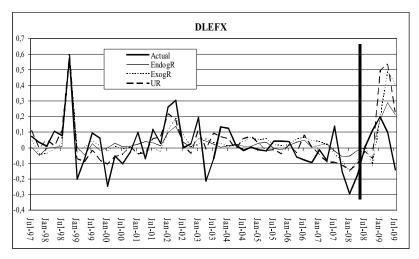


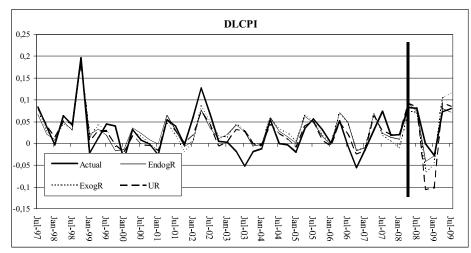
# Impulse Response to 1% Shocks to BOI Interest Rate:

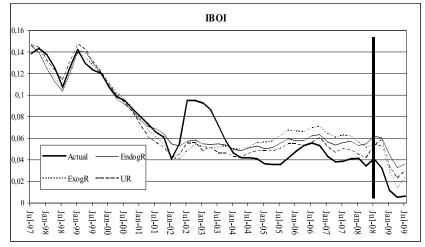


# **Appendix 8: The Dynamic Simulation**



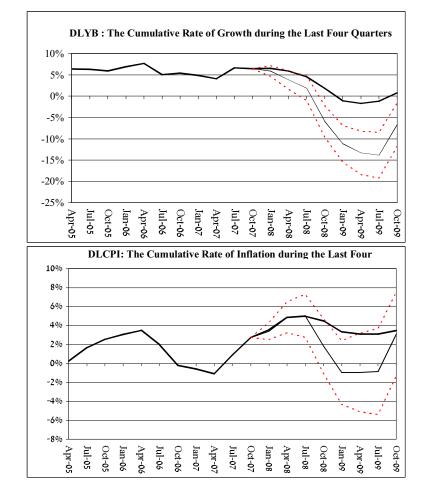


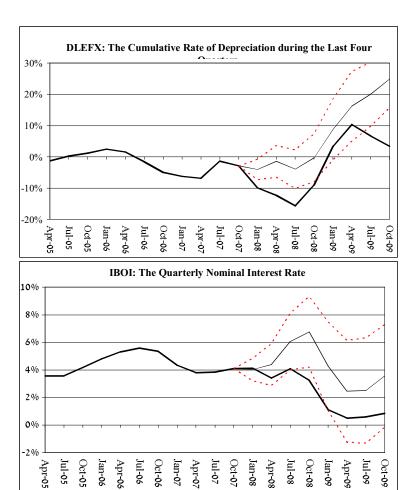




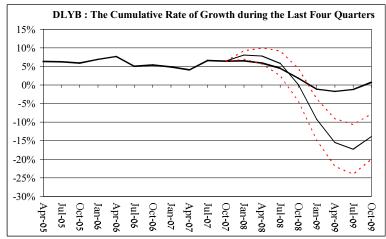
Appendix 9: Out of Sample Forecasts with one S.D. Confidence Interval 2008q1-2009q4

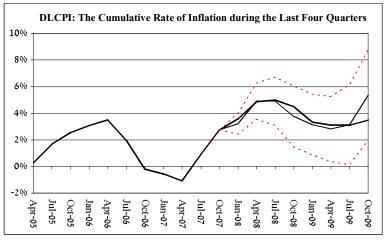
#### The Unconstrained Model, UR

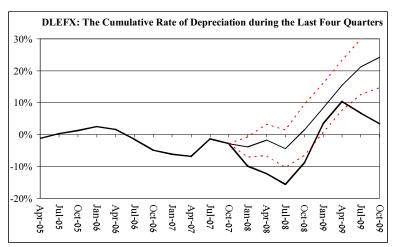


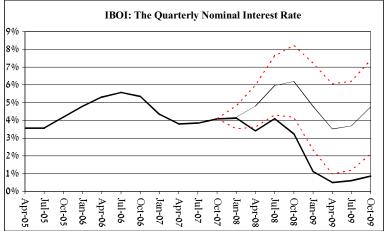


#### The Model with Constraints on the Exogenous Variables, EXOGR

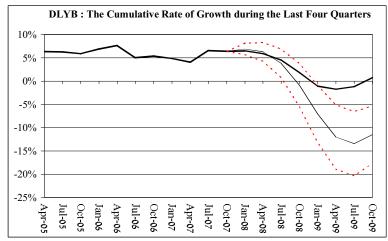


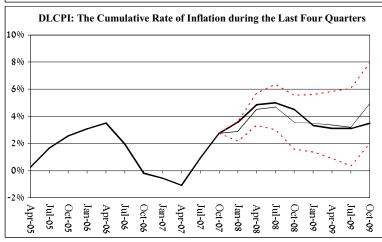


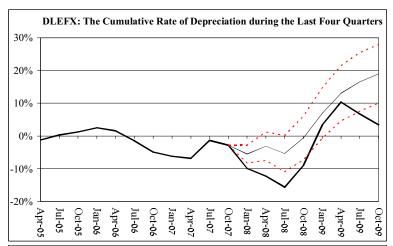


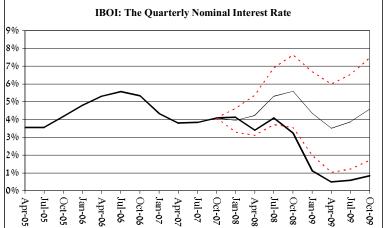


#### The Fully-Constrained Model, ENDOGR

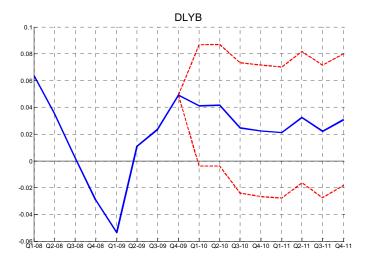


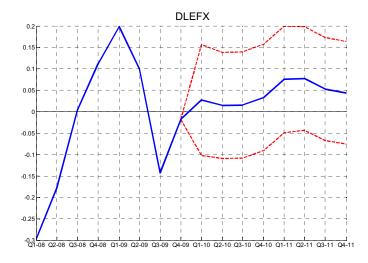


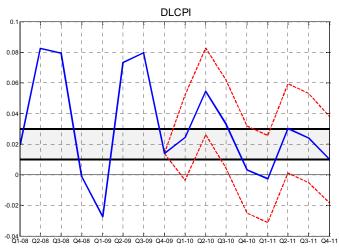


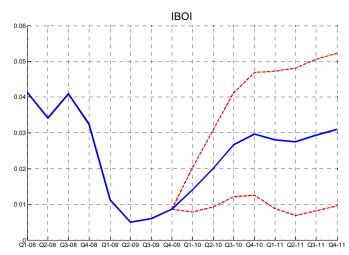


# Appendix 10: Forecasts for 2010 and 2011, ENDOGR Model (Annualized Rates, ±1 S.D.)









## **Appendix 11: The Fully-Constrained Model with Model Consistent Expectations**

FIML ESTIMATION OF RATIONAL EXPECTATIONS MODEL

Date: 26-Apr-2010 Time: 09:52 Sample (adjusted): Q3-1997 Q3-2008

Included observations: 45 after adjustments

Computation time: 29 seconds

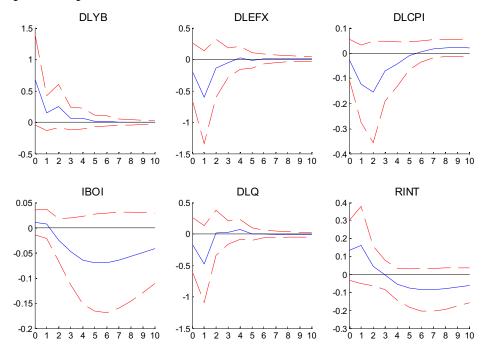
Standard errors in parenthesis, Z-stats in square brackets

Standard errors	in parenthesis	, Z-Stats in	square brack	ets
SS Values:	DLYB 0.0400	DLEFX 0.0000	DLCPI 0.0200	IBOI 0.0400
DLYB	-1.230022 ( 0.666963) [ -1.844214]		-1.502444 ( 2.003100)	-0.072204
DLEFX	0.046696 ( 0.162673) [ 0.287056]		0.556599 ( 0.518956) [ 1.072537]	
DLCPI	-0.046696 ( 0.162673) [ -0.287056]			0.368323 ( 0.097249) [ 3.787414]
IBOI	-0.961072 ( 0.447864) [ -2.145902]	( 6.609379)		
Et[DLYB (t+1)]	-0.230022 ( 0.666963) [ -0.344880]			
Et[DLCPI(t+1)]	0.961072 ( 0.447864) [ 2.145902]			-0.563923 ( 0.273361) [ -2.062925]
Et-1[DLYB (t)]	0.230022 ( 0.666963) [ 0.344880]			
Et-1[DLCPI(t)]	-0.961072 ( 0.447864) [ -2.145902]			
DLYB (-1)	( 0.241444)	( 0.439579)	-1.446886 ( 1.985293) [ -0.728802]	( 0.063561)
DLYB (-2)		-0.351154 ( 0.402349) [ -0.872759]	-0.371099 ( 0.377007) [ -0.984329]	0.033880 ( 0.026860) [ 1.261360]
DLEFX(-1)		0.374245 ( 0.222206) [ 1.684226]	-0.221672 ( 0.436195) [ -0.508195]	0.015305 ( 0.025710) [ 0.595277]

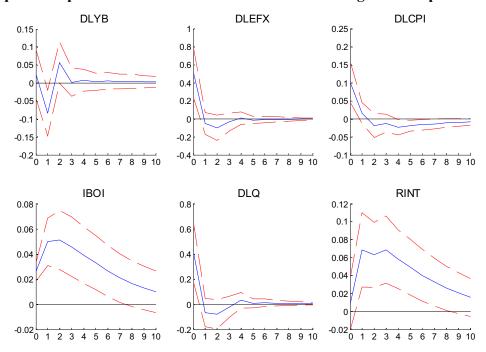
```
DLEFX(-2)
                   0.111225
                              -0.388164
                                           0.024854
                                                        0.001168
                ( 0.081542) ( 0.232770) ( 0.090527) ( 0.013742)
                [ 1.364023] [ -1.667582] [ 0.274546] [ 0.084980]
DLCPI(-1)
                   0.048064
                                1.593863
                                           -0.351239
                                                        -0.002736
                ( 0.117178) ( 0.940876) ( 1.328174) ( 0.085353)
                [ 0.410180] [ 1.694020] [ -0.264453] [ -0.032056]
                   0.229649
                               0.434527
                                           0.370511
DLCPI(-2)
                                                        0.112317
                ( 0.262906) ( 0.759290) ( 0.453372) ( 0.058322)
                [ 0.873503] [ 0.572280] [ 0.817234] [ 1.925815]
IBOI (-1)
                   0.961072
                              21.286562
                                           -2.648427
                                                        0.780259
                 ( 0.447864) ( 6.806816) ( 3.810144) ( 0.203998)
                [ 2.145902] [ 3.127242] [ -0.695099] [ 3.824829]
                             -4.019239
IBOI (-2)
                   0.016293
                                         -0.875544
                                                        0.047461
                 ( 0.212643) ( 1.587791) ( 1.572734) ( 0.152030)
                [ 0.076624] [ -2.531341] [ -0.556702] [ 0.312185]
TARGET
                                           10.642184
                                                         0.105811
                                          ( 13.180361) ( 0.257210)
(SS = 0.0200)
                                          [ 0.807427] [ 0.411378]
DSECURITY
                  0.135420
                              -0.193495
                ( 0.098996) ( 0.133966)
[ 1.367940] [ -1.444356]
(SS = 0.0000)
EFGR
                   0.913136
(SS = 0.0320)
                ( 1.363167)
                [ 0.669864]
EFGR m1
                   0.860719
(SS = 0.0320)
                (0.740811)
                [ 1.161860]
DLEFCPI
                   0.046696
                                            9.037515
                ( 0.162673)
                                         (9.528646)
(SS = 0.0200)
                [ 0.287056]
                                         [ 0.948457]
DLEFCPI m1
                  -0.048064
                                            2.954026
(SS = 0.0200)
                (0.117178)
                                         (2.803349)
                [-0.410180]
                                         [ 1.053749]
                              18.897234
IEF
(SS = 0.0400)
                             ( 6.609379)
                             [ 2.859154]
IEF m1
                              -21.286562
(SS = 0.0400)
                             (6.806816)
                             [-3.127242]
DUM98Q4
                  -0.136941
                                1.222470
                                            0.157215
                                                       -0.026128
               ( 0.111880) ( 0.259466) ( 0.139739) ( 0.015101)
(SS = 0.0000)
                [-1.223994] [ 4.711478] [ 1.125064] [-1.730192]
```

DUMQ2 (SS = 0.2500)			( [	0.018136 0.020951) 0.865642]	
DUMQ3 (SS = 0.2500)			(	-0.003502 0.024256) -0.144354]	
STD of Eq. Log-likelihood Akaike information Schwarz criterion Hannan-Quinn criter	criterion	0.154629	4 - -	0.046660 13.653213 15.806809 13.478222 14.938736	0.009231

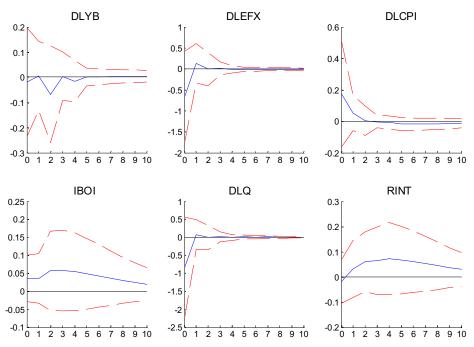
# Impulse Response to 1% Shocks to the Business Sector GDP Growth Rate:



## Impulse Response to 1% Shocks to the Nominal Exchange Rate Depreciation:



# Impulse Response to 1% Shocks to the CPI Inflation Rate:



# Impulse Response to 1% Shocks to BOI Interest Rate:

