ESTIMATING THE PREMIUM IMPLICIT IN THE YIELDS OF TREASURY BILLS

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This study examines the variation in the premium implicit in the yield curve of 12-month Treasury Bills during the period 1992–2002. This premium is measured using a number of econometric methods, one of which allows for the estimated premium to react to uncertainty as measured by the variance of the excess forward return. The findings indicate that during the disinflationary process the premium was characterized by a downward trend which corresponded to the decline in the risk of inflation. During the period from the end of 2000 until the end of 2002, the premium increased as a result of the Intifada which began in the last quarter of 2000 and the crisis of confidence in economic policy which characterized the first half of 2002. The study also examines additional factors that influence the premium. Among those found to have a significant influence were the gap between inflation and the inflation target, the interest rate gap between Israel and aboard, the proportion of Treasury Bills held by the public and the Bank of Israel interest rate (and its standard deviation).

1. INTRODUCTION

This study examines the variation in the premium implicit in the yield curves of 12-month Treasury Bills during the period 1992–2002. This period was characterized by a gradual process of disinflation that eventually led to price stability. Since the premium is not an observable variable, but rather is derived from the forward returns on Treasury Bills, it is estimated using a number of methods. The goal is to analyze its variation over time and to identify the factors that influence it. The estimation of the premium and the analysis of variables that affect it are of critical importance to the management of monetary policy since Treasury Bill yields are currently one of the main indicators used in evaluating the path of the interest rate expected by savers in the economy during the coming year. By neutralizing the changes in the yield curve due to the premium, the path of the interest rate as derived from the yield curve can be more accurately estimated.

The method used in this study is consistent with the expectations theory which attributes the structure of the yield curve and its fluctuations to, among other things, individuals' forecasts of the interest rate (on the assumption that no profits are possible from arbitrage

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between short-run and long-run investments).[1] There are a number of definitions of the expectations theory in the literature. For example, the "pure" expectations theory holds that the premium demanded by individuals on assets with yield is zero (Lutz, 1940) or that the premium is constant over time (Hicks, 1939).[2] According to this definition, the fluctuations in the yield curve are due only to the changes in individuals' forecasts of the future interest rate. According to a more liberal version, the expectations theory does not rule out the possibility that the premium can change over time and not just between the various horizons (Cook and Hahn, 1990).

Studies that have tested the pure expectations theory have not found broad empirical support for it. Thus, for example, the results of the most accepted study — which tested whether the slope of the yield curve explains the expected change in the spot interest rate (the Fama regression in Fama, 1984; see discussion below)[3] — were ambiguous and differed across countries, sample periods and the terms of the bonds. In the US, for example, most research has shown that the yield curve does not predict the change in interest rates (Fama, 1984; Mankiw and Summers, 1984; Mankiw and Miron, 1986; Campbell and Shiller, 1991; and others). In contrast, the results for other countries (such as England, France, Germany, Canada and Japan) were mixed (Minkiw, 1986, Hardouvelis, 1994; Bekaert, Hodrick and Marshall, 1995).[4]

In the absence of empirical support, a number of alternatives to the pure expectations theory have been suggested over the years. In the US, for example, one of the explanations is based on the monetary policy implemented during the period 1915–79 whose declared goal, among others, was the stabilization of the interest rate (Mankiw and Miron, 1986). In this situation, the expected changes in the interest rate were negligible and the behavior of the interest rate was characterized by a random process that could not be forecasted.

Another explanation of the failure of the expectations theory in its pure form is related to the assumption of the rationality of expectations which the estimation is based on. A number of studies tried to deal with the possibility of a certain amount of bias in expectations through the use of surveys of individuals' expectations regarding future interest rates. Using direct measures of these expectations, Friedman (1979) and Froot (1989) found that forward returns derived from the yield curve did not contribute to explaining the interest rate expected by individuals.

The most common explanation in the literature for the low correlation between the slope of the yield curve and changes in the interest rate is based on the possibility that the

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[1] The theory also assumes that there are no transaction costs. Thus, for example, individuals view investing in an asset for 12 months as equivalent to investing for 6 months and then at maturity investing again for the same period.

[2] This approach does not rule out the possibility that the premium varies between horizons. Thus, for example, according to the liquidity preference theory, individuals derive utility from high liquidity and therefore, the longer the term of the asset, the higher the premium that they will demand for the loss of liquidity.

[3] These tests were based on the Rational Expectations Model of Term Structure (RETS) which assumes that the ex-post change in the spot interest rate reflects, on average, the change that was expected at the time of the valuation of the forward return.

[4] A comprehensive survey of the empirical findings of research on the expectations theory can be found in Anderson et al. (1996) and Cook and Hahn (1990).

premium changes not only between horizons but also over time and it is this explanation which paved the way for the liberal version of the expectations theory. According to this approach, the validity of the theory should be tested by removing the fluctuations in the premium demanded by investors from the fluctuations in the yield curve. There are a number of theories that explain the variation of the premium over time, one of which attributes the variation to the assessed risks in holding the asset due to, for example, uncertainty regarding inflation (for an unindexed asset) and/or changes in the interest rate. This variation will be the result of, among other things, changes in confidence in the monetary regime and its goals. Changes in the demanded premium are likely to also be the result of fluctuations in economic activity. Kessel (1965), for example, claimed that due to the smoothing of private consumption, there is a negative correlation between the premium implicit in yields and the business cycle, such that during booms (when disposable income is high) investors demand a lower premium than during recessions (when disposable income is low). Fama and French (1989)'s findings support this hypothesis. According to the Preferred Habitat theory (Modigliani and Sutch, 1966), the premium will also vary as a result of changes in the tastes of savers and borrowers in the economy.[5]

Since the premium implicit in forward returns is not directly observable, a number of methods were developed in order to estimate its variation over time. These include: the ARCH-M method which was suggested by Engle, Lilien and Robins (1987) according to which the premium is dependent on the conditional variability of interests rates; the Single Factor Estimation method which is based on the assumption that the premium increases monotonically with the length of the horizon (Tzavalis and Wickens, 1997; Gordon, 2003)[6]; and the Lower Bound Estimation method which tests the minimal level of variability of the premium according to the variability of the excess forward return[7] on bonds (Startz, 1982).

Recently, a number of studies have focused on the Kalman filter as a way of removing the forecast error from excess forward return. The major advantage of this method is that it enables one to test how the premium has changed over time without first identifying the variables that affected it during the sample period. It should be mentioned that identifying these variables can be particularly difficult in the absence of a structured theoretical model that specifies the factors which determine the level of the premium. This is also due to the difficulty in quantifying a number of unobservable variables that are likely to have some influence on the premium, such as the political climate and the level of confidence in economic policy. Among the researchers who have used this method are Iyer (1997), who found that US data provided evidence of significant variability in the premium; Gravel and Morely (2004) who tested the premium using Canadian data under a number of specifications and found a strong connection between the estimated premium on the one hand and variability in the interest rate and a number of political variables on the other; and

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[5] According to this theory, individuals in an economy are likely to have different preferences for the horizon of loans and for the horizon of savings. In this case, surplus demand and supply in the capital market will influence the demanded premium.

[6] Evidence of the monotonicity of the premium can be found in a number of studies, such as McCulloch (1987).

[7] The excess forward return is defined as the difference between the forward return and the expected spot interest rate.

Gordon (2003) who tested the validity of the theory in an inflation target regime in which yields reflect not only inflationary pressure but also the reaction of the central bank to that pressure. He found significant evidence that in New Zealand the premium varies according to a stationary time series process while his findings for the variability of the premium in Australia were not as conclusive.

As in previous studies, we use a number of methods to estimate the premium implicit in Treasury Bill yields. In the first method, we test the possibility that the premium is fixed over time. This involves estimating the premium from the excess forward return using an ARIMA model and Fama (1984)'s equation. The use of Fama's equation also makes it possible to test whether there are indications of variation in the premium over time. In the second method, we estimate the premium using a Kalman filter. Since the estimation of the premium by simple smoothing of excess forward returns does not allow for the estimated premium to react to the level of uncertainty, the estimation was divided into two stages: during the first stage, the Markov Switching Regime was utilized which enables the mapping of the sample period into episodes of high and low levels of uncertainty regarding future interest rates/inflation. In the second stage, the premium is estimated from the excess forward returns such that its level is influenced by, among other things, the level of uncertainty estimated in the first stage. We include an additional component in this stage which reflects the persistent bias in the public's expectations of the interest rate. This bias is known as the "Peso Problem" and is related to the level of confidence in the central bank (see, for example, Bekaert, Hodrick and Marshall, 1995). Later in the study, a number of macroeconomic variables are tested which influence the level of the premium and its variation.

The main findings of the estimation indicate that there is a risk premium implicit in the yields on 12-month Treasury Bills that varies over time. Thus, the premium declined during the disinflationary process in the 90s as the risk of inflation diminished. From the end of 2000 until the end of 2002, the premium increased, primarily due to the Intifada which broke out in October 2000 and the crisis of confidence in economic policy during the first half of 2002 (as a result of the unexpected reduction in the monetary interest rate and the sharp increase in the budget deficit). Among the factors which influence the premium over time are changes in the Bank of Israel interest rate (and the standard deviations of these changes), the gap between inflation in the previous 12 months and the inflation target and the interest rate spread between Israel and abroad.

The study is structured as follows: Section 2 describes the main assumptions of the estimation and the calculation of excess forward returns on Treasury Bills. These are later used to estimate the premium. Section 3 describes the results of estimating the premium using an ARIMA model and the Fama equation. Section 4 focuses on estimating the premium using a relatively simple Kalman filter which separates between the premium and the forecast error of the excess forward return. Section 5 expands the Kalman filter estimation method and allows the estimated premium to react to the level of uncertainty regarding the risk of inflation. Section 6 tests whether the expanded model is preferable to the more restricted one. Section 7 tests which macroeconomic factors influence the premium and Section 8 presents conclusions.

2. THE FORWARD RETURN AND ITS CALCULATION AND THE BASIC ASSUMPTIONS OF THE ESTIMATION

The main assumption of the expectations theory is that the demanded yield on a long-term asset reflects an average of the yields on shorter-term assets such that there is no possibility of arbitrage profits through the buying and selling of assets of various terms to maturity. This assumption makes it possible to derive the monthly required forward return ($f_{t,t+i}$) from Treasury Bill yields (R_{t+i}) for various terms to maturity:

(1) $\qquad (1+R_{t,t+i})^i = (1+r_{t,t+1})(1+f_{t,t+2})(1+f_{t,t+3})........(1+f_{t,t+i})$,

where the demanded return in period t for an asset whose date of maturity is in period $t+i$ is a geometric average of the forward returns (until period $t+i$):

From equation (1) we can derive the forward return in the following manner:

(2) $\qquad f_{t,t+i} = \dfrac{(1+R_{t,t+i})^i}{(1+R_{t,t+i-1})^{i-1}} - 1.$

The forward return that is demanded for the period $t+i$ ($f_{i,j+i}$) reflects the spot interest rate expected during this period [$E_t(r_{t+i})$], and a premium (τ_i) which reflects the risk in holding an unindexed asset (which is due to the uncertainty surrounding inflation/the interest rate), as well as the compensation for the loss of liquidity as a result of investing for the long term. Thus:

(3) $\qquad f_{t,t+i} = E_t(r_{t+i}) + \tau_i.$

On the assumption that expectations are rational[8], the forecast error $u_{t,t+i}$ with respect to the spot interest rate behaves like white noise:

(4) $\qquad r_{t+i} = E_t(r_{t+i}) + u_{t,t+i} \qquad\qquad u_{t,t+i} \sim (0, \sigma_u^2)$

3. THE VARIOUS METHODS FOR ESTIMATING THE PREMIUM ON THE ASSUMPTION THAT IT IS FIXED OVER TIME

3.1. The estimation of the premium from the excess forward return

As pointed out earlier, the expectations theory in its classic form assumes that the demanded premium is fixed (or equal to zero). One of the approaches to estimating a premium that is fixed over time is to use Excess Forward Returns (EFR) which are calculated from the forward return ($f_{t,t+i}$) less the ex-post spot interest rate:

(5) $\qquad EFR_{t,t+i} = f_{t,t+i} - r_{t+i}.$

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[8] According to the Rational Expectations Model of Term Structure. A study by Elkayam and Ilek (2004) found evidence that the Israeli public's expectations are rational.

In this calculation we assume, as already mentioned, that expectations are rational so that the ex-post interest rate reflects an average of the expectations of the interest rate in previous periods (see equation (4) above). By using equations (3), (4) and (5), the excess forward return can be expressed as a sum of the premium and the forecast error, as follows:

(5') $EFR_{t,t+i} = \tau_i + u_{t,t+i}$,

where (τ_i) represents the demanded premium on the asset at maturity in another i months. Before presenting the estimation results, we will examine the statistical characteristics of the excess forward returns that were calculated for all horizons. These are presented in Table 1 below. Column (1) presents the excess forward returns calculated for all horizons; columns (2) and (3) present the average and standard deviation of the excess return; and column (4) presents the results of the test for the existence of a unit root in the EFR data.

Sample Period: The estimation used monthly data on Treasury Bill yields for various horizons during the period January 1993 to December 2003.[9] January 2002 was chosen for the starting date of the (effective) sample period since at that time the Bank of Israel began to announce the desired direction of the interest rate on monetary loans[10] and thus

Table 1
Statistical Characteristics of Excess Forward Returns (EFR), January 1993 to December 2003

EFR_{t+i}[1]	Mean	Std. Dev	P.P.Statistic[2]
(1)	(2)	(3)	(4)
$i = 2$	-0.661	0.771	-4.950
$i = 3$	-0.536	0.999	-5.722
$i = 4$	-0.453	1.222	-4.543
$i = 5$	-0.259	1.561	-4.346
$i = 6$	-0.405	1.851	-4.327
$i = 7$	-0.367	2.049	-4.271
$i = 8$	-0.208	2.249	-3.859
$i = 9$	-0.329	2.365	-3.876
$i = 10$	-0.532	2.503	-3.936
$i = 11$	-0.533	2.633	-4.339
$i = 12$	-0.117	2.651	-4.054

[1] Column (1) presents the horizon of the Treasury Bill which was used in the calculation of the excess forward return.

[2] Column (4) presents statistical values for the unit root test using the Phillips-Perron method with width i-1, where i is the Treasury Bill horizon. The critical values for rejection of the existence of a unit root are –2.89 (5%) and –3.48 (1%).

[9] Since the study uses 12-month excess forward returns, the effective sample period during which the premium is estimated (at the time of its valuation) is January 1992 to December 2002.

[10] See the article by Blass and Jabra (1996).

 provided the public with clearer information regarding the management of monetary policy. (This can also be seen in the Treasury Bill yields in previous periods which are characterized by a high level of volatility.) The sample period finishes at the end of 2003 prior to the imposition of a tax on Treasury Bill yields (in January 2004). Source of data: Bank of Israel.

    A number of conclusions can be drawn from the table. First, as the horizon increases, the standard deviation grows (monotonically) which is an indication of high uncertainty with respect to the future interest rate. Second, in all ranges, the excess forward return is stationary. This finding indicates that if the risk premium changes over time, it is likely that the process is not a random walk but rather mean-reverting. The figures also indicate that in all ranges the average excess forward return is negative which means that the ex-post interest rate was on average higher than the demanded forward return. This finding can have two possible explanations: the public systematically underestimated future interest rates and/or that during certain periods the public demanded a negative premium for holding Treasury Bills. This issue will be discussed further below.[11]

    Figure 1 describes the changes in EFR for horizons of 3, 6, 9 and 12 months. It can be seen that as the horizon of the forecast gets longer, the return to the mean becomes slower. In addition, the error becomes more persistent since the forecast errors follow a moving average (MA) process. Thus, as the horizon of the EFR becomes longer, the forecast errors

Figure 1
Changes in the Excess Forward Return for Various Horizons
 (percent)
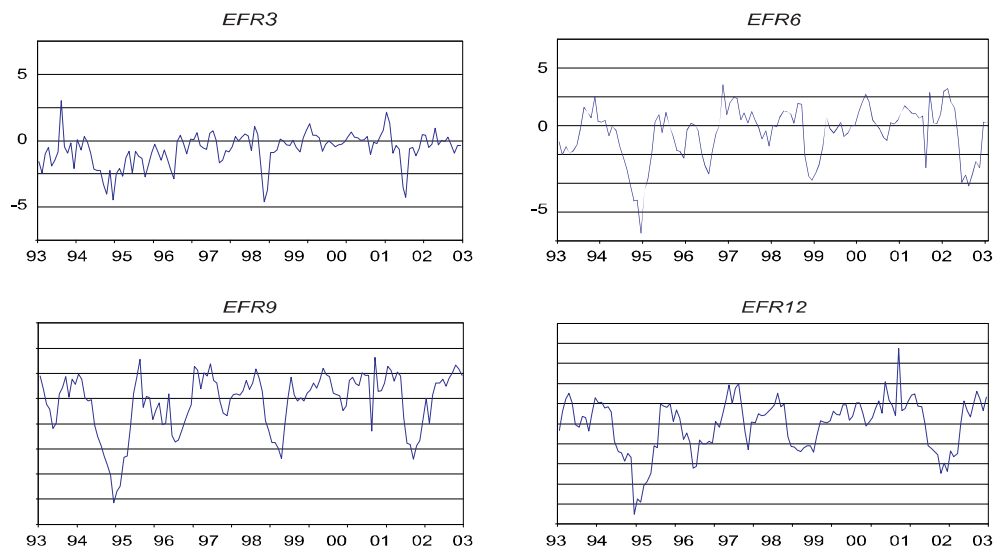
[11] The possibility of a negative premium within Treasury Bill yields is not likely and we have reservations in this regard. Below we will suggest other specifications for estimating the premium.

take on a larger magnitude. In addition, it can be seen that as the horizon of the EFR shortens, its variance decreases. This indicates that the premium for short horizons is almost zero or fixed and that all the variation in EFR is a result of error in the forecast of the interest rate.

Equation (5') implies that the expected excess forward return is equal to the premium implicit in Treasury Bill yields (on the assumption that it is constant). However, it is possible that the estimated premium is not identical to the expected excess forward return due to the moving average process in the forecast error[12]. The reason for this is that the expectation of the estimated moving average process in the final sample is not necessarily equal to zero. Therefore, in order to estimate the level of the premium (assuming it is constant) we estimated equation (5') as follows:

$$(5'')\qquad EFR_{t,t+i} = c_0 + u_{t,t+i}, \quad u_{t,t+i} = \sum_{j=1}^{i-1} \theta_j e_{t+i-j} + e_{t+i}, \quad e_{t+i} \sim N(0, \sigma_e^2),$$

where c_0 is the level of the premium and u_{t+i} represents the forecast error which is characterized by a moving average process.

Table 2 below describes the results of the estimation (carried out using the Maximum Likelihood method).

Table 2
Estimation of the Fixed Premium from Excess Forward Returns, January 1993 to December 2003

EFR^1	C_0^2
(1)	(2)
	-0.420
$i = 3$	(0.140)
	-0.153
$i = 6$	(0.348)
	-0.080
$i = 9$	(0.535)
	0.503
$i = 12$	(0.773)

[1] Column (1) presents the horizon of the Treasury Bill for which the excess forward return was calculated.

[2] The values in parentheses are the standard deviations of the coefficients.

[12] The data are monthly while the forecast period is longer than a month. This can be illustrated as follows: assume that in period t there was an unexpected shock in the interest rate. As a result, the (monthly) forecast errors of the public for a period of one year ahead (from period t-11 till period t) will be dependent on one another. This phenomenon has been dealt with in the literature in a similar manner (see, for example, Gravelle and Morley, 2005 and Gordon, 2003).

The results of the estimation show that the estimated premium in this specification is not significantly different from zero (except in the case of short horizons). For a horizon of three months, the premium is significant though its sign is negative (as in the case of the estimated premium for horizons of 6 and 9 months). Since the existence of a negative premium was not found in most of the empirical work, it is possible that this finding is evidence of bias in the above estimation. This possibility will be assessed below.

3.2 Estimation of the premium using Fama's approach

Fama (1984) suggested that the expectations theory (and through it the behavior of the premium) be tested using the explanatory power of the expected changes in the interest rate relative to the actual changes. The equation used by Fama for estimation can be obtained by adding the expression $(r_{t+i} - r_t)$, which represents the actual change in the spot interest rate between period t and period $t+i$, to both sides of equation (3):

(6) $$(r_{t+i} - r_t) + f_{t,t+i} = E_t(r_{t+i}) + \tau_i + (r_{t+i} - r_t)$$

After reorganizing the terms of the equation, we obtain:

(7) $$(r_{t+i} - r_t) = -\tau_i + (f_{t,t+i} - r_t) + u_{t,t+i}$$

where $u_{t,t+i}$ represents the forecast error under the assumption of rational expectations (see equation (4) above).

The expectations theory in its classical form requires that the premium τ_i be equal to zero or constant. Thus, equation (7) implies that as long as the assumption of rational expectations holds, the expected change in the interest rate is an unbiased estimator of the ex-post change in the interest rate. In order to test this hypothesis, equation (7) is estimated in the following form:

(8) $$(r_{t+i} - r_t) = \alpha + \delta(f_{t,t+i} - r_t) + u_{t,t+i}$$

The validity of the expectations theory, as mentioned earlier, requires that the coefficient δ not be significantly different from 1. (In this case, the constant in the equation represents the estimated premium when $\tau_i = -\alpha$.) Cook and Hahn (1990) showed that if the premium τ_i is not constant over time, then the estimated coefficient δ will be biased downward and significantly different from 1. In the extreme case, in which the standard deviation of the premium is equal to the standard deviation of the expected change in the interest rate, there will be a maximal bias and the estimated coefficient δ will be equal to 0.5. If the coefficient δ is in fact smaller than 1, the classical assumption of the expectations theory (which implies a constant or zero premium) is put into doubt and/or the assumption of rational expectations on which the estimation is based can be rejected. Before we begin the analysis of the results, it is worth mentioning the moving average process in the forecast errors that were described above. Ignoring this process causes a bias in the variances of the estimated coefficients and as a result statistical inferences will be incorrect. There are a number of accepted ways of dealing with this

problem. We have chosen the Newey-West (1987) approach which suggests estimating the model using OLS and adjusting the standard deviations of the coefficients.

Table 3 above presents the results for the estimation of equation (8). Column (1) presents the period of the forecast. Columns (2) and (3) present the estimated coefficients with the adjusted standard deviations according to the Newey-West method. Columns (4) and (5) present the p-value for the test of the hypothesis that δ is equal to 1 and 0.5, respectively. (The value $\delta = 0.5$ represents the lower bound of the possible bias in the coefficient as a result of the variation in the premium.) Column (6) presents the goodness-of-fit of the regression.

Table 3
Tests of the Classical Expectations Theory according to Fama's Approach

$$r_{t+i} - r_t = \alpha + \delta(f_{t,t+i} - r_t) + u_{t,t+i}$$

Period of the Forecast	α	δ	$\delta = 1$	$\delta = 0.5$	R^2
(1)	(2)	(3)	(4)	(5)	(6)
$i = 2$	0.268 (0.075)	0.442 (0.071)	0.000	0.414	0.23
$i = 3$	0.452 (0.168)	0.864 (0.143)	0.343	0.012	0.26
$i = 4$	0.628 (0.241)	1.303 (0.184)	0.102	0.000	0.38
$i = 5$	0.317 (0.269)	1.138 (0.174)	0.430	0.004	0.28
$i = 6$	0.410 (0.389)	1.010 (0.230)	0.973	0.029	0.22
$i = 7$	0.309 (0.443)	0.903 (0.216)	0.654	0.064	0.21
$i = 8$	0.178 (0.469)	0.936 (0.219)	0.772	0.049	0.16
$i = 9$	0.256 (0.532)	0.883 (0.243)	0.632	0.116	0.15
$i = 10$	0.341 (0.661)	0.782 (0.278)	0.434	0.313	0.13
$i = 11$	0.233 (0.748)	0.669 (0.335)	0.325	0.615	0.11
$i = 12$	-0.029 (0.635)	0.722 (0.261)	0.289	0.397	0.13

The standard deviations of the coefficients after the Newey-West adjustment appear in parentheses.

Several conclusions can be drawn from the results of the estimation presented in Table 3. First, in almost all of the tests, the hypothesis that $\delta = 1$ cannot be rejected (at a reasonable level of confidence). However, as the period of the forecast becomes longer, the hypothesis that $\delta = 0.5$ cannot be rejected either. This finding raises the possibility that the premium on Treasury Bill yields, particularly for longer horizons, is not constant over time. It should be mentioned that even the cases in which the values of δ are found to be between 0.5 and 1 may indicate that the premium varies over time. Second, as the period of the forecast increases, the model's goodness-of-fit declines. On one hand, this finding points to the difficulty in predicting interest rates as the horizon increases and, on the other hand, may indicate the model's lack of specification which is apparently manifested in a variable premium. The estimation of the premium by means of the Fama equation was also carried out on the assumption that forward returns, which are calculated from the Treasury Bill curve, contain measurement error (for example, as a result of a random change in the preferences of individuals regarding the term of their investment). The results of the estimation provide significant evidence for the validity of the expectations theory in its classical form for short terms while for longer terms the results are ambiguous, as was the case for the findings in Table 3 (see Appendix A3).

In conclusion, the results of the Fama regression imply that the validity of the classical expectations theory (according to which the premium is either zero or constant) cannot be rejected for short-term Treasury Bills. However, the tests for longer terms do not provide unambiguous evidence in support of the theory.

These findings provide the motivation to search for other frameworks which can be used to test whether the premium is constant or variable over time.

4. ESTIMATION OF THE PREMIUM USING A KALMAN FILTER

One of the accepted ways of estimating a premium that varies over time is to use a Kalman filter. This is due to the fact that the premium is an unobservable variable and the process of identifying it involves making certain assumptions regarding its stochastic behavior.

In order to estimate the premium using the Kalman filter method, we assume that its movements are governed by an autoregressive process of the following form:

(9) $\quad EFR_{t,t+i} = \tau_{t,i} + u_{t,t+i}$,

$$\tau_{t,i} = \alpha_0 + \alpha_1 \tau_{t-1,i} + \varepsilon_t$$

$$u_{t,t+i} = \sum_{j=1}^{i-1} \theta_j e_{t+i-j} + e_{t+i} \quad , \quad e_{t+i} \sim N(0, \sigma_e^2),$$

where the parameter $\tau_{t,i}$ represents the evaluated premium in period t for an asset which matures in i months. $\dfrac{a_0}{1 - a_1}$ represents the unconditional expectation of the premium while the parameter ε_t represents the shock to the premium in period t. We allow the forecast error ($u_{t,t+i}$) to vary according to a moving average process of order i-1.

Since there are indications that the premium will vary only for long horizons (see the Fama estimation), we restrict the discussion in the following sections to the premium implicit in the yields of 12-month Treasury Bills. Table 4 and Figure 2 below describe the results of the estimation of the premium using the Kalman filter together with the excess forward returns. We would mention that the premium (appearing as a bolded line) is presented according to its valuation 11 months previously.

Table 4
Results of the Estimation of the Parameters from the Excess Forward Return Using a Kalman Filer, January 1993 to December 2003

Sum of the squared errors	α_0	α_1
1.187	0.007 (0.039)	0.945 (0.017)

The values in parentheses are the standard deviations.

The results of the estimation show that during the disinflationary process there were extended periods in which the ex-post interest rate was higher than the ex-ante forward returns demanded previously. As a result, the premium estimated was negative during those periods. Although in the literature there are theoretical explanations for the existence of a negative premium[13], we believe that the negative gap created between the forward return and the spot interest rate does not reflect a negative premium but rather a systematic error in expectations which sometimes characterizes a disinflationary process (Gravelle and Morley, 2005; Gordon, 2003). This phenomenon, which is sometimes described in the literature as the Peso Problem, is explained primarily by the uncertainty with regard to policy and its goals.[14]

In our sample, the Peso Problem was apparently the result of the uncertainty regarding the commitment of the monetary regime to achieving inflation targets. In this situation, it appears that the public took into account the possibility that the rate of interest would in the future be lower than that required to achieve the inflation target.[15] Indeed, one can see from Figure 2 that the negative gap between the forward return and the spot interest rate mainly characterizes the periods in which there was a low level of confidence in monetary policy with respect to its commitment to bring down inflation.[16] Thus, for example, for most of the first two and a half years of the inflation target regime (from the second half of 1994 until the end of 1996)[17], during which there was a certain amount of uncertainty with regard to

[13] For example, according to the Preferred Habitat theory, when individuals prefer to invest in long-term assets and to lend for shorter terms, a negative premium will result.

[14] The Peso Problem was first mentioned in relation to the uncertainty regarding the foreign exchange regime in Mexico prior to the currency crisis in that country in 1976.

[15] Evidence during the 90s of an anti-inflationary reaction which was stronger than that expected by the public and which had been agreed upon with the government can be found in a study by Zussman (2004).

[16] We would point out that the surprise in the nominal Bank of Israel interest rate was the result of an unexpected rise in the real interest rate rather than the result of inflation. In periods of uncertainty, the central bank deviated from its normal behavior (the Taylor Rule) and raised the real interest rate in a more aggressive manner in order to bring down inflation and thus regain the confidence of the public.

[17] Inflation targets were formally adopted by the government in the second half of 1994.

Figure 2
The Estimated Premium on 12-month Treasury Bills
(percent)
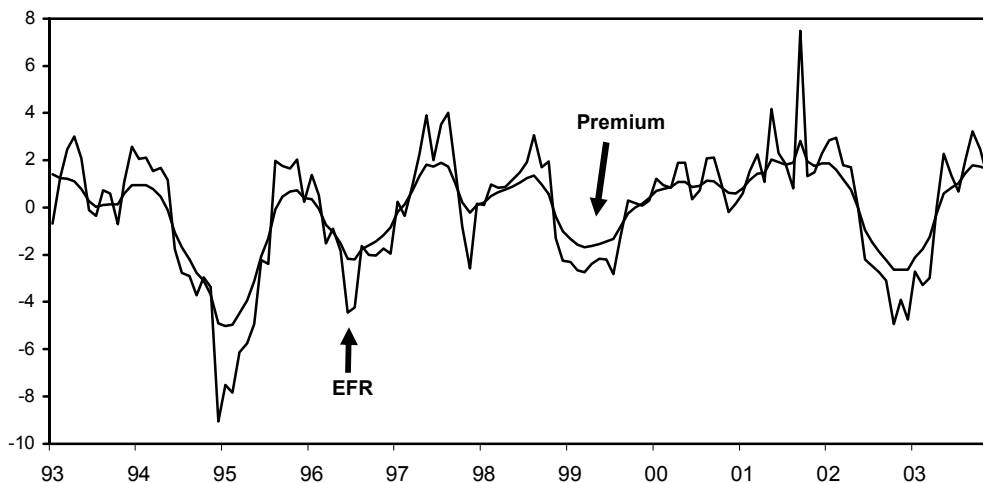
the degree of the central bank's commitment to moderating the rate of inflation, the excess forward return was negative as a result of an unexpectedly tight monetary policy.

Two additional episodes in which the excess forward return was negative occurred after sharp reductions in the interest rate in August 1998 (1.5 percent) and December 2001 (2 percent). These reductions, which were accompanied by inflationary pressures and the undermining of confidence in monetary policy, required subsequently raising the interest rate well beyond its expected level. This move, which was required in order to restore confidence in monetary policy, led to a persistent bias in individuals' expectations of movements in the interest rate. We would point out that the bias in expectations during these periods does not indicate that expectations were not rational but rather reflects the uncertainly regarding the central bank's willingness to persevere in achieving the inflation target.

As mentioned above, the major advantage in using the Kalman filter is that it allows the estimated premium to vary within the sample period according to a pre-determined stochastic process. In our sample, which contains a number of episodes of persistent bias in expectations (beyond the moving average process in the forecast errors), this advantage is also a disadvantage since the bias resulting from the Peso Problem is not identified in estimation and is interpreted as a decline in the premium to a negative level.[18]

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[18] A similar path for the estimated premium was obtained by single factor estimation which assumes that at any point in time, the premium implicit in the excess forward return is influenced by only two components: the time horizon of the asset and the risk in the timing of the valuation of the asset. This approach is essentially based on cross-section estimation which, at any point in time, utilizes the information implicit in the excess forward returns for all horizons with the main assumption being that the premium increases monotonically — though at a decreasing rate — as the asset's horizon becomes longer (see Tzavalis and Wickens, 1997 and Gordon, 2003).

Another disadvantage in this type of estimation is that the premium is derived only from the level of the excess forward return without taking into account changes in its variance. Using a theoretical framework, Engle et al. (1987) showed that higher uncertainty leads to a demand for a higher premium by savers. This relationship is also supported by the results of empirical testing.

In the following sections, we will deal with the two points mentioned above — the Peso Problem and the reaction to the level of uncertainty — while estimating the premium.

5. THE ESTIMATION OF THE PREMIUM, ITS REACTION TO UNCERTAINTY AND THE SYSTEMATIC BIAS IN EXPECTATIONS

During the disinflationary process in the Israeli economy, there were a number of periods of high uncertainty with respect to the path of the interest rate and the inflationary environment. It is likely, given the low level of confidence in policy, that this uncertainty led to a higher premium on financial assets and at the same time created a persistent bias in expectations. Therefore, in order to identify the demanded premium from the excess forward return, the estimation is carried out in two stages: In the first stage, we will identify the periods in which uncertainty was particularly high. This is necessary since later in the estimation we will allow the premium to react to the level of uncertainty. In the second stage, we will include a third component in the excess forward return — in addition to the premium and the forecast error — which will capture the persistent bias in expectations. This third component will be identified by making a number of assumptions with regard to its stochastic behavior.

5.1 Identification of the periods of high uncertainty

The level of uncertainty with regard to the path of the interest rate/inflation can be identified from the variance of the excess forward return since it represents, among other things, the gap between individuals' forecast of the future interest rate and the ex-post interest rate. We would stress that one of the weaknesses of this method of identification is that the level of uncertainty is likely to also be affected by outlying events which were not taken into account at the time of the valuation of the premium and perhaps should not have affected its level. Thus, for example, it can be claimed that the decline in the interest rate at the beginning of 2002, which led to a sharp increase in the variance of the excess forward return, had not been expected and therefore there is no reason that the premium would react before the event. In order to moderate the influence of these outlying events on the estimated premium, an additional component will be added during the second stage of the estimation which captures the possible bias in expectations of the path of the interest rate.

The mapping of the sample according to level of uncertainty was done using the Markov Switching Regime approach, as follows:

$$(10) \quad EFR_{t,t+i} = \mu_{S_t} + u_{t,t+i}, \quad u_{t,t+i} = \sum_{j=1}^{i-1} \theta_j e_{t+i-j} + e_{t+i}, \quad e \sim (0, \sigma_{S_t}^2)$$

$$\mu_{S_t} = \mu_0 + \mu_1(1 - S_t), \qquad S_t = \{0,1\}, \quad \Pr[S_t = 0] = p, \quad \Pr[S_t = 1] = q,$$

$$\sigma_{S_t}^2 = \begin{cases} \sigma_0^2 & \text{iff} \quad S_t = 0, \\ \sigma_1^2 = \dfrac{\sigma_0^2}{(1 + hS_t)} & \text{iff} \quad S_t = 1, \quad h < 0. \end{cases}$$

where $\mu_0 + \mu_1$ represents the expectation of the excess forward return in a situation of low uncertainty (variance) (S=0), while μ_0 alone represents the expectation of the excess forward return in a situation of high uncertainty (variance) (S=1). A priori, when the expectation of the excess forward return reflects the expectation of the premium alone, a positive correlation would be expected between the two. Thus, the higher the level of uncertainly, the higher the premium will likely be that the public demands for the risk in holding an asset of this sort. However, during the sample period, in which the expectation of the calculated excess forward return was influenced by the existence of a moving average process in the forecast errors and apparently also reflected a persistent downward bias in expectations (the Peso Problem), there may have in fact been a negative correlation between them.

The results of the estimation of the parameters μ_0, μ_1, the high variance σ_1^2 and the decrease in variance h are presented below in Table 5 and Figure 3.

Table 5
Results of the Estimation of the Parameters Using the Markov Switching Regime Method

μ_0	μ_1	σ_1^2	h
-0.134	0.123	3.908	-0.881
(0.096)	(0.178)	(0.854)	(0.052)

The values in parentheses are the standard deviations.

The results of the estimation in fact differentiate between two regimes since during the sample period there were a number of episodes in which the level of uncertainty rose significantly. This finding is also supported by the high probability of being in the regime with high variance. (In certain periods this probability is close to one; see Figure 3 below.) At a later stage, we will use these probabilities to map the situations of uncertainty during the sample period. This mapping will make it possible to link the extent of uncertainty to the level of the premium in the next stage of the estimation.

Figure 3
The Excess Forward Return and the Probability of a Regime with High Variance
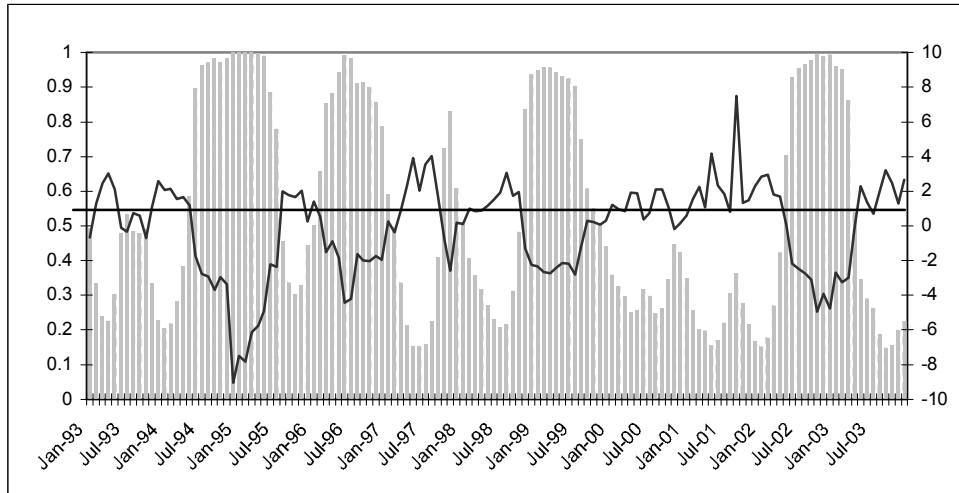
The excess forward return for a 12-month period ———

The probability of being in a regime with high variance (left axis) ═══

However, we would mention that no evidence was found of a significant change in the expected excess forward return between the two regimes. In our estimation, this finding is likely an indication of two opposing forces which cancel each other out: on the one hand, the high level of uncertainty which apparently led a higher premium and, on the other hand, the persistent downward bias in the expectations of the interest rate.

5.2. Estimating the premium

At this stage we will expand the empirical testing in two directions: First, we will allow the premium to react to the level of uncertainty in the economy according to the mapping performed in the first stage, whereby the estimation of the premium will take into account periods of high uncertainty. We assume that the increase in the level of uncertainly between the two regimes is the result of forecast error while the variance of the premium was equal in both situations. Second, we will include a third component (in addition to the premium and the forecast error) in the excess forward return which will represent the persistent bias in the expectations of the interest rate (as a result of the Peso Problem). The optimal way of removing the bias from the excess forward return is to select an exogenous variable which is a good proxy for the bias. However, since it is very difficult to find such a variable (which is not correlated with the premium), the bias in expectations will be identified through an assumption regarding its stochastic process over time. The components of the excess forward return will be identified in the following manner:

(11) $EFR_{t,t+i} = \tau_{i,t} + k_{t,i} + u_{t,t+i}$

$$\tau_{i,t} = \alpha_{0S_t} + \gamma_{S_t} \log \sigma_1^2 + \alpha_{1S_t} \tau_{i,t-1} + \varepsilon_t \ , \quad \gamma_{S_t} = \begin{cases} \gamma, & iff \quad S_t = 1 \\ 0, & iff \quad S_t = 0 \end{cases}$$

$$k_{t,i} = \beta_1 k_{t-1,i} + \eta_t, \qquad \eta \sim N(0, \delta_\eta^2),$$

$$u_{t,t+i} = \sum_{j=1}^{i-1} \theta_j e_{t+i-j} + e_{t+i}, \quad e_{t+i} \sim N(0, \delta_{S_t}^2), \quad \varepsilon_t \sim N(0, \sigma_{S_t}^2),$$

where the parameter $k_{t,i}$ represents the persistent bias in expectations or, in other words, the gap between the expectations of the path of the interest rate, which were created in period t, and the realization of the interest rate i periods later. We assume that this bias behaves like a first-order autoregressive process which captures the gradual learning process among the public with respect to the policy being implemented. It is taken into account that the bias in expectations ($k_{t,i}$) does not include a constant because in the long run, on the assumption that expectations are rational, the bias disappears. The equation for the premium ($\tau_{t,i}$) contains a first-order autoregressive process (as in the previous estimation) and the variance of the excess forward return when uncertainty is high (σ_1^2). Therefore, the parameter γ_{S_t} is likely to receive a positive value only in situations of high uncertainty ($S=1$ according to the mapping done in the first stage). In the rest of the situations, this parameter will receive a value of zero. This specification, which allows the amount of variance to influence the expectation of the premium according to the ARCH-M method of Engle, Lilien and Robins (1987), is included within the Kalman filter estimation in a situation of regime change, as in Smith (2002). In addition, the forecast error ($u_{t,t+i}$) behaves according to a moving average process, as in the previous specification.

The results of the estimation, which are presented in Table 6 and Figure 4, show that the premium is stationary (with jumps in expectation as a result of the regime change) and the value of the autoregressive coefficient, which is somewhat larger than 0.9, indicates that the process of convergence to the mean is relatively slow. On average, the estimated premium (during the whole sample period) is 0.7 percent while during periods of high uncertainty, the value of the premium rises to even beyond 1.5 percent.

Table 6
Results of the Estimation of the Parameters from the Excess Forward Return,
**January 1993 to December 2003, Expanded Model**

State of Nature	α_0	α_1	γ_s	β_1 [2]
$S = 1$ (High Variance)	0.103 (0.377)	0.938 (0.000)	0.637 (0.086)	0.909 (0.000)
$S = 0$ (Low Variance)	0.266 (0.021)	0.915 (0.000)	-----	0.933 (0.000)

[1] The values in parentheses are p-values.

[2] The path of the bias in expectations of the interest rate is presented in Table A4 in the Appendix.

As shown in the table, the premium declined gradually during disinflation, apparently as a result of the reduction in the risk of inflation, and reached a minimum of 0.2 percent from 1999 until the third quarter of 2000. From the end of 2000 onwards, the increase in the premium resumed due to two outlying events: The first was the outbreak of the Intifada in October 2000 which apparently raised fears of a financial crisis and did in fact lead to a dramatic rise in Treasury Bill yields.[19] The second event, which led to a sharp increase in the premium, occurred during the first half of 2002 and included a sharp depreciation of the sheqel (against the background of an unexpected drop in the interest rate at the end of 2001) and a significant deficit in the government budget. A sharp depreciation in the sheqel also occurred in the second half of 1998. However, unlike in 2002 when the background to the rise in the premium was diminished public confidence in economic policy, in 1998 the premium did not rise significantly, apparently due to the relatively rapid and credible reaction of the central bank. (An additional indication of the high credibility of the central bank during this period can be seen in the low variance of the 12-month forward returns. See Figure A3 in the Appendix.)

Figure 4
The Changes in the Premium Implicit in 12-month Treasury Bill Yields (at the time of their valuation)

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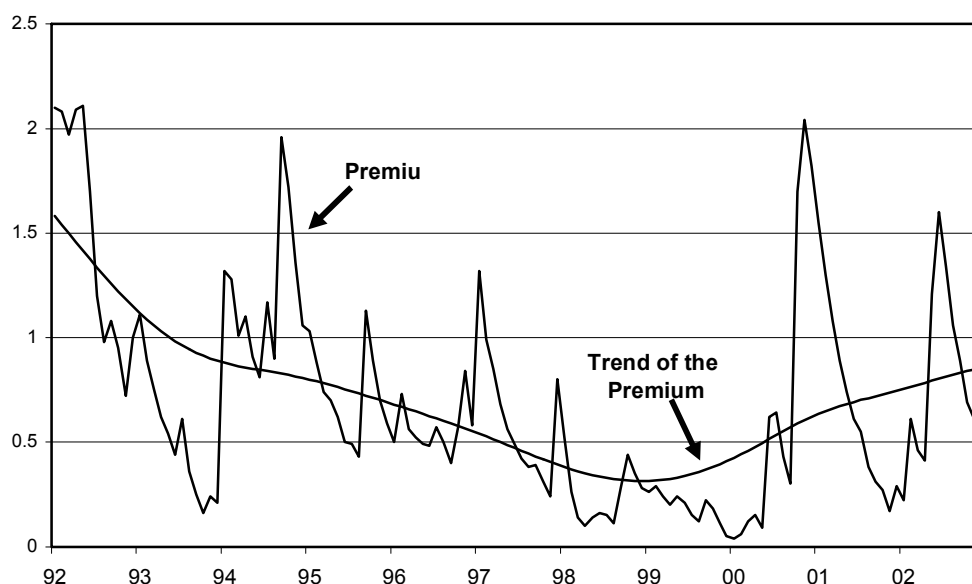
[19] The fear of a crisis was also reflected in the General Share Index which dropped by a monthly average of 4.5 percent during the last quarter of 2000 in comparison to an increase of 1.5 percent during the first three quarters of that year.

6. TESTING THE GOODNESS-OF-FIT

In order to test which of the three models estimated (constant premium, variable premium using a simple Kalman filter and variable premium using a Kalman filter that includes the reaction of the premium to variance) is preferred in identifying the premium, a comparison can be made between the values of the log likelihood obtained in each estimation using a Wald test. The question is whether the constraints imposed on the general model to obtain the more restricted model reduce the estimation's goodness-of-fit. In order to provide an answer, a third model which is presented below can be treated as the general model (equation 11) from which the two more restricted models (ARIMA estimation and estimation by a simple Kalman filter) are derived by imposing the appropriate restrictions. Table 7 below compares the values of the log likelihood of the estimations.[20]

Table 7
Testing the Goodness-of-fit of the Estimation of the Premium Implicit in 12-month Treasury Bills, January 1992 to December 2002

Specification of the Estimation	Estimation of a Constant Premium	Simple Kalman Filter Estimation	Kalman Filter Estimation including Uncertainty and the Bias in Expectations
	(1)	(2)	(3)
Sum of Squared Residuals	553.81	323.16	249.45
Log-Likelihood	-171.57	-154.73	-136.91

Wald Test

	Constraints	Statistical Value	Critical Value – at a 5% Level of Confidence
(2) compared to (1)	$\alpha_1 = \sigma_{\varepsilon}^2 = 0$	33.68	5.99 (2 degrees of freedom)
(3) compared to (2)	$\alpha_{S_t} = \gamma_{S_t} = \beta_1$ $\sigma_{\eta}^2 = \sigma_{S_t}^2 = 0$	35.64	11.1 (5 degrees of freedom)

* Wald Test values are calculated as follows:

2* (Log-Likelihood$_{unrestricted}$ − Log-Likelihood$_{restricted}$). These values have a χ^2 distribution and the number of degrees of freedom is equal to the number of constraints imposed in the transition from the expanded model to the more restricted model.

[20] These values are calculated as follows:

$$\text{Log Likelihood} = -\frac{T}{2}\left[(1 + \log(2\pi) + \log(\frac{SSR}{T}))\right]$$

The table shows that the null hypothesis can be rejected. In other words, the estimation of the broadest model (which includes the reaction of the premium to the level of uncertainty and the bias in expectations as a third component in the excess forward return) is preferable to a significant extent over the previous estimations which include more restricted estimation frameworks.

7. THE PREMIUM AND THE FACTORS WHICH INFLUENCE IT

In this section, we will attempt to determine which factors have influenced the premium over time. This test is an important one since if we succeed in explaining the variation in the premium using economic factors, this will constitute strong evidence for the validity of the estimation of the premium carried out above, primarily using technical methods (a Kalman Filter with regime change). In order to do this, we chose a number of variables which were available to the public at the time of the valuation of the premium and which provide some indication of the path of the interest rate and inflation until the Treasury Bill's date of maturity. It is reasonable to assume that the most relevant variables for valuing the premium are the public's expectations of future economic developments but since there is no data on these variables (that is not derived from the yield curves), we use the change in the variables at the time of the valuation of the premium on the assumption that they in part also reflect the public's expectations of future developments. Among the variables used was the gap between inflation during the previous 12 months and the inflation target during that period (*PAAR*). This variable is meant to capture the degree of confidence in monetary policy and its commitment to the disinflationary process. Additional variables include: the gap between the Bank of Israel interest rate and the Fed interest rate in the US (*GAPRATE*) which is meant to capture the forces operating in the foreign currency market; the average change in the Bank of Israel interest rate during the previous three months (*DIEF3*) which is meant to capture the reaction of the Bank to inflationary pressures; the standard deviation of these changes during the previous six months (*SKV6DIEF*) which is meant to capture the reaction of the central bank to outlying events; and the average change in the rate of holdings of Treasury Bills by the public during the previous 12 months (*DMAK_PUB12*) which provides an indication of the depth of the Treasury Bill market. In addition, we added two dummy variables: a dummy for the last quarter of 2000 which captures the effect of the outbreak of the Intifada (*DUM_INT*)[21] and a continuous dummy which receives a value of 1 from the second half of 1997 and onward and 0 otherwise (*DUM97AFT*). This variable is meant to capture the drop to a new plateau in the inflationary environment.[22]

We would point out that the estimated premium is derived from the excess forward return calculated to the Treasury Bill's day of maturity. Therefore, all the explanatory

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[21] We used a dummy variable since the premium in the last quarter of 2000 cannot be fully explained by economic factors (and most of which we attribute to the outbreak of the Intifada).

[22] Evidence of the drop to a new plateau in the inflationary environment can be found in the study by Liviatan and Melnick (1998).

variables were given a lag of 12 months since this constitutes the information available at the time the premium was valued. The results of the estimation are presented in Table 8.

Table 8
The Factors Influencing the Variation in the Premium, January 1992 to December 2002

	(1)	(2)	(3)	(4)	(5)	(6)	(7)[1]
C	0.869 (0.00)	0.832 (0.00)	0.729 (0.00)	0.505 (0.00)	1.341 (0.00)	1.434 (0.00)	1.644 (0.00)
DUM97AFT	-0.443 (0.00)	-0.392 (0.00)	-0.359 (0.00)	-0.305 (0.00)	-0.619 (0.00)	-0.666 (0.00)	-0.706 (0.00)
DUM_INT	1.350 (0.00)	1.453 (0.00)	1.403 (0.00)	1.450 (0.00)	1.210 (0.00)	1.945 (0.00)	1.169 (0.00)
PAAR		0.033 (0.13)	0.035 (0.04)	0.013 (0.45)	0.039 (0.00)	0.029 (0.03)	0.037 (0.02)
PAAR^2			0.011 (0.00)	0.011 (0.00)	0.014 (0.00)	0.012 (0.00)	0.007 (0.06)
SDV6DIEF				0.326 (0.00)	0.382 (0.00)	0.452 (0.00)	0.098 (0.43)
GAPRATE					-0.101 (0.00)	-0.111 (0.00)	-0.101 (0.00)
DIEF3						0.105 (0.12)	0.243 (0.00)
DMAK_PUB12							-0.155 (0.02)
R^2-*Adjusted*	0.32	0.35	0.39	0.53	0.65	0.66	0.63

The figures in parentheses are the p-values adjusted using the Newey-West method.

[1] Due to the lack of reliable data on the proportion of Treasury Bills held by the public before 1992, the sample period for the estimation in column (7) is only from January 1993 to December 2002.

The table shows that that the coefficients of the explanatory variables and their signs do not change significantly with the addition of other explanatory variables. In addition, although the valuation of the premium is related to factors which are difficult to quantify, such as psychological factors (which are dependent on, among other things, the political climate) and individuals' expectations of future economic developments, the goodness-of-fit obtained (above 60 percent) is satisfactory. The model's goodness-of-fit is presented in Figure A1. It can be seen that the rise in the premium during the first half of 2002 is in fact partially explained by the model. This is because some of the explanatory variables in the model represent the public's level of confidence in policy which, as mentioned above, significantly diminished during this period.

With regard to the influence of the variables, it can be seen from the coefficients of *PAAR* and *PAAR^2* that the gap between inflation and its target had a symmetric effect on the premium though not centered around zero. Thus, for example, the positive deviation of inflation from its target will act to raise the premium at an increasing rate while a negative deviation will work towards lowering the premium but only to a certain point. A deviation beyond this point (which according to the estimation is a negative deviation of 1.2 percent) will work to raise the premium (see Figure A2 in the Appendix). In our opinion, this is a manifestation of the level of confidence in the central bank's willingness to achieve price stability following the disinflationary process. Therefore, a positive deviation from the target apparently lowers the level of confidence and contributes to a higher premium while a negative deviation (up to a certain amount) is a signal to the public of a high commitment to the disinflation process and therefore contributes to reducing the premium. As in the case of a positive deviation, a significant negative deviation also diminishes confidence and raises the premium. Three other variables which have a positive influence on the premium are the standard deviation of the Bank of Israel interest rate during the previous six months, the average change in the interest rate during the previous three months and the dummy variable for the last quarter of 2000 during which the Intifada began.

Three variables were found to have a negative influence on the premium: The first is the gap between the Bank of Israel rate of interest and the Fed rate of interest which, as mentioned earlier, is meant to capture the future pressures in the foreign exchange market and their effect on future inflation and interest rates. The second is the dummy variable for the latter period of the disinflation process (from the second half of 1997 onward) during which there was a significant decline in the level of inflation.[23] The third is the average change in the rate of holdings of Treasury Bills by the public during the previous year which, as mentioned earlier, is an indication of the depth of the Treasury Bill market and therefore acts to reduce the demanded premium. As mentioned above, the estimation of the model which takes these variables into account (column 7) was carried out with a smaller sample since the data was available only from 1993 onward.

8. DISCUSSION AND CONCLUSIONS

In this study, we have attempted to estimate the premium implicit in 12-month Treasury bill yields at the time of their valuation. The sample period used was 1992–2002 during which the Israeli economy was characterized by a process of disinflation. Since the premium is not observable, use was made of a Kalman filter which makes it possible to identify the premium by means of certain assumptions regarding its stochastic process. As part of these assumptions, we allowed the premium to react to the level of uncertainty which was derived from the variance of the excess forward return.

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[23] During the estimation, we introduced a number of additional explanatory variables, such as the level of inflation, the standard deviation of inflation, the standard deviation of changes in the exchange rate, the standard deviation of the output gap, a dummy variable for the Knesset elections and a dummy variable for the removal of the Treasury Bill ceiling. These variables were found not to have a significant effect on the premium.

The results of the estimation show that the premium is positive and was equal to 0.7 percent on average during the sample period. Furthermore, the premium varied over time which is not compatible with the classical theory of expectations. The analysis of the results shows that the premium was characterized by a downward trend during the 90s against the background of a process of disinflation which took place in the economy during this period. This process led to a decline in the risk of inflation. From the end of 2000 until the end of 2002, there was a noticeable reversal in this trend and the premium began to increase. There were two major events during this period: the deterioration in the security situation towards the end of 2000 and the crisis of confidence in economic policy during the first half of 2002 (an unexpected reduction in the interest rate and a sharp increase in the budget deficit).

These findings point to the problematic nature of using Treasury Bill yield curves in identifying the level and changes in public expectations of the future path of the interest rate and as a result are liable to create a certain bias in the management of monetary policy. Thus, for example, policymakers are liable to fully attribute a sharp increase in the slope of the Treasury Bill yield curve to an increase in the expectations of the future path of the interest rate when in actuality this increase also reflects in part an increase in the risk premium demanded by the public.

It was also found that the level of the premium is influenced by economic conditions and the level of uncertainty at the time the public values the Treasury Bill asset. In particular, it was found that a positive deviation of inflation from its target works to increase the premium and that a negative deviation has a similar effect though only if it is larger. The estimation also shows that positive (negative) changes in the Bank of Israel interest rate have a significant positive (negative) influence on the premium. This is perhaps because these changes reflect the reaction of the central bank to inflationary pressures. The standard deviation of these changes, which reflects the reaction of the central bank to outlying events, was also found to have a significant effect on the premium. Among the variables found to have a negative effect on the premium were the gap between domestic and foreign interest rates (a reduction in the interest rate gap will lead to an increased premium), which reflects the reduced inflationary environment and the reduction in the risk of inflation derived from it, during the second half of 1997 and onward (as shown by the continuous dummy variable). Another variable with a negative effect on the premium is the average change in the rate of holdings of Treasury Bills by the public which indicates the depth of the Treasury Bill market.

APPENDIX A1: IDENTIFICATION OF SITUATIONS OF UNCERTAINTY

Equation (10), which makes it possible to identify situations of uncertainty, presents the excess forward return as a random process in which the behavior of the conditional expectation and variance can be described by a Markov chain with two situations — high variance and low variance.[24]

Since the system has a large number of parameters, the estimation of their joint distribution is complex and time-consuming. Therefore, we estimated the parameters using the simultaneity method (Gibbs sampling) which interprets them as random variables. Thus, it is possible to sample their values from the appropriate distribution. The sampling processes are organized in such a way that, at each iteration, one of the parameters is sampled based on the rest of the parameters which were obtained in the previous iteration. It has been shown that the sampling process, when carried out a large number of times, converges to the parameters that would have been estimated from the joint distribution. The estimate of each parameter is obtained as an average of its samplings and the standard deviation between the samplings represents the significance of the estimate. In order to neutralize the influence of the initial values, a large number of the initial samplings are omitted. For a detailed theoretical explanation of the sampling process, see Chib and Greenberg (1996).

We grouped the parameters of Equation (10) into six groups: $g1$ to $g6$. If we designate the iterations by k, we progress through the samplings from group to group such that the sampling of each group is based on the values of the parameters sampled in the previous iteration and therefore on the data in the sample (data on the excess forward return). The grouping of the parameters was as follows:

$$g1 = \{S_t^{(k)}, t = 1, \dots T \mid p^{(k-1)}, q^{(k-1)}, \sigma_0^{2(k-1)}, h^{(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, \theta^{(k-1)}, efr_T\}.$$

$$g2 = \{p^{(k)}, q^{(k)} \mid S_T^{(k)}$$

$$g3 = \{\sigma_0^{2(k)} \mid h^{(k-1)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, S_T^{(k)}, \theta^{(k-1)}, efr_T\}.$$

$$g4 = \{h^{(k)} \mid \sigma_0^{2(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, S_T^{(k)}, \theta^{(k-1)}, efr_T\}.$$

$$g5 = \{\theta^{(k)} \mid h^{(k)}, \sigma_0^{2(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, S_T^{(k)}, efr_T\}.$$

$$g6 = \{\mu_0^{(k)}, \mu_1^{(k-1)} \mid h^{(k)}, \sigma_0^{2(k)}, S_T^{(k)}, \theta^{(k)}, efr_T\}.$$

The initial values for the case of $k=1$ were obtained from an ARIMA estimation: $\mu_1 = 0.5$, $\mu_0 = 0$ (as an alternative value for the constant which is not significant), $\sigma_1^2 = 3.08$ and $h = -0.5$ so that the initial ratio between the variances is doubled. The coefficients of the moving average, which were obtained by ARIMA and were used for the

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[24] The restriction according to which a low expectation is connected necessarily to high variance was not imposed a priori.

initial values of the θ coefficients, are presented in Table A1. A detailed description of the sampling processes for each group of parameters together with a description of the appropriate distributions can be found in Kim and Nelson (1999).

A total of 1,000 iterations were carried out. The results for the first 500 were removed and the parameters estimated on the basis of the latter 500 iterations.

Table A1
ARIMA and Filter Parameters used as Initial Values

Moving Average (MA) Coefficients		
Simple Kalman Filter	ARIMA	Lag (in Months)
0.466 **	0.704 **	1
0.238 **	0.656 **	2
0.145 **	0.537 **	3
0.143 **	0.592 **	4
0.182 **	0.548 **	5
0.107 *	0.424 **	6
0.022	0.323 **	7
0.007	0.234 **	8
0.000	0.255	9
0.101	0.096	10
0.000	0.074	11
Variance of the residuals		
3.086		
(the premium) 0.513		
(the moving average) 1.189		

* Significant at a level of 10 percent

** Significant at a level of 5 percent.

APPENDIX A2: ESTIMATION OF THE PREMIUM AND THE BIAS IN EXPECTATIONS BY MEANS OF AN EXPANDED FILTER

Equation (11), which is used for the estimation of the premium and the bias in expectations, can be represented using a state-space presentation:

(11.1) $\quad EFR_{t,t+12} = Hz_t$,

(11.2) $\quad z_t = A_{S_t} + Fz_{t-1} + B_{S_t} x_t + \xi_t$,

where the state vector $z_t = (\tau_t, k_t, u_{t+12}, ..., u_{t+2})$ is composed of three unobservable variables: the premium, the bias in expectations and a moving average process with 11 lags. These components represent the full decomposition of the excess return. Therefore, matrix

H in the measurement equation (11.1) is a row vector with values of 1 in the first three places and zeros in the rest.

The transition equation (11.2) describes the changes in the state vector, z_t, as an inertial process which in certain periods is influenced by the exogenous factor of high uncertainty in the financial markets. These periods, according to equations (10) and (11), are identified by a situation variable, $S_t = 1$, and are characterized by high variance, σ_1^2, where the exogenous variable, x_t, in equation (11.2) is the log of the high variance: $x_t = \log(\sigma_1^2)$.

Some of the parameters in the transformation equation (11.2) are dependent on the situation or, in other words, they change in situations of high uncertainty: these are the vector of constants, $A_{S_t} = (\alpha_{S_t}^0, 0,..., 0)'$, in the transformation equation of the premium and autoregressive coefficients in the variation of the premium in matrix F_{S_t}.

The variances of the vector of residuals, ξ, of the transformation equation (11.2) also change in situations of high uncertainty and make up a situation-dependent diagonal matrix:

$$\operatorname{var}(\xi_t) = diag(\delta_{S_t}^2, \delta_\eta^2, \sigma_{S_t}^2,....\sigma_{S_{t-11}}^2)$$

in which only the variance, δ_η^2, of the residual in the equation for the bias in expectations is not situation-dependent.

The estimation process of a similar system, which includes change in situation according to a Markov chain, can be found in Kim and Nelson (1999). The initial values for estimation are based on the parameters estimated for a simple filter (10) which does not include switching between situations of uncertainty. The initial values are presented in Tables A1 and 4. A mapping of the situations of uncertainty and the appropriate variances of those situations are obtained from equation (11). After estimating the premium, the bias in expectations and the moving average process using a Kalman filter, the initial parameters were recalculated on the basis of SUR equations.

APPENDIX A3: ESTIMATION OF THE FAMA EQUATION WITH MEASUREMENT ERROR IN THE CALCULATION OF FORWARD RETURNS

We will now assume that there is a certain amount of measurement error (denoted by ε_t) in the specification of the forward return as it appears in equation (3) above. Thus,

(12) $f_{t,t+i} = E_t(r_{t+i}) + \tau_i + \varepsilon_t.$

After adding the term $(r_{t+i} - r_t)$ to both sides of equation (3) and reorganizing the terms, we obtain:

(13) $(r_{t+i} - r_t) = -\tau_i + (f_{t,t+i} - r_t) + u_{t,t+i} + \varepsilon_t.$

As we saw above, the test of the hypothesis regarding the absence of bias in the expected changes in the rate of interest with respect to the actual changes can be represented in the following form:

(14) $\quad (r_{t+i} - r_t) = \alpha + \delta(f_{t,t+i} - r_t) + u_{t,t+i} + \varepsilon_t.$

If there are measurement errors in the model, the estimation of equation (14) using OLS will provide biased estimates of the parameters as a result of the dependence between forward returns and the residuals in the model (i.e. $\operatorname{cov}(f_{t,t+i}, u_{t,t+i} + \varepsilon_t) \neq 0$ because, from equation (12), $\operatorname{cov}(f_{t,t+i}, \varepsilon_t) \neq 0$).

In order to overcome this problem the forward return can be transferred to the left side of the equation:

(15) $\quad r_{t+i} - r_t - \delta_1 f_{t,t+i} = \alpha - \delta_2 r_t + u_{t,t+i} + \varepsilon_t.$

We will estimate model (15) and test the validity of the expectations theory (in its classical form) by testing the hypothesis that $\delta_1 = \delta_2 = 1$.

Another way of testing this hypothesis is through the following equation:

(16) $\quad r_{t+i} - r_t - \delta^{guess} f_{t,t+i} = \alpha - \delta r_t + u_{t,t+i} + \varepsilon_t.$

In the first stage, we guess the parameter δ^{guess} and based on that guess we estimate the parameter δ by OLS. We carry out the estimation in a number of iterations until we obtain $\delta^{guess} = \delta$. In the second stage, we test the hypothesis that $\delta = 1$.

Table A3 presents the results of the estimation of equations (15) and (16) with forecast horizons of 3, 6, 9 and 12 months. The second column of the table presents the parameters estimated in equation (15) and the third column presents the p-values for testing the hypothesis that $\delta_1 = \delta_2 = 1$. The fourth column presents the estimated parameters

Table A3
Results of the Fama Equation taking into Consideration Measurement Errors in the Calculation of Forward Return

pvalue $(\delta = 1)$ (5)	$(\delta^{guess} = \delta)$ α	δ (4)	pvalue $(\delta_1 = \delta_2 = 1)$ (3)	α	δ_1	δ_2 (2)	Forecast Horizon (1)
0.78	0.067 (0.662)	1.02 (0.052)	0.07	-0.189 (0.602)	1.482 (0.215)	1.402 (0.199)	3 months
0.11	0.27 (1.339)	0.838 (0.099)	0.90	0.091 (1.423)	0.938 (0.243)	0.918 (0.203)	6 months
0.00	1.01 (1.547)	1.537 (0.115)	0.12	1.951 (1.585)	0.609 (0.208)	0.759 (0.215)	9 months
0.00	0.68 (1.693)	2.00 (0.130)	0.06	2.28 (1.609)	0.573 (0.222)	0.774 (0.233)	12 months

The figures in parentheses are standard deviations with the Newey-West adjustment.

(according to equation 16) at the end of the process of convergence of the coefficient δ to the coefficient we guessed previously ($\delta^{guess} = \delta$). The last column presents the p-values for testing the hypothesis that $\delta = 1$.

Figure A1
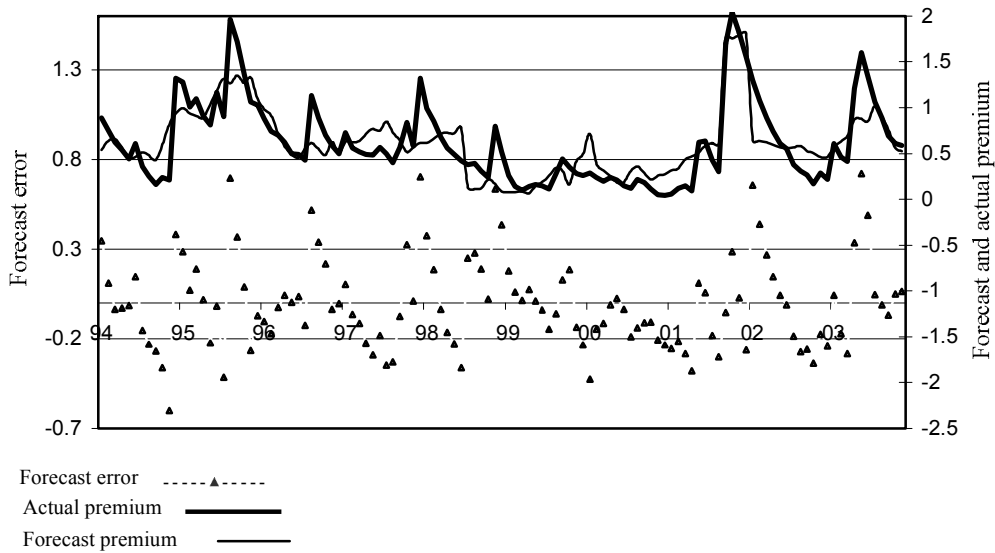**The Variation in the Premium – Actual and Forecast**
(percent)

Forecast error   - - - - ▲ - - - -
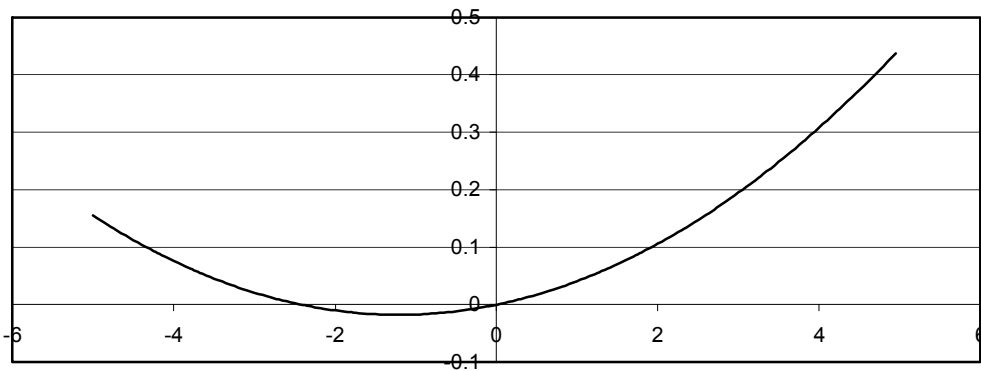Actual premium   ━━━━━
Forecast premium ─────

Figure A2
The Effect on the Premium of the Gap between Inflation during the Previous 12 Months and the Inflation Target

The Gap between Inflation and the Inflation Target   ─────

Figure A3
The Standard Deviation of the Forward Return for a 12-month Period
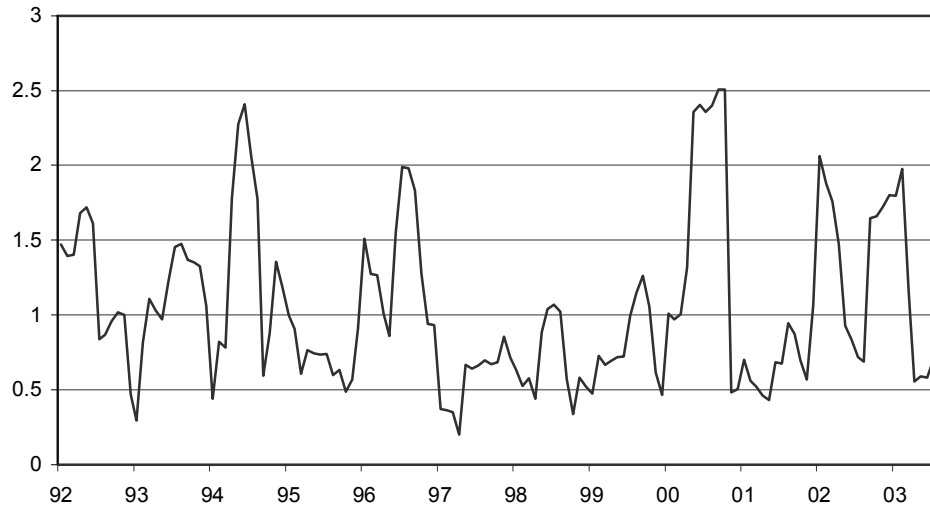**(percent)**

Figure A4
The Estimated Bias in Expectations of the Interest Rate Path ($K_{t,12}$)
(percent)

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