THE NAIRU IN ISRAEL: AN UNOBSERVED COMPONENTS APPROACH

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The Non-Accelerating Inflation Rate of Unemployment (NAIRU) is estimated for the post-stabilization period, as an unobserved stochastic variable, using state-of-the-art State Space Models. The NAIRU is identified by a Phillips curve equation, and is assumed to follow a random walk. The basic model is augmented by an equation that captures the persistence of the unemployment gap. We also use the joint system first introduced by Apel and Jansson (1999) to estimate potential output and the NAIRU simultaneously. Confidence intervals around the NAIRU were computed by a jackknife technique. The results indicate that the actual variation of unemployment has only a minor effect on the NAIRU, which remained relatively stable throughout the sample period. The state variables have sufficiently stable characteristics to be successfully predicted at least one step ahead. However, policy implications that may be derived are sometimes limited, as there is substantial uncertainty around the estimated NAIRU. No evidence for hysteresis was found. The estimates show that the disinflation process during the 1990s did not cause an increase in the NAIRU.

1. INTRODUCTION

Unemployment in Israel varied substantially during the 1990s. This was the result of both the business cycle and a huge immigration influx during the first years of the decade, as well as other structural changes. During that era, the Bank of Israel followed a declining- inflation-target regime. These changes suggest that some of the basic properties of the economy have changed as well.

The perception that some fundamentals of the economy lie under the veil of the business cycle is one of the major concepts of economic thought. While some of these fundamentals are backed up and well defined by theoretical models, empirical work that aims to recover these forces has encountered substantial obstacles.

One major problem is that some of these obstacles are unobserved: they are blurred due to numerous shocks that cause deviations from long run equilibria. Hence the economy is constantly moving in the neighborhood of these fundamentals, but the complexity of the economy and the realization of new shocks does not allow for exact identification.

Among these obscured fundamentals, the natural rate of unemployment has played a key role for decades. This variable is of interest for two reasons: first, it enables us to investigate the factors responsible for its changes (e.g., the labor-market structure and demography).

Second, policy implications can be derived from it, especially in the context of the unemployment/inflation trade-off. When using the term natural rate of unemployment it is essential to define in what sense this rate is natural: current economic theory generates more than one definition (see Rogerson, 1997). One of these definitions, and the focus of this work, is the Non Accelerating Inflation Rate of Unemployment (NAIRU). As its name suggests, the NAIRU is the unemployment rate that is consistent with a stable inflationary process, in the absence of supply shocks, or simply the rate of unemployment that does not exert pressure on the price level. A comprehensive theoretical and empirical review of the NAIRU framework was presented in a special issue of the Journal of Economic Perspectives (1997).

Knowledge of the NAIRU may have important policy implications, since it provides a better understanding of inflationary pressures, especially the influence of real factors such as the unemployment gap on inflation. The simple, not to say simplistic, approach to the NAIRU is to assume it has a fixed rate over time: on that account, the textbook NAIRU in the US was long quoted to be 6 percent (see Blanchard, 1997). Recent business-cycle episodes in some OECD markets disprove the fixed NAIRU assumption. For instance, unemployment went to historically low levels both in the UK and the US during 2000, without any visible inflationary pressure, which may be interpreted as a decline in the NAIRU. In the US, where a 6 percent NAIRU was common, unemployment went down to 4 percent with only minor signs of pressure on wage or price inflation. With regard to Israel, evidence for a changing NAIRU during the post-stabilization era has been found by Yotav-Solberg (1997), and Sussman and Lavi (1999), who suggested that an I(1) process is reasonable.

There are several possible estimation strategies for identifying the NAIRU. The classic way is to construct a price-price equation where the level of unemployment affects inflation. Using some restrictions on the inflation process that ensure that it is stable, it is possible to use the estimated coefficients in order to generate the NAIRU. This framework was used by Yotav-Solberg (1997), who found that even when restrictions on the inflationary process are imposed, the generated NAIRU tends to move much more than one would consider reasonable. The solution to this problem usually used in these models is to smooth the estimated NAIRU till a reasonable NAIRU volatility is achieved. In countries with stable inflation rates, this framework seems to work better (see Tulip, 2001).

The present study uses the unobserved-components (UC) approach. The essence of this approach is to treat the fundamentals, namely the NAIRU, the unemployment gap and potential output as latent variables. Assumptions regarding the stochastic processes that describe the evolution of these latent variables over time enable them to be identified with accuracy. This approach reflects the fact that not much is known about the factors that determine them (Apel and Jansson, 1999; hereafter, AJ). Since the determinants are unknown, they are treated as random. The unobserved variables are recovered through a set of identifying equations that define the relation between these variables and observed variables. The identifying equations may be structural in the sense that they have some theoretical grounds or, alternatively, identities that generate the statistical decomposition of actual data into these variables.

In order to estimate the NAIRU one has to have a well specified inflation process. Following Gordon (1997) this study uses a variant of the triangle version of the Phillips curve, where the inflation process depends on three factors: inertia, demand and supply. The basic framework is extended by specifying the unemployment gap as an AR process: this fits Friedman's concept that deviations from the natural rate cannot be permanent (see Laubach, 2001). This framework

is extended further in order to estimate the NAIRU and potential output simultaneously (see AJ, 1999).

The UC approach has been used extensively in recent years. Statistical decompositions of GDP were presented in Harvey (1989). Estimates of the NAIRU based on the Phillips curve were presented by Gordon (1997), and recently by Laubach (2001) A joint system for estimating the NAIRU/output gap was first introduced by AJ (1999). This system serves nowadays as the IMF workhorse, and was also estimated for Israel (Bal-Gündüz, 2001).

This study uses different State Space Models to estimate the NAIRU. An iterative procedure was adopted, involving at each step Kalman-filter and SUR-equation routines that sequentially improve the model parameters and expected values of latent (state) variables, according to maximum likelihood criteria.

The results indicate that the NAIRU is surprisingly stable. Changes in the actual unemployment rate have only a minor effect on the level of the NAIRU. The estimated potential output varies significantly, suggesting that the immigration influx plays an important role. In order to build a confidence interval for the NAIRU we applied the jackknife technique. As in Laubach (2001) the uncertainty around the NAIRU and potential output estimates is substantial. During the investigated period, however, the unemployment rate differed significantly from the NAIRU in three episodes—as for example during the business boom of 1995–96.

2. THE MODEL

2.1 NAIRU as unobserved component

We follow previous works quoted above, and use a Phillips curve specification to define the NAIRU. Specifically, we use the Gordon's Triangle equation. The general structure of the triangle is:

$$(1) \qquad \pi_t = \alpha(L)\pi_t + \rho(L)(u_t - u_t^n) + \beta x_t + \varepsilon_t^\pi,$$

where p_t is the inflation rate, u_t is the unemployment rate and u_t^n is the NAIRU. Note that the causality in this specification is somewhat counter-intuitive. Usually, we think of the unemployment/inflation trade off as the ability to boost real activity by an unexpected price shock. Instead, in this specification the unemployment gap is used as an explanatory variable—a proxy for excess demand. As explained by Gordon (1997) one can justify this handling by empirical findings that suggest that unemployment Granger-causes inflation (see King and Watson, 1994), where inflation is a function of three factors: inertia, demand and supply. The excess demand is captured by the unemployment gap, while the x's capture supply shocks. This equation implies that when unemployment persists under a certain natural level, other things being equal, the inflation rate will rise. This natural level is the NAIRU. The NAIRU is well defined in this context if and only if the sum of the lagged inflation coefficients equals 1.

This formulation assumes that in the absence of supply shocks, and if actual unemployment equals the NAIRU, the inflation rate converges to a constant long-run equilibrium level. This assumption does not match the inflation trend during the estimated period (1990s), due to the declining-inflation-target regime that was implemented (successfully) at that time. Hence, we estimated the model in first differences of inflation.

(2) $\Delta\pi_t = \alpha(L)\Delta\pi_t + \rho(L)(u_t - u_t^n) + \beta x_t + \varepsilon_t^{\Delta\pi}$

This specification does not assume an implicit long-run equilibrium (Bal-Gündüz, 2001), and therefore is more appropriate in the case of Israel, but still defines a meaningful NAIRU (see AJ, 1999). An alternative specification may use the unexpected inflation, namely the difference between actual and expected inflation. In the case of adaptive expectations, the two different specifications are similar.

The first term in the triangle is the inertia term that is captured by lags of the dependent variable. An alternative specification that we use is a moving average specification:

(3) $\Delta\pi_t = \rho(L)(u_t - u_t^n) + \beta x_t + (1 - w(L))\varepsilon_t$

The justification for this specification is empirical: the equations are not equivalent, even if the AR process is invertible, since other components remain unchanged. However, the residuals that are generated by the different models suggest that this specification captures inertia as well.

The atheoretical part of the model specifies the stochastic process of the NAIRU and the unemployment gap. We now turn to complete the system with some assumptions about the dynamics of the unobserved components. Equation ($u_t^n = u_{t-1}^n + \varepsilon_t^n$) specifies the NAIRU process as a random walk. Although in the long run one would not expect the NAIRU to follow a random-walk process since it is bounded, this process seems to be a good approximation for short-run movements. As cited in the introduction, evidence for an I(1) specification was found by Sussman and Lavi (2001).

Together, the system is composed of equations (4)-(6).[1] The theoretical part of the model is captured by a triangle relationship (the measurement equation (4), whilst the atheoretical part of the model specifies the stochastic process of the NAIRU and the unemployment gap (transition equations (5)-(6)).

(4) $\Delta\pi_t = \alpha(L)\Delta\pi_t + \rho(L)(u_t - u_t^n) + \beta x_t + \varepsilon_t^{\Delta\pi}$;

(5) $u_t^n = u_{t-1}^n + \varepsilon_t^n$;

(6) $\left(u_t - u_t^n\right) = \delta_1\left(u_{t-1} - u_{t-1}^n\right) + \delta_2\left(u_{t-2} - u_{t-2}^n\right) + \varepsilon_t^{gap}$.

A parsimonious version, composed of equations (4)—(5) only, is the specification used by Gordon (1997). As we describe later, this specification tends to generate a volatile NAIRU, unless a restriction is imposed on ε_t^n. In the extreme case the volatility is restricted to zero and the NAIRU is fixed: in other cases, an assumption of the smoothness of the NAIRU is required. As a result, the smoothness of the NAIRU achieved by this specification is an assumption rather than a result. In order to avoid this difficulty, an augmented system with equation (6) was constructed. This specification requires that the unemployment gap follows an autoregressive process. Note that the AR parameters are estimated simultaneously without restrictions. Hence, the process is not restricted to being stationary. The augmented system

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[1] Actually, the system is closed by an additional equation, an identity that restricts the sum of unobserved components to being equal to actual unemployment, and imposes the requirement of consistency on the decomposition (see Appendix 2).

generates a relatively stable NAIRU as a result, without any further direct restrictions. Moreover, the estimated δ's suggest that the unemployment gap is stationary, although the results of an ADF test applied to the estimated series are inconclusive.

This equation has some economic reasoning since sticky labor markets cause inertia in the unemployment gap. It also imposes an indirect restriction on the NAIRU process; thus, no additional direct assumptions on the NAIRU volatility are required in order to get a relatively smooth NAIRU path.

2.2 Potential output as unobserved component

The system below generates a pure statistical decomposition of actual output. The measurement equation is an identity (with no error term) that defines actual output as a sum of two components: potential output and the output gap. The transition block that is composed of equations (8) and (9) defines the stochastic properties of the unobservables. Potential output is assumed to follow a random walk plus drift process (8), while the output gap is assumed to be an AR(2) process (9). This captures the persistence of the business cycle. This system was also estimated by de Brouwer(1998). Likewise, an AR(2) process was estimated, with no further restrictions on the residuals.

(7) $\quad y_t = y_t^p + y_t^{gap};$

(8) $\quad y_t^p = y_{t-1}^p + d + \varepsilon_t^p;$

(9) $\quad y_t^{gap} = \vartheta_1 y_{t-1}^{gap} + \vartheta_2 y_{t-2}^{gap} + \varepsilon_t^{og}.$

The estimated parameters ($\vartheta_1 = 0.59$, $\vartheta_2 = 0.19$) indicate that the output gap is stationary. Hence, the system decomposes actual output into a stationary and a non-stationary process.[2] This means that the shocks to potential output are permanent, while the shocks to the gap process are transitory (see Yakhin and Menashe, 2001).

2.3 Simultaneous system for NAIRU and potential output

As first introduced by AJ (1999), it is possible to simultaneously estimate the NAIRU (and therefore the unemployment gap) and potential output (and the derived output gap). This section describes this augmented model.

The term potential output is in a way indefinite, and mirrors the equivocal nature of the term ''natural rate''. A thorough review of this issue has been presented recently by Yakhin and Menashe (2001). As for the output gap estimated here, it is simply a NAIRU output gap, meaning that potential output in this study is the level of output that does not cause inflationary pressure.

The system ties the NAIRU and potential output by augmenting the measurement block using Okun's law (equation 12) that relates cyclical unemployment to cyclical output. Since a

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[2] This decomposition is not unique, however. It is possible to decompose output into two non-stationary processes: this depends on the initial values of the ρ's.

new unobserved variable is added, an additional assumption about its dynamics is required. Consequently, the system is completed by augmenting the state block with an equation that describes potential output dynamics as a random walk plus drift (equation (15)). The drift captures the growth of the labor force as well as improvements in productivity.

$$(10) \quad \Delta\pi_t = \alpha(L)\Delta\pi_t + \rho(L)\left(u_t - u_t^n\right) + \beta x_t + \varepsilon_t^{\Delta\pi};$$

$$(11) \quad y_t = y_t^p + \gamma_1\left(u_t - u_t^n\right) + \gamma_2\left(u_t - u_{t-1}^n\right) + \varepsilon_t^y;$$

$$(12) \quad u_t^n = u_{t-1}^n + \varepsilon_t^n;$$

$$(13) \quad \left(u_t - u_t^n\right) = \delta_1\left(u_{t-1} - u_{t-1}^n\right) + \delta_2\left(u_{t-2} - u_{t-2}^n\right) + \varepsilon_t^{gap};$$

$$(14) \quad y_t^p = y_{t-1}^p + d_t + \varepsilon_t^p.$$

3. ESTIMATION TECHNIQUE

In order to estimate the system's parameters we first cast it in a state-space form (SSF). The essence of this representation is to express a dynamic system with two equations (or, in the multivariate case, by two blocks of equations written in a matrix form).

The measurement equation describes the dependence of observed components (such as inflation and output) on a set of variables, some of which may be unobserved (state variables), and others observed regular (exogenous) variables. This block may be based on theoretical grounds (hence, structural), or on identities (this will be clarified in the next sections).

$$(15) \quad \underset{n\times 1}{Y_t} = \underset{n\times m}{H}\,\underset{m\times 1}{Z_t} + \underset{n\times k}{B}\,\underset{k\times 1}{X_t} + \underset{n\times 1}{\xi_t},$$

where

y_t is a vector of n response (dependent) observed variables at (and up to) time t
Z_t is a vector of m unobserved (state) components at time t,
H is a measurement matrix, assumed to be time invariant,
X_t is a vector of k exogenous or lagged dependent variables with coefficient matrix B,
ξ_τ is a vector of serially uncorrelated disturbances with $E(x_t) = 0$ and $Var(x_t) = R$.

The transition equation describes the evolution of the unobserved components over time, i.e., the transmission from observation $t-1$ to t. The transition equation is based on a set of atheoretical assumptions, reflecting the fact that our knowledge about the factors that determine their evolution is limited (AJ, 1999). These assumptions are based on some economic grounds, or may be justified empirically, such as the evaluation of potential output as a random walk plus drift.

The transition equation describes the dynamic process of the unobserved components, Z_t and takes the form of a first order Markov process:

$$(16) \quad \underset{m\times 1}{Z} = \underset{m\times m}{F}\,\underset{m\times 1}{Z_{t-1}} + \underset{m\times 1}{\eta_t},$$

where

F is an m x n transition matrix, assumed to be time-invariant,

η_t is a vector of serially uncorrelated disturbances with $E(h_t) = 0$ and $Var(h_t) = V$.

For example, the SSF for the system of equations (11) to (16) is given below (a full set of SSFs for the different models used throughout this work is given in Appendix 2):

$$(17) \quad Y_t = \begin{bmatrix} y_t \\ u_t \\ \Delta\pi_t \end{bmatrix}, \quad Z_t = \begin{bmatrix} y_t^p \\ u_t^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \\ d_t \end{bmatrix};$$

$$(18) \quad H = \begin{bmatrix} 1 & 0 & \gamma_1 & \gamma_2 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & \rho_1 & \rho_2 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \delta_1 & \delta_2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$(19) \quad X_t = \begin{bmatrix} \Delta\pi_{t-1} \\ \Delta\pi_{t-2} \\ \dots \\ x_t^1 \\ x_t^2 \\ \dots \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0\dots00 & 0 & 0 & 0\dots0 \\ 0 & 0 & 0\dots0 & 0 & 0 & 0\dots0 \\ \alpha_1 & \alpha_2 & \cdots & \beta_1 & \beta_2 & \cdots \end{bmatrix};$$

$$(20) \quad \xi_t = \begin{bmatrix} \varepsilon_t^y \\ 0 \\ \varepsilon_t^{\Delta\pi} \end{bmatrix}, \quad \eta_t = \begin{bmatrix} \varepsilon_t^p \\ \varepsilon_t^n \\ \varepsilon_t^{gap} \\ 0 \\ 0 \end{bmatrix}.$$

The aim is to obtain expected values of the state components $Z_t^1, \dots Z_t^m (t = 1, \dots T)$ and of their variances, as well as measurement and transition coefficients.

Note that had $Z^1, \dots Z^m$ been observed, the matrices H and F would have been immediately evaluated by straightforward regression estimates. On the other hand, if the system matrices H, F, R and V were known, the unobserved components $Z_{t|t-1}$ would be calculated by one pass of the Kalman filter (a full description of the Kalman filter and smoother is given in Appendix 3).

In order to solve this dual estimation problem, we apply the estimation maximization (EM) algorithm of Watson and Engle (1983). This is a derivative free, iterative algorithm that consists of a Kalman filter pass and SUR estimation. The idea of this method is to maximise the expected likelihood function, by improving the set of unknown parameters and state variables, using information only on their first two moments. This means that the maximization of the likelihood function is achieved not through its derivatives but step by step, indirectly.

The likelihood function L of the unknown parameters in (15) and (16) depends on the innovations $Y_t - E\left(Y_t | Y_{t-1}, \ldots, Y_1, X_t, \ldots, X_1\right)$ and their variance C_t:

$$L(\theta) = \frac{1}{2} \sum_{t=1}^{T} \log|C_t| - \frac{1}{2} \Sigma \left(Y_t - HZ_{t|t-1} - BX_t\right) C_t^{-1} \left(Y_t - HZ_{t|t-1} - BX_t\right),$$

where θ is the vector of unknown parameters.

The innovations and their variances are calculated previously by a Kalman filter pass.

When the best parameter values are obtained, a new pass of Kalman filter is run, generating an improved set of state variables $Z^1, \ldots Z^m$ and their variances. This is the estimation step of the algorithm, while the first step was maximization.

More precisely, the estimation starts with the initial matrices H, F, R and V, and initial values for the mean and variance of the state variables. It allows the first pass of the Kalman filter, generating $Z^1, \ldots Z^m$ by one-step prediction, i.e., $Z_{t|t-1}$. While the system (15)-(16) holds for conditional expected values of Z_t, the last can be evaluated using the smoothing procedure that recursively calculates $Z_{t|T} = E\left(Z_t | Y_T, Y_{T-1}, \ldots, Y_1, X_T, X_{T-1}, \ldots, X_1\right)$ and their mean square error matrices. The values are the best estimates for expected values of Z_t given the information available till T, and the parameter set q. The smoother runs backward from the last to the first observation of each Z variable.

Once the Z's are generated, they may be considered as 'regular' observed variables. Next, the maximization step can be carried out. When the system is unrestricted, ordinary least square estimates are sufficient, that is :

$$H = \left(Z'Z\right)^{-1}\left(Z'Y\right) + \left(Z'X\right)\left(X'Y\right), \ B = \left(X'X\right)\left(X'Y\right) + \left(X'Z\right)\left(Z'Y\right)$$

and $F = \left(Z'_{-1}Z_{-1}\right)^{-1}\left(Z'_{-1}Z\right),$

where Z consists of smoothed state variables $Z^1_{t|T}, \ldots Z^m_{t|T} \quad \left(t = 1, \ldots T\right)$ and Z_{-1} of their lagged values.

In our case the system contains parameter restrictions, such as zeros and ones in H and F matrices. Some equations of our system are fully restricted, having no degrees of freedom, so OLS estimates for these equations cannot be obtained.

Assuming that residuals among partially restricted and not restricted equations may be correlated, at each iteration we solve a reduced SUR system, which includes only full rank equations.

In other words, having $n + m$ equations of which l are fully restricted, we compose at each iteration a SUR system of $n + m - l$ equations and obtain reduced matrices H^*, F^* and B^*. To complete this step, we need only to enlarge the system, by resubstituting the 0s and 1s in their original positions.

When the parameters of *H, F* and *B* are updated and a new series of residuals x_t and h_t are available, together with their variance matrices *R* and *V*, we can switch to the next pass of the Kalman filter, constructing an improved set of *Z*s.

Therefore, one iteration of the EM algorithm involves solving SUR equations (maximization step), one pass of the Kalman filter, which calculates one-step predictions of state variables and one pass of the Kalman smoother that evaluates their conditional expected values (estimation step).

The process converges when the relative changes of parameters, the likelihood function, and state variables become negligible.

Our experience shows that this method is sensitive to some initial values, while to others it is rather indifferent. For instance, we found that even when the guesses about mean *Z* values are very poor (even a zero vector was tried) we reached convergence near the same region. The sensitivity to initial values of the transition matrix *F* and of the measurement matrix *H* (that were based on ad hoc considerations) is low, too.

Residual variances (*R* and *V* initial matrices) depend also on the initial assumptions about *B* coefficients, the influence of the exogenous variables on the response. They were approximated by OLS, assuming the NAIRU was fixed over time and equal to the sample average.

The algorithm tends to be quite sensitive to the initial variance of *Z*, however. The solution to this problem was to use a diffuse prior (see Harvey, 1989): this method calculates the initial variances under lack of stationarity, assuming the variance to be infinite. The results support this treatment, as some of the eigenvalues of *F* are near unity. Alternatively, we tried to enlarge the variances till convergence was reached: this method was inferior to the previous one in terms of convergence speed. The reason for this difference is that we do not have a good guess for the relative variance of the state variables.

4. EMPIRICAL RESULTS

Three main models were used: the first version (hereafter, V1) is the basic form, composed of equations (4) and (5), without unemployment gap as a state variable. The second model specifies the unemployment gap as an AR process (equations (4) to (6)) and the third model is the AJ model (V2 and V3 respectively). This order is not random: each model is in fact a nested parsimonious form of the next model. In addition, statistical decompositions of output are presented.

This section describes the estimation results of the models, and is organized as follows. First we give a short description of the data. Then we discuss the results for two partial models for the NAIRU, and then switch to the results of the joint NAIRU/output model. We also compare the parameters of the model obtained from the different versions.

The data are composed of quarterly data from 1987:I to 2001:I. Since the model assumes that the inflation process is at least partially determined by real factors, the sample period includes only the post stabilization period. Before that, it is clear that the process was almost completely dominated by nominal factors.[3] $\Delta\pi$ is the second difference of seasonally adjusted

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[3] However, Sussman and Lavi (1999) found that even during the high inflation period (1975-1985) real factors had a significant influence on inflation.

log CPI.[4] The unemployment rate is seasonally adjusted chained unemployment rate. Supply side shocks are captured by the relative price of imports excluding oil and diamonds, the relative price of oil, deviations of labor productivity from its trend and the relative contribution of new immigrants to population growth. Other variables that have been checked include the IMF's Real Exchange Rate[5] (RER), and the interest rate. A complete description of the data is given in Appendix 1.

Table 1
System Estimation Output

Parameters		version			
		V1	V2 (AR)	V2 (MA)	V3
Measurement equation (H)	γ_1	–	–	–	–0.000509
	γ_2	–	–	–	–0.011246
	ρ_1	–0.187071	–0.334002	–0.201102	–0.295034
	ρ_2	–	0.1609596	0.111102	0.1846966
Transition equation (F)	δ_1	–	1.1217391	1.1395334	1.1930171
	δ_2	–	–0.202972	–0.224058	–0.275905
	ω_1	–	–	–0.211654	–0.47757
	ω_2	–	–	–0.2794	–0.477553
Error term variances	$\sigma^2(\varepsilon_t^y)$	–	–	–	0.000099
	$\sigma^2(\varepsilon_t^{\Delta\pi})$	1.23439	1.24033	0.44071	1.44365
	$\sigma^2(\varepsilon_t^p)$	–	–	–	0.000127
	$\sigma^2(\varepsilon_t^n)$	0.01	0.01769	0.02049	0.0452
	$\sigma^2(\varepsilon_t^{gap})$	1.74649	0.21398	0.20492	0.17764
	$\sigma^2(\varepsilon_t)$	–	–	1.35459	1.04236
Average and standard deviation of unobserved variables	y_p	–	–	–	11.02 (0.20)
	d	–	–	–	0.01194 (0)
	u_n	8.49 (0.29)	8.44 (0.14)	8.66 (0.19)	8.70 (0.36)
	$(u-u_n)$	–0.1 (1.37)	–0.04 (1.50)	–0.27 (1.51)	–0.31 (1.40)
	ε	–	–	–0.06 (1.18)	–0.005 (0.05)
Average SE	y_p	–	–	–	0.000139
	d	–	–	–	0
	$u_{n,}(u-u_n)$	0.064978	0.261481	0.54929	0.651889
	ε	–	–	0.33331	1.516338
Likelihood		174.688	53.150	58.520	–322.976
Convergence (iterations)		5	6	13	44

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[4] Both X12 and SABL procedures were applied. Major differences between the seasonally adjusted series appeared during the first two years of the sample. After 1989 the differences are negligible.

[5] Although this variable was used as a supply shock in several previous works, the Real Exchange Rate may be confounded by demand shocks.

Table 1 presents the results obtained by the different models. Our model selection strategy was to choose a specification for which the state space model performance was best in terms of stability, and to eliminate the effect of the unemployment gap on inflation. Thus, we over-controlled for this variable, and even insignificant lags of the unemployment gap are included, clearing also possible level effects (on prices), and not only change effects (see AJ, 1999). We, nevertheless, applied the parsimony principle when selecting the exogenous variables.[6] The models were estimated using two alternative specifications: an AR specification, where inertia is captured by four lags of $\Delta\pi$, and a specification where inertia is captured by two moving average terms. When using lagged variables of the dependent variable, they were treated as "regular" exogenous variables.[7]

The estimates, as presented in Table 1, are in accordance with economic theory. The sums of the coefficients of the unemployment gap in the Phillips curve equation (ρ_1 and ρ_2) are negative for all specifications, as is the sum of the coefficient in Okun's law equation (in V3). The transition matrix coefficients imply that the unemployment gap process is stationary. There is a clear trade-off between augmenting the system and the average standard errors of the unobserved components as calculated by the Kalman smoother.[8] The estimated error term variance of the NAIRU varies between 0.017 and 0.045. This interval, although shifted downward, overlaps with some of the restricted values in Gordon (1997), where an SD of 0.2 was used. Since the models were not selected on the basis of the levels of significance of the explanatory variables, we present later single equations estimates that were constructed using standard considerations with the relevant t-statistics.

4.1 The NAIRU

The most notable result is the stable pattern of the NAIRU during the sample period. Although the unemployment rate varied from a peak of 11.3 percent to a low of 5.8 percent, the NAIRU during this period changed by no more than one percentage point, as shown in Figure 1.[9] Note that this result holds for the models that were generated without any direct restrictions on the NAIRU variance (V2 and V3 below), confirming the variance constraint in the first model.

The second result is that the level of the NAIRU is quite high: in fact, in all the specifications the average level of the NAIRU is higher, by 0.1 to 0.3 percentage points, than the average level of unemployment (8.39 percent). This result may reflect the fact that during the first years of the sample period, nominal factors still dominated the inflation process.[10]

[6] An alternative strategy was used by AJ (1999), and by Bal Gündüz (2001) where the system includes five lags of all exogenous variables. Our treatment of inertia and the unemployment gap is identical, however.

[7] For a justification of this treatment see Hamilton (1994) or Harvey (1989). Alternatively, the lagged variables may be treated as state variables.

[8] The average SEs are calculated by averaging the diagonal elements of the $P_{t/T}$'s for t = 1...T (see Appendix 3).

[9] V1 was generated by the assumption that the NAIRU follows a random walk only. V2 is a model where an additional assumption was made about the process of the gap (AR(2)). V3 is the augmented AJ model that includes output.

[10] As shown by Sussman and Lavi (2001), money growth (or alternatively, monetary policy as reflected by the real interest rate) dominated the inflation process during the first years of the sample.

Figure 1
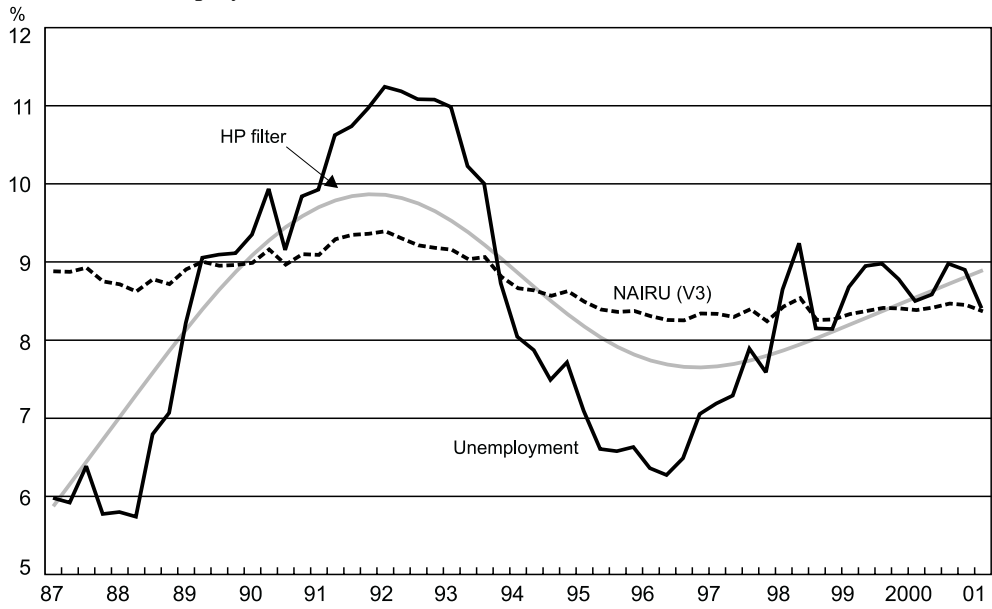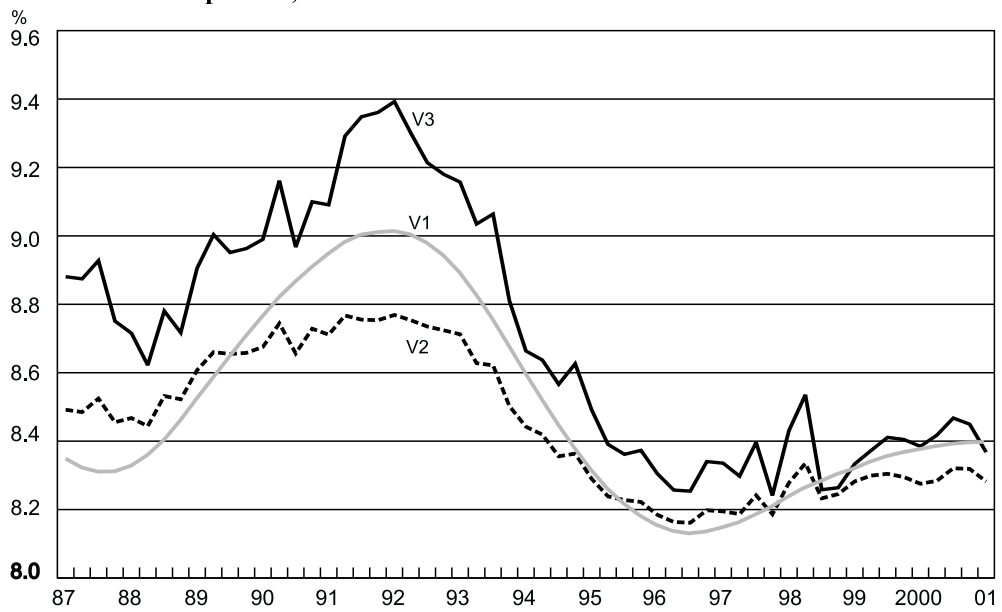**Actual Unemployment and the NAIRU, 1987–2001**

Figure 2
NAIRU Comparison, 1987–2001

The results from the different systems are given below. As presented in Figure 2, the NAIRU generated by the different systems resemble each other, though some differences emerge. As claimed before, the smoothness of V1 is more an assumption than a result. The variance of the NAIRU in V1 was restricted so that its amplitude resembles that of the unrestricted V2. Although the pattern is similar, V1 generates a smoother NAIRU. The smoothed pattern of V1 is the result of both the assumption about its (low) variance and the assumption about the stochastic process. The jagged pattern of V2 and V3 is the result of the decomposition of the unemployment rate into two different components, where the second component (the unemployment gap) is generated by an AR(2) process (not like V1 where the random walk of the NAIRU is the only assumption). This creates the jagged NAIRU as these systems impose the restriction that the unemployment gap to be smoothed, and therefore the movements of actual unemployment are reflected more closely in the NAIRU.

Digging deeper: At first sight the NAIRU estimates, especially V1, look as if they could have been generated by a univariate filter such as the HP filter. A closer look at the results shows that these series differ substantially. The first obvious difference is the series average: while the HP filtered series has the same average as the original series, the generated NAIRU series is above average. As presented also by Yakhin and Menashe (2001), it is interesting to check the correlations between the computed series and an HP filtered series. Table 2 shows that the HP series is highly correlated with the original unemployment series while the state space NAIRU estimates are less correlated. Not surprisingly, the correlation coefficient drops as the system includes a larger information set. Hence when the NAIRU is constructed using inflation and unemployment only, the correlation coefficient is still high (but still below the univariate HP filter). When using the assumption that the unemployment gap follows an AR process the coefficient drops to 0.66 : and when introducing output the coefficient drops further to 0.64.

Table 2
Correlations between State Space NAIRU

	Estimates and HP filtered unemployment					
	U_hp	U	V1	V2_MA	V3	V2_AR
U_hp	1.00					
U	0.90	1.00				
V1	0.82	0.83	1.00			
V2_MA	0.58	0.66	0.92	1.00		
V3	0.55	0.64	0.91	0.98	1.00	
V2_AR	0.66	0.74	0.95	0.99	0.97	1.00

Although V2 and V3 are highly correlated, their levels differ, especially during the 10 first years of the sample. Adding output to the system (V3) increases the NAIRU volatility and changes its reaction to shocks: for instance, during the early 1990s, when unemployment went above 11 percent (the immigration influx from former Soviet Union countries was at its peak then), the augmented form (V3) reacted much more to changes in the actual rate (an 0.5

percentage point increase) while the reduced form (V2) indicates that the NAIRU hardly changed: this leads to a difference of 0.8 percentage points between the two estimates during 1992. The reason may be that when taking information on output into account, the model interprets the output rise in 1991 as a permanent shock, i.e., as a rise in potential output; As a result, the NAIRU increases more dramatically (during this episode unemployment rose as well, without any downward pressure on prices, after controlling directly for immigration). The immigration shock caused a simultaneous rise in output and unemployment. It seems that the augmented model handles the immigration shock better, as the new immigrants permanently increased the labor force. It is reasonable to assume that this shock caused the NAIRU to increase in the short run.

Minor differences between the versions are apparent after immigration has slowed down.

4.2 The unemployment gap

The derived unemployment gap reflects the business cycle state. As such, the gap should follow some regularities, namely the persistence and stationary nature of the business cycle. The parameters of the transition matrix F of all estimated models point out that the unemployment gap follows a stationary process, although the parameters show that this is a borderline-case.[11] Note that the unemployment gap is modeled as an unrestricted AR(2) process. When checking the gap itself the results are inconclusive, however. An ADF test shows that the unit root hypothesis may be rejected based on high critical values only (i.e., the ADF test statistic is near the 5 percent critical value).

4.3 Potential output and the output gap

This section presents the potential output and imputed output gap derived from the 3rd version (AJ model). The estimated drift is 1.19, reflecting a 4.76 annual growth rate. The system output is compared with both HP filtered series and a Kalman filter series generated by the non-structural system of Section 2.2.

The estimated output gap is highly correlated with the unemployment gap: this result is not surprising, as the output gap, by construction, is related to the unemployment gap only. Yet, the NAIRU that is estimated by this system is different (V3 in Figure 2). The reasons for the different NAIRU are the information set that now includes also actual output, and the extra assumption on the stochastic process of potential output that imposes a restriction on the evolution of the output gap, reflected in the NAIRU via equation (12).

Figure 4 presents the different output gaps derived from the augmented system (V3), the output gap derived from a pure statistical decomposition (KF), and the output gap as a difference between actual and HP filtered output (HP). The main difference that meets the eye is the interpretation of each method of the 1992 mass immigration period. The HP filter gap indicates that this period was a business boom, while the State Space Model shows that potential output rose significantly during this period, leading to the conclusion that this movement of output was not cyclical. The statistical decomposition (KF) lies in between: the interpretation is that

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[11] Some of the eigenvalues of the transition matrix F equal one.

Figure 3
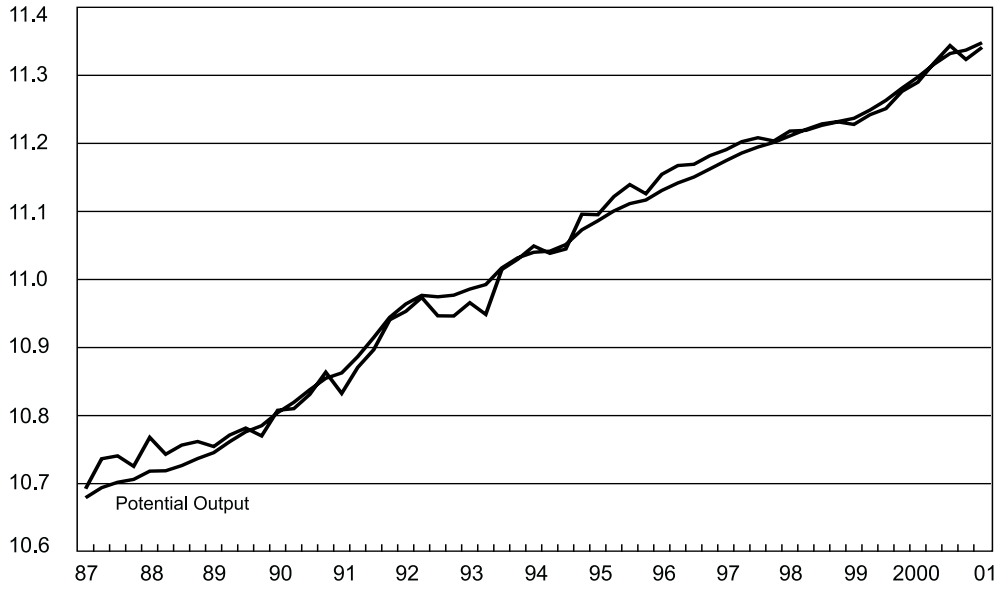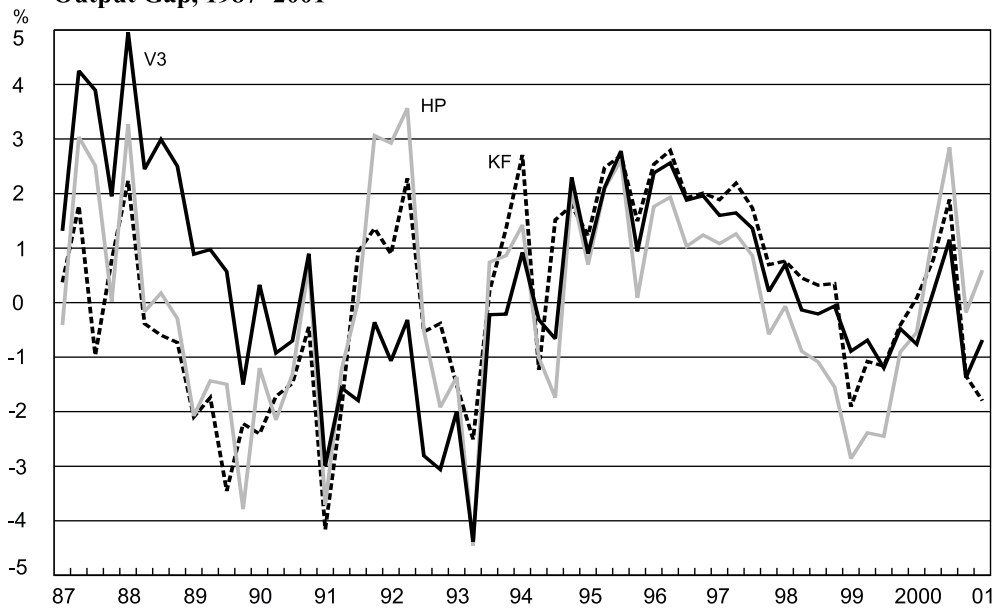**Actual and Potential (log) Output, 1987–2001**

Figure 4
Output Gap, 1987–2001

this model better captures permanent shocks than a simple HP estimate. A closer look at the gaps reveals the following results: the correlation between HP and KF is 0.77, and the correlation between HP and V3 is 0.54; the ACF of HP gap indicates that only the first two terms are significant, while the ACF of the other methods contain 6-7 significant terms. This implies that these methods comply with our perception of the business cycle, while a simple HP gap does not exhibit a cycle at all.

Table 3
Single Equation Phillips Curve Estimates

	V1	V1	V2(AR)	V2(AR)	V2(MA)	V3
U_gap_t	–0.30		–0.27		–0.10	–0.09
	(–2.02)		(–2.1)		(–2.07)	(–2.23)
U_gap_{t-1}		–0.24			–0.22	
		(–1.60)			(–1.65)	
$\Delta\pi_{t-1}$	–0.48	–0.41	–0.48	–0.41		
	(–3.39)	(–2.82)	(–3.40)	(–2.82)		
$\Delta\pi_{t-2}$	–0.53	–0.49	–0.53	–0.49		
	(–3.20)	(–2.83)	(–3.22)	(–2.83)		
$\Delta\pi_{t-3}$	–0.25	–0.26	–0.24	–0.26		
	(–1.67)	(–1.77)	(–1.68)	(–1.77)		
$\Delta\pi_{t-4}$	–0.28	–0.25	–0.28	–0.25		
	(–2.18)	(–1.91)	(–2.20)	(–1.90)		
MA(1)					–0.67	–0.63
					(4.37)	(–4.55)
MA(2)					–0.13	–0.33
					(–0.85)	(–2.39)
$Immigration_{t-1}$	2.24	2.79	2.45	2.83		
	(2.14)	(2.47)	(2.31)	(2.53)		
$Imports_t$		20.81		21.06		
		(2.79)		(2.83)		
Oil_{t-3}	3.91	3.88	4.03	3.89	3.04	2.44
	(2.63)	(2.63)	(2.73)	(2.64)	(2.34)	(1.90)
$Productivity_{t-1}$	–14.15	–12.66	–13.55	–12.61		–3.41
	(–2.00)	(–1.81)	(–1.95)	(–1.81)		(–2.40)
RER_t	–17.8		–18.77		–19.14	–15.82
	(–2.80)		(–2.97)		(–3.23)	(–2.50)
R^2_{adj}	0.45	0.44	0.46	0.44	0.43	0.41
DW					1.91	2.05
LM_test	9.01	9.07	8.90	9.56		
(probability)	(0.34)	(0.33)	(0.35)	(0.20)		

4.4 Confidence intervals

Being conditional expectations, estimated state variables have two sources of uncertainty. The first, due to parameter uncertainty, reflects the uncertainty around the estimated parameters in *H* and *F* matrices. This type of uncertainty would remain even if the state variables were

observed and model was estimated using standard regression methods. The second type of uncertainty is the filter uncertainty and reflects the fact that the estimated Z-values represent conditional expectations of true unobserved values. This uncertainty is due to Kalman filter estimation and would be present even if the true values of the matrices H and F were known. A decomposition of mean squared error into these components is shown in Hamilton (1994, 397-399).

The question is what is the pivotal statistic that enables us to build the confidence interval around the estimated state variables, taking into account both sources of uncertainty. Obviously, the use of a prediction interval in this case will overestimate the uncertainty since it relates to out-of-sample error terms.

In order to obtain a rough estimate of the confidence interval around the unobserved NAIRU we applied the jackknife technique.[12] The idea of jackknifing is to modify the sample by randomly deleting one or a group of observations and to measure the generated bias in the statistic of interest. The bias measured by repeating this procedure, and normalized in a special way (see Appendix 4), should have an approximate t distribution, and constitutes a pivotal statistic for robust interval estimation (Miller, 1974).

This simulation enables us to get an idea of the aggregate uncertainty around the unobserved components directly, that is, without estimating each source separately.

During the last two decades the resampling techniques (bootstrapping rather than jackknifing) were applied in a time-series domain for model selection (Veall, 1992), as well as parameters and standard error checks (see, for example, Li, 1994; Stoffer and Wall, 1991; and Brownstone, 1990).

Since our observations are serially correlated, instead of deleting the row of observations, we replaced it by the average of its neighbors. Such interpolation resembles the deletion of rows because the modified observation is not independent and does not contain any new information. The deleted rows were chosen by a random counter. After this treatment, we re-estimated the model. Thus, new estimates of the state variables were obtained, together with the new system matrices H and F. Repeated twenty-two times, this simulation created twenty-two versions of each state-variable for each date. The details about the data processing are given in Appendix 4. The outcome of this experiment is that the confidence bands, computed by jackknifing, are much narrower than the prediction intervals by the Kalman filter. Yet, the confidence bands, obtained by jackknifing, contain both sources of uncertainty, as discussed before.

The figures below show the standard 95 percent confidence band for the NAIRU for the three different models(V1, V2, V3). Model V3 was used to estimate the confidence bands for potential output. As presented in the figure below, the uncertainty around the NAIRU estimates depends on the model that was used. Augmenting the model by new state variables increases the uncertainty around the NAIRU estimates.

The conclusion from model V3 is that in the standard confidence band, actual unemployment was significantly different from the NAIRU only during three episodes. This reflects the limited power of the unobserved components approach. Similar results were reported by Laubach (2001), where for most countries that were checked the unemployment rate from 1970 to 1998 varied within the 95 percent confidence band.[13] This led Laubach to the conclusion that "The question whether at any point in time the actual unemployment rate is above or below

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[12] We thank I. Muchnik for suggesting this strategy.
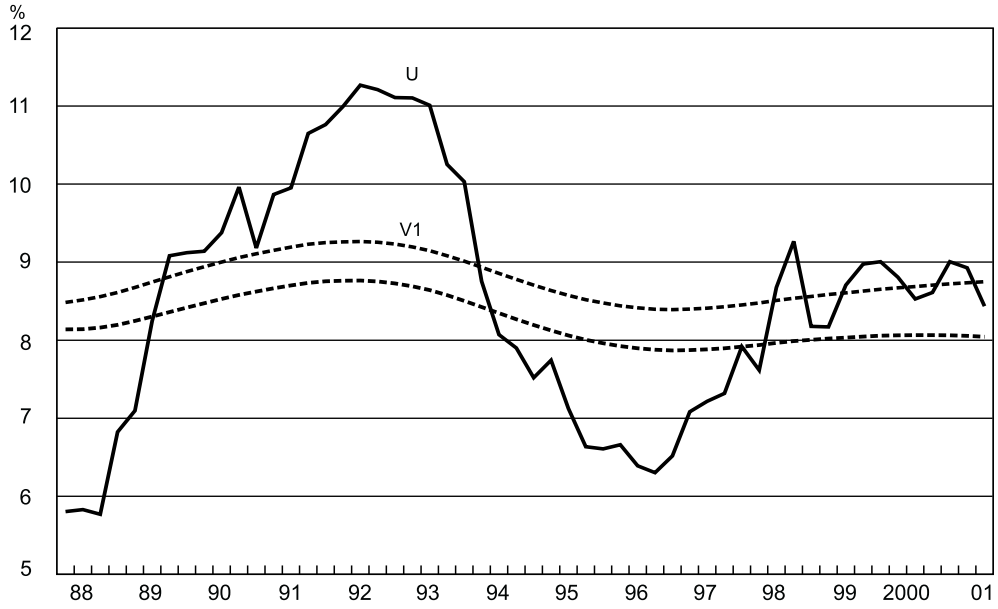
Figure 5a
NAIRU Confidence Band, 1988–2001

Figure 5b
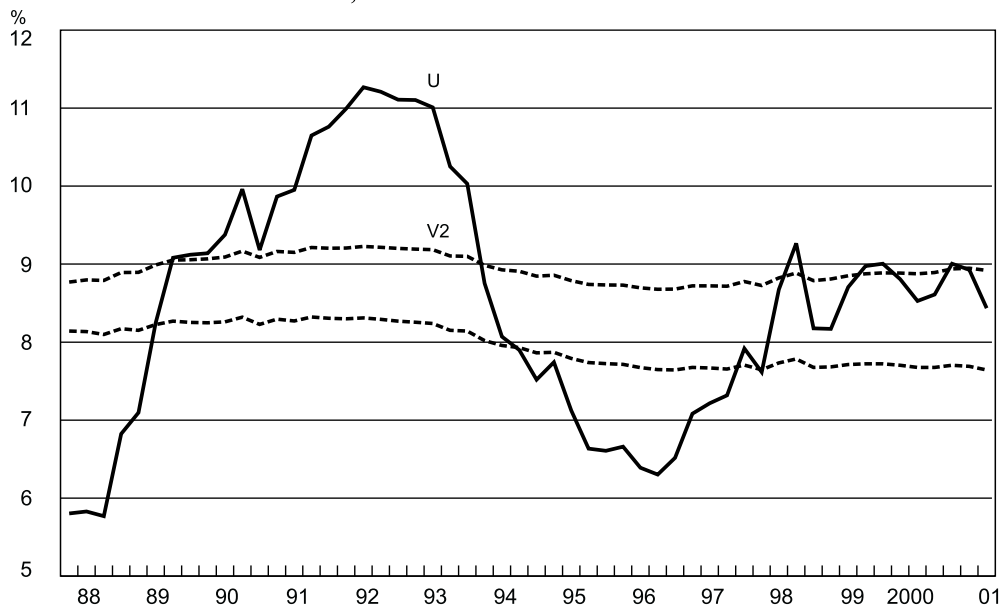**NAIRU Confidence Band, 1988–2001**

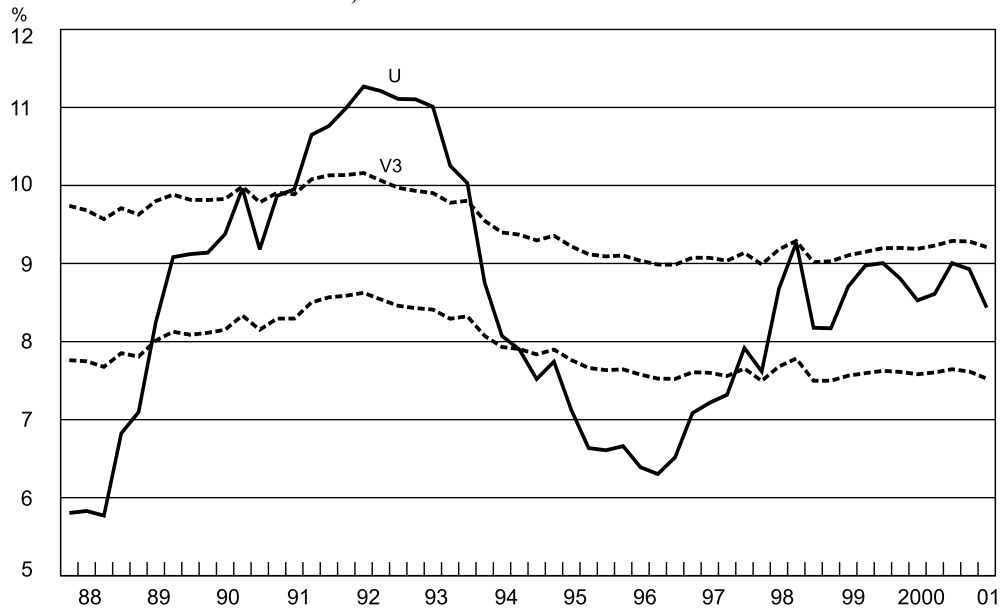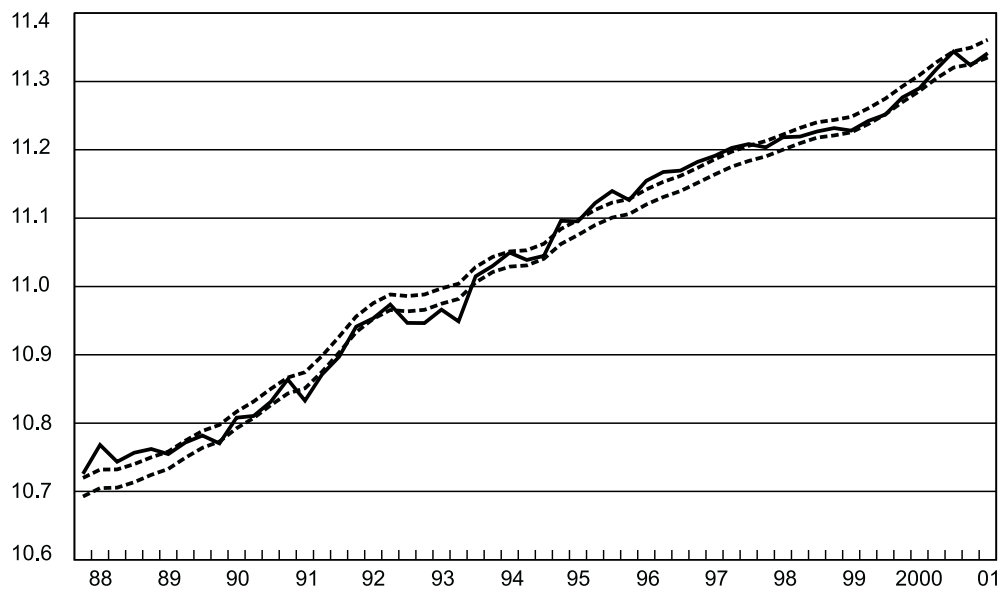**Figure 5c**
NAIRU Confidence Band, 1988–2001

Figure 6
Potential (log) Output Confidence Band, 1988–2001

the NAIRU can rarely be answered at conventional confidence levels.'' Note that the parsimonious models V2 and V1 generate lower uncertainty around the estimated NAIRU. This result is straightforward, as augmenting the State Space representation by new unobserved components increases uncertainty.

The limited uncertainty regarding the NAIRU estimates are mirrored in the uncertainty around potential output estimates. Again, using the standard confidence bands, actual output was significantly different from potential output only during three episodes. The high growth rate during 2000 for example, is still within the confidence band for potential output.

4.5 Sample effects

Although the sample period that we used for estimation was relatively short, one could question our assumption that the links between the variables are constant, or technically, that the system matrices H and F do not vary with time.

The problem with this approach is that it is clear that during this period (1987-2001) some structural changes, especially in the labor market, took place. Since wage determination is one of the channels that the Phillips curve works through, these changes pose the question whether the effect of the unemployment gap on prices has changed.

On the one hand, major changes in the labor market took place due to the influx of immigrants. This supply-side shock reduced the bargaining power of workers both directly and indirectly, as the new workers, most of them unorganized, reduced the bargaining power of labor unions. On the other hand, the transfer-payments policy became more generous, offsetting the effect of the immigration on the reservation wage, and leaving the total effect on the NAIRU unknown.

In addition to these effects, the number of foreign workers increased dramatically during the 1990s. Note that all these factors may also affect the participation in labor force.

In order to answer these questions we also estimated the NAIRU for a sub-sample starting in 1995—after the influx of immigrants. The results imply that the total effect of unemployment gap on prices is similar, but the lag structure is different: changes in inflation are much more sensitive to the contemporaneous unemployment gap, rather than to a lagged gap as for the whole sample. This may be the result of a less rigid labor market.

The estimation results of the augmented system (V3) imply that the same holds for equation (11), Okun's law, where the link between unemployment and output gap becomes contemporaneous rather than lagged. Again, this may reflect a more flexible labor market in which changes in output are mirrored immediately in the unemployment rate.

Figure 7 compares the NAIRU generated using the whole sample with that generated by the new reduced 1995-2001 sample. The results show that the NAIRU is stable with respect to sample effects. Note that the new sample is enlarged to end-2001 (three more observations compared to the full sample); this enables us to check the end-sample effect. We therefore check simultaneously for two different sample effects. The end-sample effect can be seen clearly after 1996, when the NAIRUs were almost identical. During 2001 the actual rate of

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[13] This refers to the bivariate model with I(2) NAIRU, that seems to capture the upward employment trend in the inspected countries better. This model implies that the NAIRU was within the band throughout the period in Canada, France, Italy and Australia, while in Germany, the US and the UK it was significantly different, at least once.

unemployment increased steeply from 8.6 percent in the first quarter to 10.3 percent in the last quarter. Since this process was not followed by acceleration downwards in the rate of

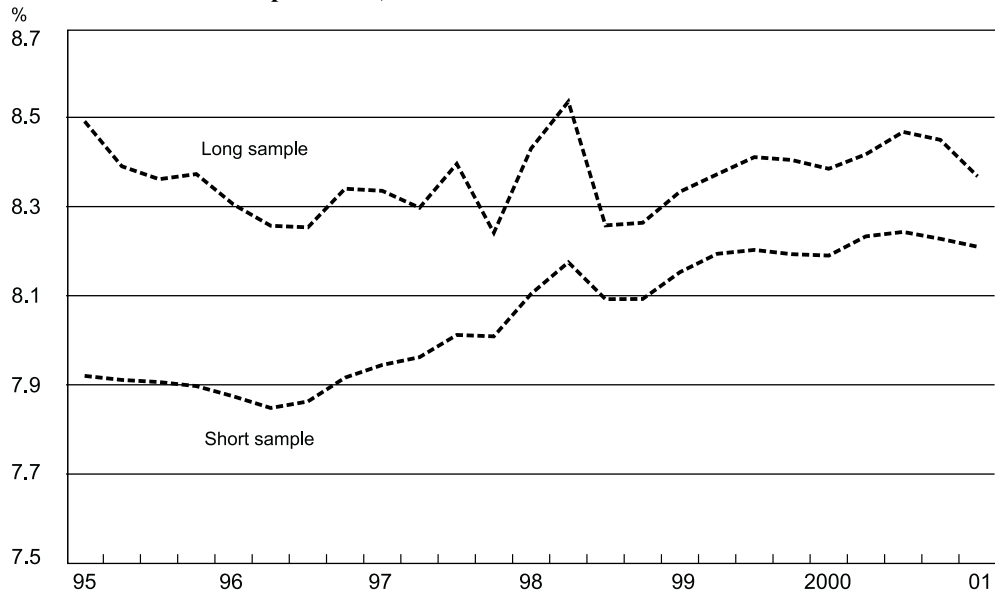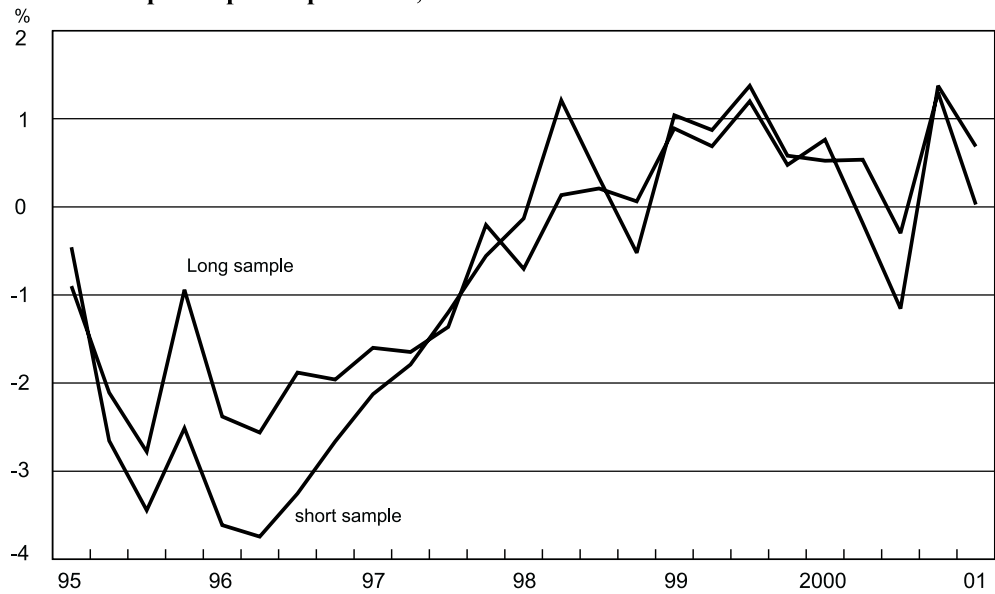**Figure 7**
The NAIRU: Sample Effect, 1995–2001

Figure 8
The Output Gap: Sample Effect, 1995–2001

inflation, the model interprets this as a rise in the NAIRU. Since we used smoothed Kalman estimates, which use the full information set, pre-2001 NAIRUs are updated upwards (the stickiness of the NAIRU does not allow for dramatic jumps). This explains the growing difference between the estimates after 1996. Still, the difference between the estimates in the first quarter of 2001 is less than 0.5 percentage points.

The sample effect on the output gap is more substantial. This is partly because the output gap is not as smooth as the NAIRU. Note that the differences in the NAIRU (and hence in the unemployment gap) are not reflected in the output gap. For example, the output gaps according to the different samples were identical during the third quarter in 2000, while the NAIRUs were different. This is the result of the changing coefficients in Okun's law equation (the H matrix). The differences between the estimates are not large, however: the largest difference is between the gaps is close to one percentage point (during the 1996 peak).

4.6 Prediction

This section describes one-step-ahead predictions of the unobserved components. In order to generate these predictions, we used the whole sample to compute the system matrices (assuming that they are time-invariant), and then applied the prediction equation of the Kalman filter (see Appendix 3) for each observation, starting from 1997:III. When comparing the predicted sub-series with previously computed expected (smoothed) values of state variables we found a very good fit. The correlation between the predicted NAIRU and its smoothed realization is 0.8.

Figure 9 presents the predicted values of the unemployment gap and its realizations for the last four years of the sample. These results point out that the system is very stable, suggesting

Figure 9
Prediction, 1997–2001
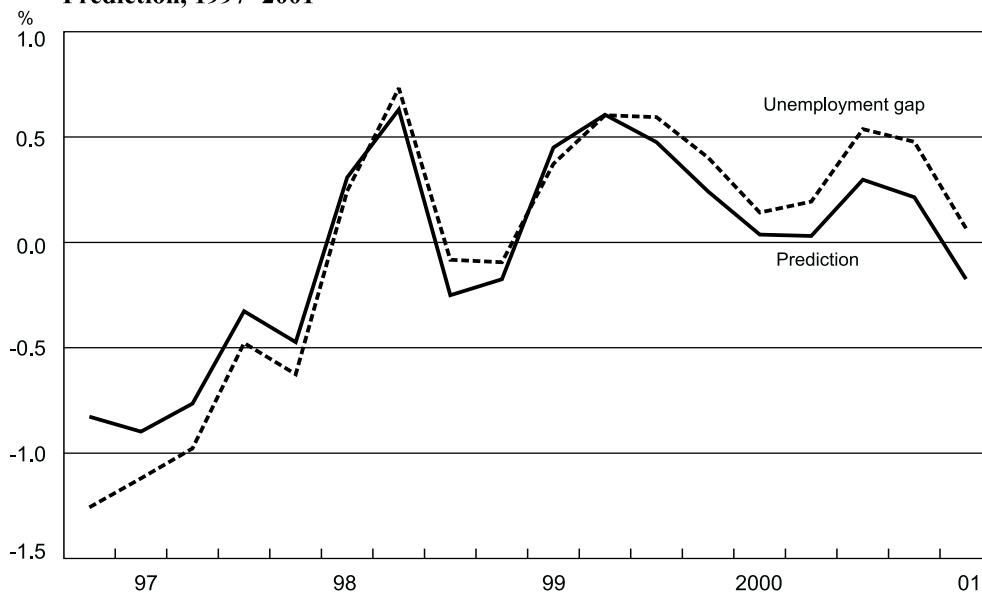
that it may be used for short-term prediction. Note that these results were achieved by using the same system matrices for the whole sample: obviously, updating the matrices at each step will improve the prediction power.

5. CONCLUSIONS

The substantial changes of unemployment rates, inflation rates, and the labor-market structure during the last decade raise the question as to what extent these changes are reflected in the NAIRU. In order to answer this we constructed State Space Models in which the NAIRU and potential output are latent variables. These variables are identified both by economic relations such as the Phillips curve and Okun's law, and by non-theoretical assumptions about their evolution over time. Following the state-of-the-art literature in this field, we start by estimating the NAIRU only with the simple univariate model (V1) suggested by Gordon (1997), and a bivariate specification (V2) as used by Laubach (2001). Next, we estimate a variant of the AJ model (1999), an augmented SSM for simultaneous estimation of the NAIRU and potential output (V3).

We use the Kalman filter, combined with a quasi-maximum-likelihood algorithm for restricted SSM estimation (EM) of Watson and Engle (1983), in order to estimate the model parameters (the system matrices), the unobserved components. In addition, we use the jackknife technique to quantify the uncertainty around these estimates—namely, the uncertainty around the NAIRU and potential output—because these variables are unobserved, and the system matrices are estimated and hence are not known without uncertainty. The fact that each model is completely nested in the next specification allows us to shed some light on the trade-off between the goodness-of-fit of the model and the uncertainty around the estimated unobserved components.

The estimation results suggest that the NAIRU path is relatively stable, with no clear evidence for hysteresis after the unemployment peak of 1991.[14] Note that this conclusion is based on models where the volatility of the NAIRU is not restricted; that is, this outcome is not a result of any direct restrictions on the NAIRU path, while the simple model (V1) required this restriction in order to reach convergence (as in Gordon (1997)). The estimated NAIRU during the sample period varied within approximately a one percentage point band, while the unemployment rate varied between 5.8 and 11.8 percent. The derived unemployment gap had a significant negative effect on the price process, as presented in the single-equation estimates of the Phillips curve. This in turn justifies the use of such an equation as an identifying equation, as it contains significant information about the unobserved components. As always, using the state-space methodology does not come without a cost: the uncertainty around the unobserved components is substantial, and therefore it is usually hard to judge the state of the economy using the conventional confidence levels, as for example holds for the period 1997-2001. This leads to the conclusion that one should use caution when deriving direct policy implications based on the NAIRU estimates.

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[14] At least as reflected in the unemployment rate. The estimates show that the disinflation process during the 1990s did not cause an increase in the NAIRU. Nevertheless, it is possible that hysteresis affected the rates of participation in the labor force.

APPENDIX 1

The data

This section describes variables used to estimate the quarterly model for Israel from 1987:I to 2001:IV.

Endogenous observed variables:

(y)	Gross Domestic Product, at fixed 1995 prices, reported quarterly by CBS, seasonally adjusted by X12 procedure, levels at natural logarithms.
(Δpai)	consumer price index, reported monthly by CBS, seasonally adjusted and log differenced at quarterly level. This variable is called pai. The variable used in the model is Dpai, that is the first difference of pai multiplied by 100.
(u)	chained unemployment rate (percent), reported quarterly by CBS, seasonally adjusted.

Exogenous variables:

(rer)	The NIS real exchange rate, reported quarterly by IFS statistics (IMF), log difference.
(mp)	the dollar import price index, excluding fuel and diamonds, PASH formula, reported quarterly by CBS on 1991 base, seasonally adjusted and log differenced.
(productivity)	labor productivity, measured as the relation of GDP to the number of employees (natural logarithm of this relation is used). Employees include Israelis, Palestinians and foreign workers. Israeli employees data, based on Labour Force surveys, have been chained through their multiple samples. GDP and employees time-series are seasonally adjusted.
(oil)	fuel import price index, PASH formula, reported quarterly by CBS on 1991 base, log differenced. No seasonality was found.
(rel_mp)	relative import price (excluding fuel and diamonds), computed as import NIS price index relatively to CPI. To convert the dollar price index into NIS, representative exchange rate is used. CPI and import price indices are seasonally adjusted. The log differences of their relations are used.
(rel_delek)	relative fuel import price computed as import NIS fuel price index relatively to CPI. To convert the dollar fuel price index into NIS, the average exchange rate is used. The CPI quarterly index is seasonally adjusted. The log difference of this index is used.
(immigration)	the share of new immigrants in population growth. Measured as quarterly new immigrant arrivals relative to total population growth. The first difference of this relation is used.

All quarterly time-series are supported and currently updated by the Bank of Israel Research Department Database.

APPENDIX 2

The systems

This section describes all the state space systems that were estimated.
All the moving average systems take the same set of exogenous variables.
The Auto Regressive systems take the same set plus four lags of Dp .
All exogenous variables are normalized to zero.

Version 1
Measurement

$$
\begin{bmatrix} u_t \\ \Delta\pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & \rho_1 \end{bmatrix} \begin{bmatrix} u_t^n \\ u_t - u_t^n \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{bmatrix} \begin{bmatrix} \Delta\pi_{t-1} \\ \Delta\pi_{t-2} \\ \Delta\pi_{t-3} \\ \Delta\pi_{t-4} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{bmatrix} \begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ x_t^4 \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{\Delta\pi} \end{bmatrix}
$$

State

$$
\begin{bmatrix} u_t^n \\ u_t - u_t^n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{t-1}^n \\ u_{t-1} - u_{t-1}^n \\ u_{t-2} - u_{t-2}^n \end{bmatrix} + \begin{bmatrix} \varepsilon_t^n \\ \varepsilon_t^{gap} \\ 0 \end{bmatrix}
$$

In this system ε_t^n must be restricted.

Version 2 (AR)
Measurement

$$
\begin{bmatrix} u_t \\ \Delta\pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \rho_1 & \rho_2 \end{bmatrix} \begin{bmatrix} u_t^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{bmatrix} \begin{bmatrix} \Delta\pi_{t-1} \\ \Delta\pi_{t-2} \\ \Delta\pi_{t-3} \\ \Delta\pi_{t-4} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{bmatrix} \begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ x_t^4 \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{\Delta\pi} \end{bmatrix}
$$

State

$$
\begin{bmatrix} u_t^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \delta_1 & \delta_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_{t-1}^n \\ u_{t-1} - u_{t-1}^n \\ u_{t-2} - u_{t-2}^n \end{bmatrix} + \begin{bmatrix} \varepsilon_t^n \\ \varepsilon_t^{gap} \\ 0 \end{bmatrix}
$$

Version 2 (MA*)*
Measurement

$$
\begin{bmatrix} u_t \\ \Delta\pi_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & \rho_1 & \rho_2 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_t^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 \end{bmatrix} \begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ x_t^4 \\ x_t^5 \\ x_t^6 \end{bmatrix} \begin{bmatrix} 0 \\ \varepsilon_t^{\Delta\pi} \end{bmatrix}
$$

State

$$
\begin{bmatrix} u_t^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \rho_1 & \rho_2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_1 & w_2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_{t-1}^n \\ u_{t-1} - u_{t-1}^n \\ u_{t-2} - u_{t-2}^n \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^n \\ \varepsilon_t^{gap} \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Note that there is no explicit error term in the measurement equation. The implicit error term ε_t is a state variable.

Version 3
Measurement

$$
\begin{bmatrix} y_t \\ u_t \\ \Delta\pi_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & \gamma_1 & \gamma_2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_1 & \rho_2 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_t^p \\ u_t^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \\ d_t \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \end{bmatrix} \begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ x_t^4 \\ x_t^5 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ 0 \\ \varepsilon_t^{\Delta\pi} \end{bmatrix}
$$

State

$$
\begin{bmatrix} y_t^p \\ u_t^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \\ d_t \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_1 & \delta_2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_1 & w_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1}^p \\ u_{t-1}^n \\ u_{t-1} - u_{t-1}^n \\ u_{t-2} - u_{t-2}^n \\ d_{t-1} \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^p \\ \varepsilon_t^n \\ \varepsilon_t^{gap} \\ 0 \\ 0 \\ \varepsilon_t \\ 0 \end{bmatrix}
$$

Note that there is no explicit error term in the Phillips curve equation. The implicit error term e_t is a state variable.

APPENDIX 3

The Kalman filter and smoother

The SSM (or SSF—state space form) allows us to distinguish between two blocks of equations: the measurement/signal block describes the dependence of observed components (such as inflation and output) in a set of variables, of which some may be unobserved, and others observed exogenous variables. This block may be based on theoretical grounds (hence, structural), or on identities (see Appendix 2). The second block, the transition/state block describes the stochastic process of the unobserved components. This block contains atheoretical assumptions describing the way the unobservables evolve over time. Though some of these assumptions may be justified empirically, such as the evolution of potential output that is proxied by a random walk plus drift process, these assumptions reflect the fact that our knowledge about these unobservables is limited (AJ, 1999).

The SSF is composed of two equations. The measurement (or observation) equation relates n x 1 vector of observable variables y_t, to α_t, an m x 1 vector of unobservable variables (the state vector).

(17) $y_t = Hz_t + BX_t + \varepsilon_t$,

where H is an n x m matrix, and ε_t is an n x 1 vector of serially uncorrelated disturbances, satisfying $E(\varepsilon_\tau) = 0$ and $Var(\varepsilon_\tau) = Q_t$. The elements of α are unobserved. X is a matrix of observed exogenous variables.

The transition (or state) equation specifies the stochastic process generating the unobservable α's as a first order Markov process

(18) $z_t = Fz_{t-1} + R\eta_t$,

where F is an m x m transition matrix $E(\eta_\tau) = 0$ and $Var(\eta_\tau) = Q_t$. The matrices H, X, Q, F and R will be referred to as the system matrices. The models that are estimated in this study assume that the system matrices do not change over time. Hence, the system is time-invariant, and the time subscripts may be omitted from the system matrices.

The assumptions below complete the specification of the state space system:
$E(\alpha_0) = a_0$, $Var(\alpha_0)(\alpha_0) = P_0$, where α_0 is the initial state vector, and P_0 is the initial state variance.

The disturbances of the measurement and transition equations are assumed to be uncorrelated with each other in all time periods, and uncorrelated with the initial state α_0.

Once rewritten in a state space form, the system may be estimated, including the unknown parameters, using the Kalman Filter and smoother. The Kalman Filter is a recursive procedure for computing the optimal estimator (thus, minimizing the Mean Square Error) at time t based on the information available at that time. This information consists of the observations up to and including y_t. The filter consists of two sets of equations: the first set is the prediction equations that generate optimal predictors for the state vector z, based on the information set at $t - 1$. Let a_{t-1} be an MSE of the state vector a_{t-1}, and P_{t-1} be the m x m covariance matrix, $P_{t-1} = E(a_{t-1} - \alpha_{t-1})(a_{t-1} - \alpha_{t-1})$, based on the information at $t - 1$, thus based on y_{t-1}. Then the prediction equations are simply given by $a_{t/t-1} = Fa_{t-1}$ and $P_{t/t-1} = FP_{t-1}F' + RQR'$.

Once a new observation y_t is available, it is possible to update the estimated state vector $a_{t/t-1}$ and its covariance matrix $P_{t/t-1}$ using the information embodied in the prediction error $e_t = y_t - Ha_{t/t-1} - SX_t$. This is done by the updating equations $a_t = a_{t/t-1} + P_{t/t-1}HF^{-1}e_t$ and $P_t = P_{t/t-1} - P_{t/t-1}HF^{-1}HP_{t/t-1}$ where $F = HP_{t/t-1}H' + Q$. The estimators for a_t and P_t that are generated using the updating equations are called filtered estimates. Note that these estimates are based on the information set up to and including y_t only, thus, the filtered estimates were generated using a one-side filter (unlike moving average filters, for example). As pointed out by Harvey (1989) and Hamilton (1994), when the state variable of interest has some economic meaning, it is better to use a full information set: thus, in order to estimate the NAIRU at time t, where, it is better to use the information up to T – NAIRU$_{t/T}$, than the truncated information set NAIRU$_{t/t}$. Thus, after the filtered estimates for the whole sample are computed, we use the Kalman smoother to generate full information set estimates. The smoothed estimates are generated using the backward recursion $a_{t/T} = a_t + P_t^*(a_{t+1/T} - Fa_t)$ and $P_t^* = P_t F P_{t+1/t}^{-1}$.

APPENDIX 4

The jackknife technique

Let $\hat{Z}_{k\ \tau}$ be the t-th observation of the k-th state variable (i.e., NAIRU), obtained as result of the estimation procedure on the whole sample of size T (in our case $T = 57$ quarters). This value constitutes a conditional expectation of the true value $Z_{k\ \tau}$.

Let $\tilde{Z}_{k\ \tau}^{(-i)}$ be the corresponding observation, estimated on the ''spoiled'' sample, when the i-th original row of data (dependent and independent variables relating to the i-th date) was deleted and substituted by averaging the neighbors around it. Suppose this procedure was repeated g times.

Consider all multiple realizations of the k-th state variable at date t and define:

$$\tilde{Z}_k \tau^{(i)} = g\hat{Z}_k \tau - (g-1)\tilde{Z}_k \tau^{(-i)}$$

and

$$\tilde{Z}_{k\ \tau} = \frac{1}{g}\sum_{i=1}^{g}\tilde{Z}_k \tau^{(i)}.$$

The length of the confidence interval $\left|\tilde{Z}_k \tau - Z_k \tau\right|$ for the $\{\tau, k\}$ observation may be found as:

$$t_{\alpha,\ g-1}\sqrt{\frac{\sum_{i=1}^{g}\left(\tilde{Z}_k \tau^{(i)} - \tilde{Z}_k \tau\right)^2}{g(g-1)}},$$

where $t_{\alpha,\ g-1}$ is taken from the Student distribution with a and $g - 1$ degrees of freedom (see Miller, 1974).

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