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Heterogeneous Discount Factor, Education Subsidy, and Inequality

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הטרוגניות בשיעור העדפת הזמן, סובסידיה להשכלה ואי שוויון

זיו נאור

תמצית

תהליך של צבירת הון אנושי במסגרת מודל של הטרוגניות בשיעור העדפת הזמן מצביע על כך שסובסידיה לצבירת ההון האנושי מגדילה את אי השוויון בכלכלה. למימון הסובסידיה באמצעות גביית מסים, ובעיקר לחלוקת המס בין סוגי הפרטים באוכלוסייה תפקיד חשוב באי השוויון.

Heterogeneous Discount Factor, Education Subsidy, and Inequality

by

Ziv Naor

Abstract

A human capital accumulation process in a heterogeneous discount factor framework shows that higher subsidy to human capital accumulation leads to more income inequality. Financing the subsidy via taxes, and the division of taxes between the types of agents play an important role in income inequality.

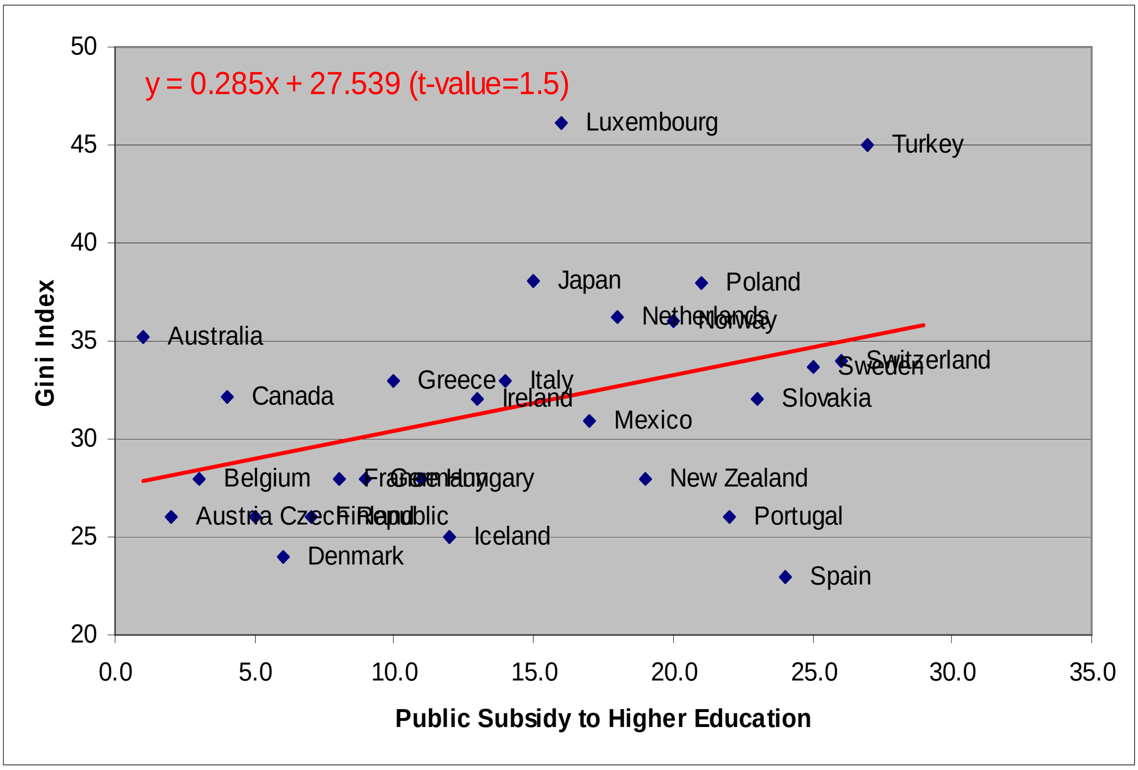
1. Introduction

Human-capital accumulation is a key element in modern economic growth and a factor of enormous influence on income inequality, especially given the positive correlation of education and income. Thus, governments tend to intervene in the human-capital accumulation decisions of the population. The optimization of government intervention in the human-capital accumulation process was vastly investigated, leading to two main approaches: subsidizing higher education and establishing student-loan programs. This paper sheds some light on the former. Obviously, the sources of heterogeneity among agents that generate heterogeneity in human capital affect the ability of a government to attain goals such as growth and the mitigation of income inequality.

A major question dealt with in the literature is whether subsidizing human-capital accumulation attains its main goal, the mitigation of income inequality. Heckman, Lochner, and Taber (1999), in a survey of empirical studies, show that partial equilibrium analysis overestimates the effect of education subsidies because it ignores possible changes in the return to education. Dynarsky (2002), in an empirical work, finds that subsidizing human capital hardly induces agents to acquire more education and has a much stronger effect on wealthier agents than on the rest of the population. Fernandez and Rogerson (1995) show that when the education subsidy is only partial, higher-income individuals may use it to exclude poorer individuals from receiving education, thereby extracting resources from them. Trostel (1996) shows that the subsidy merely reduces the distortion created by income taxes on optimal human-capital accumulation.

OECD data show a positive cross-country correlation between the size of public subsidy for higher education (as a fraction of its cost) and the Gini index (Figure 1).[1]

Figure 1: Gini Index and the Public Subsidy for Higher Education in OECD Countries[2]



In this paper, I build a model that shows that, as expected, subsidizing the cost of human-capital accumulation induces agents to acquire more education. However, since rich agents acquire more education than others and since the subsidy lowers the return to education in general equilibrium, the subsidy exacerbates inequality. In this model, the source of heterogeneity across agents is the agents' subjective discount factor. Specifically, I assume that the economy consists of two types of agents, each maximizing lifetime utility from consumption and leisure given an intertemporal

[1] The causality of the correlation is unclear: either a higher subsidy leads to higher income inequality, as predicted by the model presented in Naor (2012), or, since governments adopt the borrowing-constraint approach, claiming that subsidizing human capital may mitigate income inequality, the greater the inequality, the more inclined governments are to subsidize higher education.

[2] Source: *Education at a Glance* (2009), *Human Development Report* (2009).

budget constraint. The agents' wages depend on their human capital, which evolves commensurate with their investment in education. This investment requires both time and financial resources. The agents may freely save or borrow subject to a borrowing constraint. The agents are taxed to finance government expenditure—human-capital subsidies in particular.

Heterogeneity in human capital is usually explained via one of two major approaches. The first follows Mincer (1958), who claims that human capital is distributed as a result of the distribution of "abilities". Agents differ in this aspect, which is usually unobservable and makes human-capital acquisition easier or more profitable. The second approach is based on borrowing constraints (Friedman, 1955; Galor and Zeira, 1993). Here, since human capital is uncollaterizable, it is either more expensive or its accumulation cannot be financed by borrowing. Therefore, agents born to rich parents go to school while others do not. By implication, wealth inequality today generates heterogeneity in human capital and, in turn, income inequality in the next generation. Some studies (e.g., Galor and Tsiddon, 1997) merge both approaches.

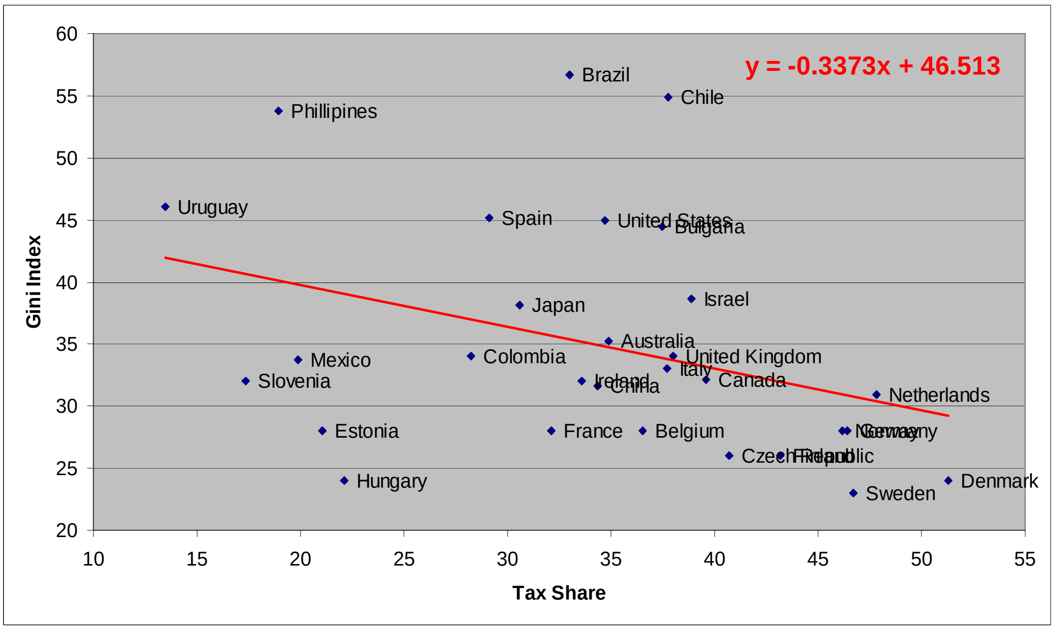
In analyzing the way a subjective time preference affects agents' decisions to acquire human capital or not, it is relevant to examine the return to education and estimate rates of time preference that might explain the differences between agents. According to the U.S. Bureau of Labor Statistics, having a college education in 1979 raised average monthly income from $707 to $908. Assuming that college entails four years of forgone income, an interest rate of 4% per year, and a forty-year working lifetime, the subjective yearly discount factor that would cause an agent to be indifferent between attending college or not is 0.97. The findings in Following Lawrance (1991) demonstrate that this is a realistic discount level.

In this paper, I also focus on how the government finances education subsidies and how its choice affects income distribution. I examine how the differential burden of taxes that finance education subsidies affect agents' levels of human capital and, in turn, income inequality.

To test the effect of taxation empirically, I build a tax-share variable. I calculate the share of taxes (direct and indirect) paid by the seven lowest income deciles in the population out of total taxes collected. This metric captures both the degree of income inequality and the progressivity of the tax system in each country.

The tax share and the pre-tax Gini index are negatively correlated across countries (t-value = –2.46), i.e., a higher tax share in the seven poorest deciles is paired with lower inequality (Figure 2). This negative correlation may reflect one of two things: the taxation of the poor induces them to work more and thus reduces inequality, or the more equal the economy is, the higher the share of taxes paid by the poor.

Figure 2: Tax Share and Gross Income Gini Index[3]



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[3] The sources of the tax-share data are specified in Table 1; the source of the Gini index is *Human Development Report* (2009) statistics and data.

The model presented here differs from that in Naor (2012) in two important assumptions. First, an investment in human capital requires not only financial resources but also time. This assumption reduces the human capital of both agents relative to Naor (2012) but, more importantly, it reduces income inequality in the model.

The second assumption is that reducing the cost of human-capital investment is accomplished not exogenously but via government subsidizes financed by taxes. Allowing different tax levels for each type of agent and examining the effect of the tax system, I find that since the impatient (poor) agent carries a higher share of the total cost of the subsidy, he works more and accumulates more human capital, narrowing the pre-tax income inequality. The net income inequality, however, rises due to this tax regime.

The rest of this paper is structured as follows: Section 2 presents the theoretical model, Section 3 presents the calibration of the model steady state and compares the results with those presented in Naor (2012), and Section 4 concludes.

2. The Model

The model resembles the one presented in Naor (2012) with two main differences. First, investment in human capital requires not only financial resources, as in Naor (2012), but also time. In each period, the agent chooses how much time to spend acquiring human capital. Hence, the cost of this investment also includes the reduction of leisure and/or labor supply. Second, I augment the model by adding a government that collects taxes to subsidize human-capital accumulation. Hence, in this model the mitigation of human-capital accumulation cost is caused by a government subsidy.

The model follows Ben-Porath (1967) and Becker (1975) with respect to the human-capital accumulation process. Human capital increases due to investment and depreciates over time. Moreover, human capital enlarges production and wages.

The rest of the model resembles that in Naor (2012). In particular, the economy is comprised of two types of agents who differ in their subjective time preference, $\hat{\beta} < \tilde{\beta}$. The population is composed of *N* agents; *m* of them patient ($\tilde{\beta}$) and *N*-*m* impatient ($\hat{\beta}$). This setup allows us to examine the income inequality created by different time preferences. Additionally, the horizon is infinite; there is a single good that may be either consumed or invested in human capital, and agents choose their paths of consumption, leisure, and investment in human capital.

2.1. Preferences

The agent obtains utility from consumption, leisure, and government expenditure as follows:

$$(1) \qquad \sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t-s_t, G_t)$$
$$U(c_t, 1-l_t-s_t, G_t) = \ln(c_t) + \varphi \ln(1-l_t-s_t) + \ln(G_t),$$

where c_t represents consumption at time *t*, l_t is time worked, and s_t is time devoted to human-capital accumulation. Total time is normalized to 1. The parameter φ captures the relative importance of leisure compared to consumption and G_t represents government expenditure on uses other than human-capital accumulation subsidies, which are discussed at greater length below.

2.2. Production

Production is based on effective labor, L_t, which depends on labor, l_t, and human capital, h_t. Defining output at time t as y_t, we obtain the following production function:

(2) $$y_t = A\left((N-m)\hat{L}_t + m\tilde{L}_t\right)^{\alpha}, \quad \hat{L}_t = \hat{l}_t\hat{h}_t^{\gamma}; \tilde{L}_t = \tilde{l}_t\tilde{h}_t^{\gamma}, \quad A > 0, \quad \alpha \in (0,1),$$

Effective labor is defined as labor multiplied by effective human capital, h_t^{γ}, where $\gamma \in (0,1)$ implies diminishing returns to human capital. This assumption follows Bils and Klenow (2000), who show that the return to education is diminishing. This also assures stability at the steady state.

Production is performed by a large number of identical and competitive firms that the agents own. Due to the diminishing returns to production, i.e., $\alpha \in (0,1)$, the firm earns positive profits:

(3) $$\pi_t = y_t - w_t((N-m)\hat{L}_t + m\tilde{L}_t),$$

where w_t is the wage per effective labor unit.

2.3. Human Capital

Human capital depreciates at rate $\delta \in (0,1)$ and evolves as

(4) $$h_{t+1} = (1-\delta)h_t + s_t \,.$$

Thus, h_t is expressed in units of time to capture the time it takes to invest in human capital. For example, these units can be interpreted as credit units in academic terms. An investment made in period *t* raises the agent's human capital in period *t*+1. Investment has also a financial cost, denoted by e_t, which enters the household's budget constraint.

2.4. Savings Mechanism

Agents may freely buy a one-period bond that earns interest rate R_t. Selling bonds, however, is subject to the borrowing constraint:

$$\hat{b}_t, \tilde{b}_t > -\phi \qquad \forall t, \qquad \phi > 0,$$

where $\hat{b}_t$ and $\tilde{b}_t$ are the amounts of bonds held by the two types of agents at the beginning of period t.

2.5. Government

The government provides public good G_t, and applies subsidy g_t, $0 < g_t < 1$, to the cost of human capital. All government expenditures are financed by lump-sum tax τ_t.

I allow for two different levels of lump-sum taxes, one for the patient agent and another for the impatient agent. I assume that the government can identify the type of agent by her choices (e.g., by her level of assets/debts). The different levels of lump-sum taxes between agents capture elements of a progressive tax system without the distortions that progressive taxation creates. It can be shown that, for high enough ϕ[4], identifying agents by their assets will not induce any agent to change her optimal choices (i.e., both agents forfeit some lifetime utility if acting as an agent of the other type).

[4] The main concern is that the patient agent would like to imitate the impatient agent. In order to prevent that, the discounted gain from imitation should be lower than its discounted cost:

$$\sum_{t=1}^{\infty} \tilde{\beta}^t \left((R_t - 1)\tilde{b}_t + \frac{1}{m}\pi_t - \tilde{\tau}_t \right) > \sum_{t=1}^{\infty} \tilde{\beta}^t \left((R_t - 1)\hat{b}_t - \hat{\tau}_t \right).$$

Thus, the budget constraint should be bigger than:

$$\phi > \frac{\tilde{\tau}_t - \hat{\tau}_t - \frac{1}{m}\pi_t}{2(R_t - 1)}, \quad \forall t.$$

Since the subjective discount factor discounts both gain and losses, it plays no role in this term.

2.6. Budget Constraint

Based on the foregoing, the budget constraint of the patient agent is:

(5) $$b_{t+1} + c_t + e_t(1 - g_t)s_t + \tau_t = w_t l_t h_t^{\gamma} + R_t b_t + \frac{1}{m}\pi_t$$

where m is the number of patient agents in the population; thus, $1/m$ represents their relative share of ownership of the firm. The parameter e translates time spent learning into resources and thus it can be interpreted as tuition. Since the model deals with a one good economy, the tuition is expressed in terms of that good.

The budget constraint of the impatient agent does not include the firm's profits. It can be shown that if he has an ownership stake in the firm, there is a price at which he will sell it to a patient agent.[5]

2.7. Households' Optimization

The patient agent maximizes her lifetime utility given her sequence of budget constraints. The problem she solves is:

(6) $$\begin{aligned} &\max \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t - s_t, G_t) \\ &s.t.: \quad b_{t+1} + c_t + e_t(1 - g_t)s_t + \tau_t = w_t l_t h_t^{\gamma} + R_t b_t + \frac{1}{m}\pi_t \qquad t = 0,1,2... \\ &\qquad \lambda_t(b_t + \phi) \geq 0, \\ &\qquad \lambda_t \geq 0, \end{aligned}$$

where λ_t is the Lagrange multiplier of the borrowing constraint.

For the impatient agent, the problem is the same except he receives none of the firm's profits.

[5] Each agent valuates the firm as its discounted future profits. Since the patient agent has a higher subjective discount factor, she observes a higher value. Thus, a price exists that is lower than the firm's value as observed by the patient agent and higher than that observed by the impatient agent.

The first-order conditions are:

$$-\frac{1}{c_t}+\frac{\beta R_{t+1}}{c_{t+1}}+\lambda_{t+1}=0,$$

$$-\frac{e_t\left(1-g_t\right)}{c_t}-\frac{\varphi}{1-l_t-s_t}+\beta\left[\begin{array}{l}\frac{\gamma w_{t+1}l_{t+1}h_{t+1}{}^{\gamma-1}+e_{t+1}\left(1-g_{t+1}\right)\left(1-\delta\right)}{c_{t+1}}+\\+\frac{\varphi\left(1-\delta\right)}{1-l_{t+1}-s_{t+1}}\end{array}\right]=0,$$

(7) $$\frac{w_t h_t{}^{\gamma}}{c_t}-\frac{\varphi}{1-l_t-s_t}=0,$$

$$\lambda_t\left(b_t+\phi\right)\geq 0.$$

The first two conditions are the Euler equations for the optimal choices of bonds and human capital. Note that the second equation differs from its equivalent in Naor (2012) because here the investment in human capital requires both time and resources. This new equation shows that the agent's concession today, in terms of utility from both consumption and leisure, equals the increase in utility gained from the return on the investment, expressed in consumption and leisure, in the next period.

The third equation is the labor-supply condition, equating the marginal utility gained from an increase in labor supply to the marginal utility cost of forgone leisure.

2.8. Firms' Optimization

Firms maximize their profits as given in Equation (3), with respect to effective labor, so that:

(8) $$w_t=\left(1-\alpha\right)A((N-m)\hat{L}_t+m\tilde{L}_t)^{-\alpha}$$

2.9. Government Budget

The government maintains a balanced budget, so that:

(9) $$g_t e_t\left(\left(N-m\right)\hat{s}_t+m\tilde{s}_t\right)+G_t=\hat{\tau}_t(N-m)+\tilde{\tau}_t m$$

2.10. Equilibrium Conditions

The three following markets must clear:

2.10.1 *Financial Market*

The financial market clears when total assets in the economy are equal to the total debt:

(10) $(N-m)\hat{b}_t + m\tilde{b}_t = 0.$

2.10.2 *Labor Market*

The labor market clears when the wage per efficiency unit equals the firm's demand for effective labor (Equation (8)) to the agents' supply of labor—Condition 7(c) solved for w_t).

(11)
$$w_t = (1-\alpha)A((N-m)\hat{L}_t + m\tilde{L}_t)^{-\alpha},$$
$$w_t = \frac{\varphi c_t}{(1-l_t-s_t)h_t^{\gamma}}.$$

2.10.3 *Goods Market*

Since there is a single good, the amount consumed by the agents and invested in human capital equals the amount produced by the firm.

(12) $y_t = (N-m)\hat{c}_t + m\tilde{c}_t + e_t((N-m)\hat{s}_t + m\tilde{s}_t).$

2.11. Steady State

It can be shown that the steady state is characterized by zero growth[6], i.e., by: $c_{t+1} = c_t; \quad l_{t+1} = l_t; \quad h_{t+1} = h_t; \quad s_{t+1} = s_t \quad \forall t.$[7]

[6] This situation may change if physical capital is added to the model; however, under the assumptions $0 < \alpha, \gamma < 1$ made here, there is no continuous growth.

As shown by Iacoviello (2005), impatient agents are borrowing-constrained in the steady state, so that $\hat{b} = -\phi, \quad \tilde{b} = \frac{N-m}{m}\phi.$ This implies that the Lagrange multiplier of the patient agents is zero, whereas the Lagrange multiplier of the impatient agents is positive: $\hat{\lambda} > 0, \quad \tilde{\lambda} = 0.$ By continuity, this holds also in the neighborhood of the steady state.

At the steady state, the interest rate that clears the market equals the subjective time preference of the patient agents. Intuitively, this means that the patient agent is indifferent between saving marginally more or not, while the impatient agent would like to borrow more. Yet since the impatient agents are constrained, the interest rate does not effect their actual borrowing decision.

The steady state does not have a closed-form solution and has to be solved numerically for specific parameter values. However, some arguments may be made to shed light on the characteristics of the steady state. From the first-order conditions and the budget constraint, the optimal levels of labor supply may be written as:

(13)
$$\hat{l} = \frac{1-\hat{s}}{(1+\varphi)} + \frac{e(1-g)\hat{s} - (R-1)\hat{b} + \hat{\tau}}{w\left(\frac{\hat{s}}{\delta}\right)^{\gamma}\left(1+\frac{1}{\varphi}\right)},$$
$$\tilde{l} = \frac{1-\tilde{s}}{(1+\varphi)} + \frac{e(1-g)\tilde{s} - \left(\frac{1}{m}\right)\pi - (R-1)\tilde{b} + \tilde{\tau}}{w\left(\frac{\tilde{s}}{\delta}\right)^{\gamma}\left(1+\frac{1}{\varphi}\right)}$$

Equations (13) show that the agents' time preferences influence labor-supply decisions not directly but via the optimal amount of time invested in human-capital accumulation, *s*, human capital, *h*, and stake in the firms' profits. Note that assets, *b*, are exogenous in the steady state because the borrowing constraint binds.

[7] I obviously assume that in the steady state the exogenous variables are constant, i.e., $e_{t+1}=e_t$, $g_{t+1}=g_t$.

As usual, the agents' assets influence their labor supply. As the agents become richer, they choose to reduce their labor supply at any fixed level of human capital, irrespective of their level of patience. The lump-sum tax levied on the agent, τ_t, has an opposite-sign wealth effect.[8]

Another important feature of Equations (13), as specified in Appendix A (Equation A.4), is that as long as income from assets (interest on bonds and profits of firms) is lower than a critical value, human capital and labor supply are positively correlated. Once income from assets (interest on bonds and profits of firms) exceeds this critical value, human capital and labor are negatively correlated.

Following Equation (7), the optimal level of human capital in the steady state may be expressed as:

$$\text{(14)} \qquad \begin{aligned} \hat{h} &= \left[\frac{w\left(\hat{\beta}\gamma\hat{l} - \hat{h}\left(1-\hat{\beta}(1-\delta)\right)\right)}{e(1-g)\left(1-\hat{\beta}(1-\delta)\right)} \right]^{\frac{1}{1-\gamma}} \\ \tilde{h} &= \left[\frac{w\left(\tilde{\beta}\gamma\tilde{l} - \tilde{h}\left(1-\tilde{\beta}(1-\delta)\right)\right)}{e(1-g)\left(1-\tilde{\beta}(1-\delta)\right)} \right]^{\frac{1}{1-\gamma}} \end{aligned}$$

Equations (14) show that the differences in human capital levels across agents result from the different time preferences as well as to labor supply.

As Appendix A shows, as long as income from assets (interest on bonds and profits of firms) is lower than a critical value, the higher β is (i.e., the more patient the agent is), the more human capital the agent chooses to have. This result resembles that in Naor (2012) and is very intuitive since human capital is one of the investments routes that this agent may pursue, although now the investment requires both time and financial resources. Given that she is more patient, she is willing to invest more in

[8] Also, as in Naor (2012), the same parameters that hold the impatient agent's labor supply below $1-\hat{s}$, ensure a positive labor supply for the patient agent. For the impatient agent to have a labor supply that is lower than the time on his hands, the parameters should satisfy: $-(R-1)b+\tau+e(1-g)s < wh^{\gamma}l$. In other words, interest payments, taxes, and the cost of holding human capital should be lower than income. By implication, the patient agent's labor supply must be positive.

human capital accumulation to generate a larger future income. However, if income from assets (interest on bonds and profits of firms) exceeds this critical value, the results will be the opposite (see Appendix A, Equation (A.8)).

Note that in the steady state the optimal time invested in human-capital accumulation for both agents is simply the amount of time needed to cover the depreciation rate:

$$(15)\qquad \begin{aligned} \hat{s} &= \delta \hat{h}, \\ \tilde{s} &= \delta \tilde{h} \end{aligned}$$

From the first-order-condition for labor supply in Equation (7), it follows that consumption should satisfy:

$$(16)\qquad \begin{aligned} \hat{c} &= \frac{w\hat{h}^{\gamma}\left(1-\hat{l}-\hat{s}\right)}{\varphi}, \\ \tilde{c} &= \frac{w\tilde{h}^{\gamma}\left(1-\tilde{l}-\tilde{s}\right)}{\varphi} \end{aligned}$$

Note that it is not clear whose consumption is higher: Although the patient agent has more human capital, it is not clear (without calibration) who has more leisure. (The impatient agent works more and invests less in human-capital accumulation.)

3. Quantitative Results

This section provides quantitative results on the effects of the exogenous variables, using simulations. The model is calibrated using OECD data presented in subsection 3.1, the specific calibration used presented in subsection 3.2. The simulation results are shown in Subsection 3.3.

3.1 The Data

Table 1 presents the OECD data on the model variables. The table shows that much the population in OECD countries invests in higher education. This investment is costly relative to annual GDP per capita and entails a rather large government subsidy. Moreover, since household debt relative to disposable income varies considerably across countries, it cannot be excluded from the discussion.

Table 1: OECD Data

	Average Level	Minimum Level	Maximum Level	Source
Annual cost of tertiary education as a function of annual GDP per capita	46%	30% (Italy)	57% (United States)	*Education at a Glance* (2009)
Household debt to disposable income	94%	34.7% (Italy)	144.5% (Denmark)	Crook and Hochguertel (2006), Girouard, Kennedy, and Andre (2006)
Subsidy of higher-education cost	18.2%	4.8% (Czech Republic)	31.0% (Australia)	*Education at a Glance* (2009)
Share of taxes paid by 7 lowest deciles	39.9%	30.5% (Japan)	51.3% (Denmark)	Forster and Pearson (2002)
Share of the 25–64 age group that has tertiary education	21.8%	12.9% (Italy)	29.6% (United States)	*Education at a Glance* (2009)

3.2. <u>Calibration</u>

As in Naor (2012), I use standard values for the year-interval parameters:

Production parameters: α is set at 0.3, A is normalized to 1.

Preference parameters: φ is set at 1.5 (Greenwood et al., 1997), $\tilde{\beta}$ is set at 0.97 and $\hat{\beta}$ at 0.9, following Lawrance (1991).

Human capital parameters: δ is set at 0.05 (Haley, 1976; Mincer and Polachek, 1974, for yearly levels). The cost of human-capital accumulation, es, is set at 0.46 of GDP, following the OECD average. Thus, $e = \frac{0.46 * y}{s}$. I set e at 1.501, which, according to the model, is the calibrated value of $e = \frac{0.46 * y}{s}$ when household debt equals 0.066 (0.94% of the impatient agent's net income) and the tax share is 0.399 (These values are the OECD averages of debt/income and tax share, taken from Table 1).

The borrowing-constraint parameter, ϕ, is set at alternative levels of 50%–150% of the impatient agents' income, as in the OECD countries.

The subsidy for human-capital accumulation, g_t, is set at 20%. I also simulate the model for subsidy levels of 5%–40% on the basis of data from the OECD countries.

I use several scenarios of tax shares between the two types of agents. The share of taxes levied on impatient agents ranges from 0 to 1, a much broader range than that reported by the OECD.

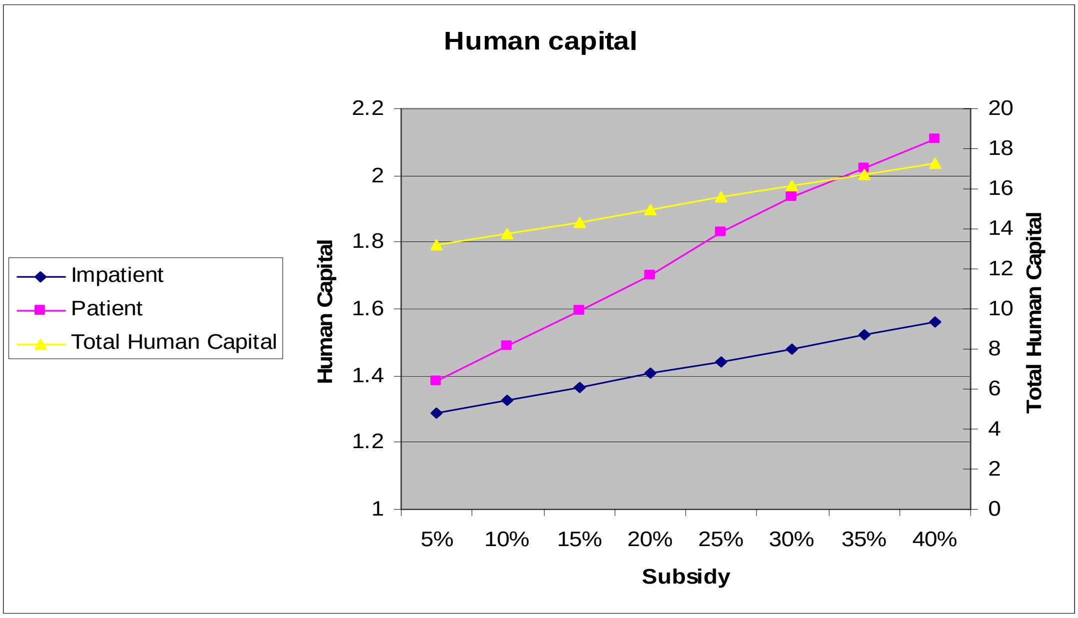
3.3 Steady State Results

First, I simulate the model for the intermediate levels of all parameters. The main characteristics of the steady state in Naor (2012) recur here. The patient agent enjoys more consumption and leisure, invests more in human capital (in both time and resources), and owns more assets than the impatient agent. The patient agent's income from labor exceeds that of the impatient agent. Obviously, this aggravates income inequality because the patient agent owns all the financial assets in the economy.

One of the main differences between this model and that in Naor (2012) is the lower pre-tax, pre-subsidy Gini index. The main cause of this lower level of inequality is the time invested in human-capital accumulation as the current setup requires. In each period, all agents have the same time limit. (Their labor, leisure, and time invested in human-capital accumulation add up to the same constant.) If time were a tradable good, the impatient agent would borrow time and reach the steady state with less time. Thus, in terms of time available, both types of agents have the same constraint. The same lower Gini index may also have been achieved by tightening the borrowing constraint, forcing the agents to borrow less and, in turn, to have less debt in the steady state.[9]

An increase in the subsidy for human-capital accumulation drives both agents to acquire more human capital and, in turn, raises the total level of human capital in the economy. However, since the effect on the patient agent is much stronger, the human-capital gap between the agents widens. The subsidy has a stronger effect on the patient agent's human capital than on that of the impatient agent for two reasons: the patient agent invests more in human capital to begin with, and she is not subject to the borrowing constraint and can therefore put the subsidy to greater use. This result is not unique to this model; it is obtained, for example, by Dynarsky (2002) and Fernandez and Rogerson (1995). These results are illustrated in Figure 3.

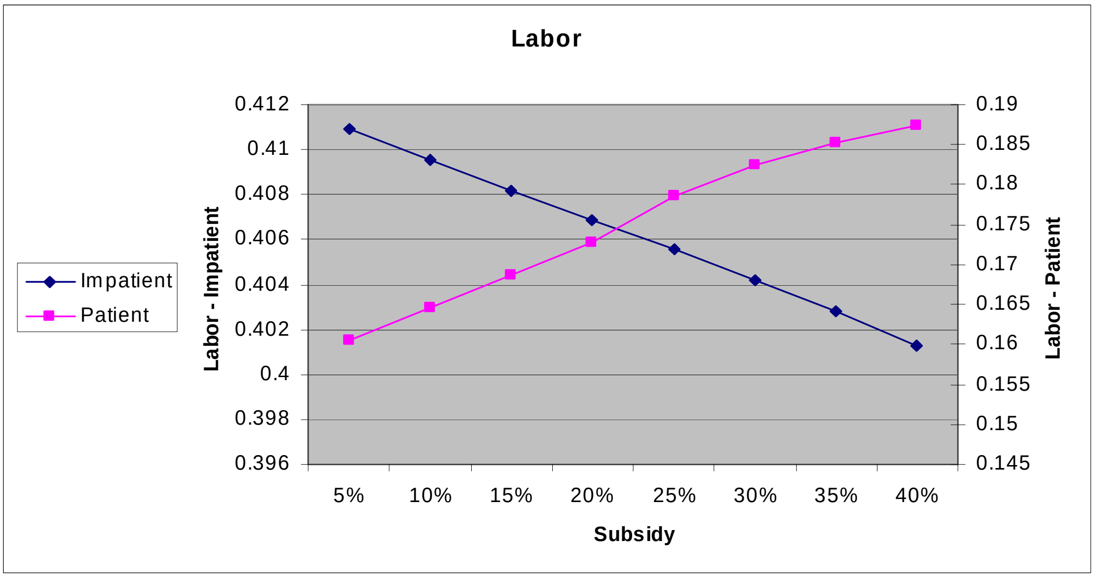
Figure 3: Human Capital as Function of Subsidy for Human Capital



[9] The interest payment on the debt drives the impatient agent to work more and acquire more human capital; this narrows the equality gap. The added income, however, is lower than the interest payment, meaning the Gini index that excludes interest payments is higher.

Moreover, the different effects of the human-capital accumulation subsidy on the two types of agents may be better understood by stressing the opposite effects of the subsidy on their labor supply: As the subsidy increases, patient agents' labor supply rises and that of impatient agents declines (Figure 4). The subsidy reduces the labor supply of the impatient agent and, in turn, his return to human capital, while, for the patient, the subsidy raises her labor supply and thus her return to human capital. The differential effect on labor supply follows from the relative strengths of the two opposite effects.

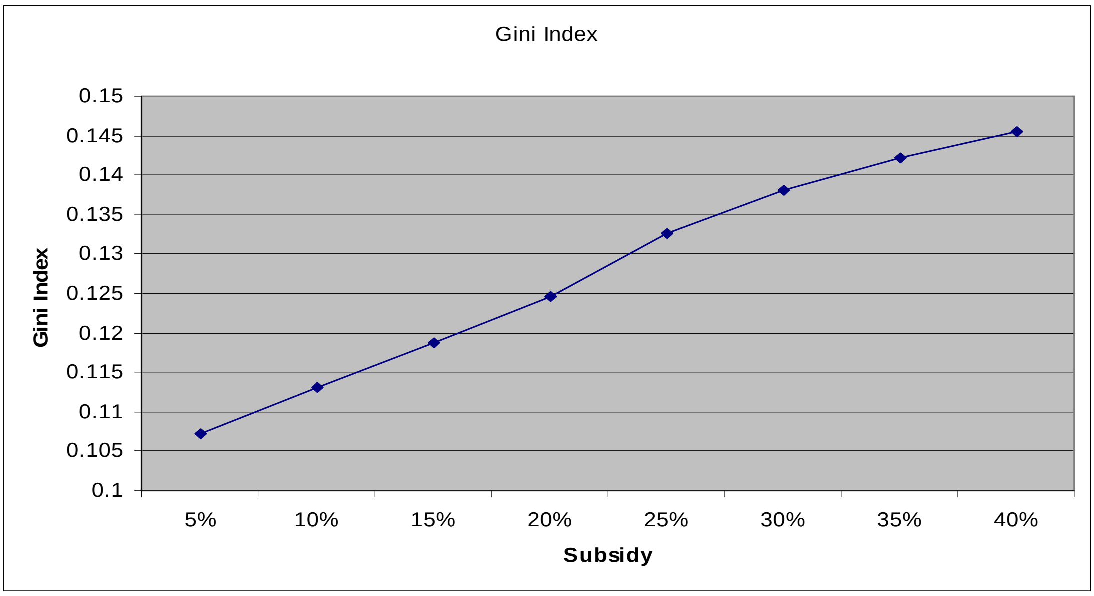
Figure 4: Labor as Function of Subsidy to Human Capital



Human capital and labor are complementary in determining wage: the return to labor supply increases in tandem with human capital. The acquisition of human capital, however, reduces leisure, and to keep leisure from being too low, labor supply may be reduced. Since the patient agent enjoys more leisure to begin with, she chooses to work more. For the impatient, the positive income effect of human capital tends to increase leisure, which is low to begin with. Note that the impatient agent's labor supply declines moderately relative to the increase in the patient agent's labor supply, so that total labor supply rises.

As the subsidy for human capital increases (for a given tax share, i.e., when the subsidy rises both agents pay more taxes, keeping the ratio of taxes constant), the gross-income Gini index also rises, meaning that the economy becomes less equal (Figure 5). This happens because as the subsidy to human capital rises, the gap between the human-capital levels of the two types of agents widens and because patient agents work more while impatient agents work less. Thus, the patient agent enjoys a greater increase in gross income than the impatient agent does. Moreover, total human capital and total labor supply raise the profits of the firm, from which only the patient agent benefits.

Figure 5: Gross-Income Gini Index as Function of Subsidy to Human Capital



The results are similar for every level of borrowing constraint and tax division between the two types of agents.

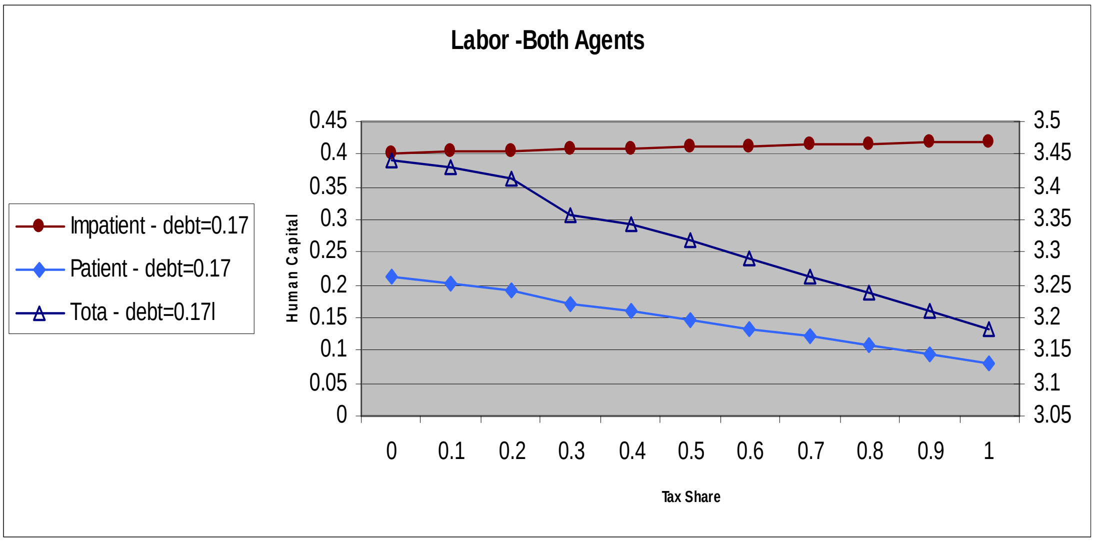
Obviously, since a subsidy requires taxes, subsidizing human-capital accumulation may have a strong effect on the Gini index of net income.[10] However, taxation also affects all agents' decisions (consumption, investment in human capital, and labor) and, in turn, the Gini index of gross income as well as that of net income.

[10] Since the subsidy affects both types of agents differently, any taxation mechanism that does not internalize it will increase the Gini index of net income.

I simulate the model for several tax-share values within the interval [0,1], with 0 meaning that patient agents remit all taxes needed to finance the subsidy to human-capital accumulation and other government expenditures; and 1 meaning that all taxes are collected from impatient agents. Thus, when the tax share goes up impatient agents pay more taxes and patient agents pay less.

As the tax share rises, the impatient agent experiences a negative income effect and thus chooses to work more. Since labor supply and human capital are complementary production factors (for each worker), this rise in labor supply increases the return to human capital. Therefore, the impatient agent also chooses to invest more in human-capital accumulation. The patient agent, in turn, enjoys a positive income effect that leads her to work less and own less human capital. Figure 6 shows the labor supply of both types of agents.

Figure 6: Labor Supply as Function of Tax Share[11]

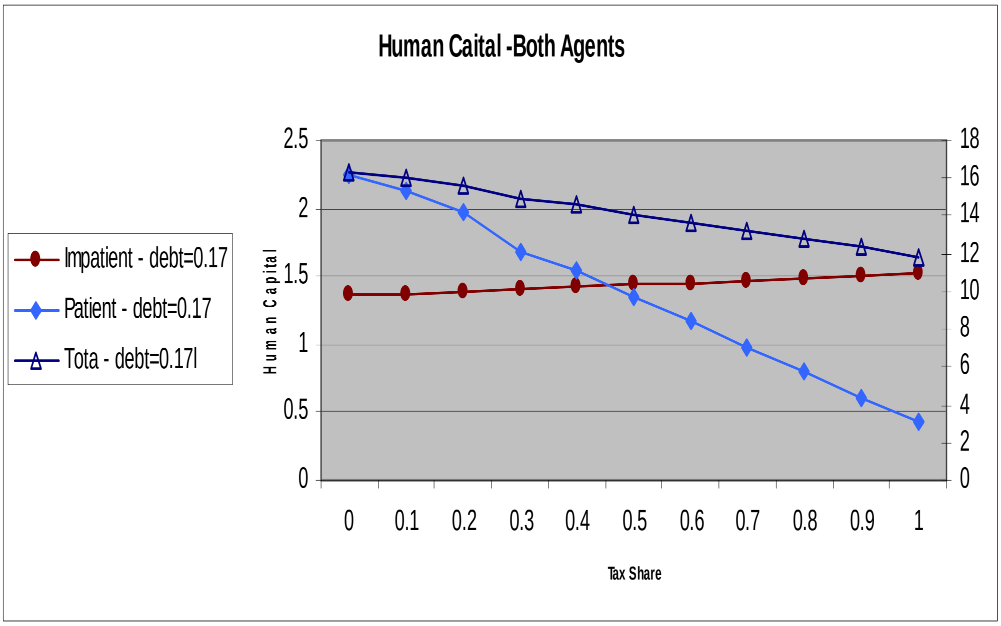


Note that the contraction of the patient agent's labor supply is much stronger than the increase in the impatient agent's labor supply. This is so because the tax-share effect is not symmetric between the types of agents for two reasons. First, there are many

[11] The scale of the labor supply for both types of agents appears on the left-hand side; that of the aggregate level appears on the right-hand side.

more impatient agents than patient ones in the population; therefore, an increase in the tax share has a stronger opposite-direction income effect on the patient agent than on the impatient agent. (The same amount of taxes is divided between more agents.) Second, since the impatient agent is a debtor, the combination of tax payments and interest payments implies that he works a lot. Thus, he has little ability to increase his labor supply even more.

Figure 7: Human Capital as Function of Tax Share[12]

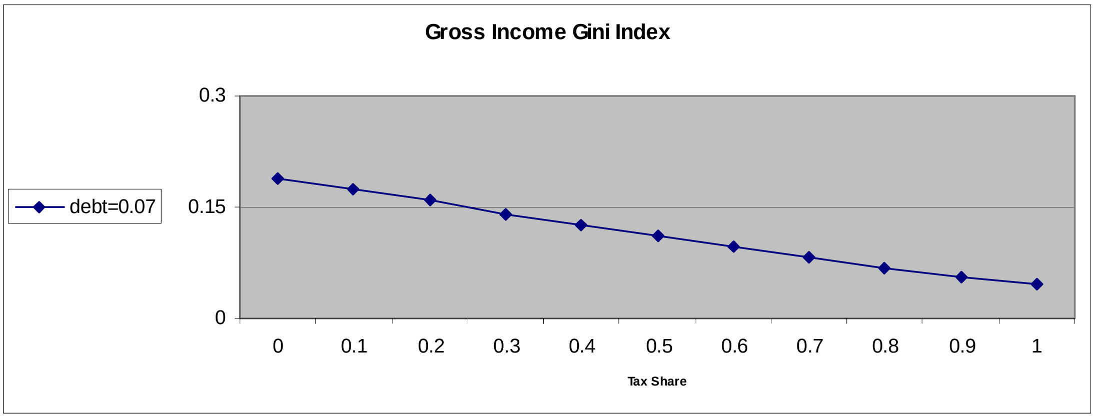


The contraction of the patient agent's human capital is stronger than the increase in the impatient agent's human capital (Figure 7). This follows from the stronger reduction in the patient agent's labor supply than the increase in the impatient agent's labor supply (Figure 6). Therefore, when the tax share is high enough, the impatient agent will have more human capital than the patient agent (Figure 7). Notably, since the tax is a lump-sum tax, there is no substitution effect. (For a discussion of distortionary taxes, see Appendix B.)

[12] The scale of the human capital for both types of agents appears on the left hand side; the scale of the aggregate level appears on the right hand side.

By implication, total human capital and total labor supply decline as taxes shift toward the impatient agent, leading to lower total production, lower wage per effective labor, and lower profits of the firm. Obviously, this pushes the gross Gini index down, indicative of a more equal economy (Figure 8).

Figure 8: Gini Index as Function of Tax Share



Another element that plays an important role in the model is the borrowing constraint. Although ostensibly a less restrictive borrowing constraint should improve the impatient agent's condition, the opposite happens in the steady state: Since the impatient agent always succumbs to the borrowing constraint, a less restrictive constraint allows him to amass more debt and, in turn, to pay more interest. Thus, in the steady state, a less restrictive borrowing constraint affects the economy much as the tax share does—it has a positive income effect on the patient agent and a negative effect on impatient one.

As for the Gini index, it may seem counter-intuitive; harming the impatient agent (by allowing him to hold a lower level of debt) actually leads to a more equal economy before interest payments are taken into account. However, the new Gini index, which takes into consideration the interest on debt repayment, shows that the higher is the debt the less equal the economy is.

4. Conclusion

In this paper I added two critical ingredients to the model presented in Naor (2012). First, to accumulate human capital, one needs to invest not only resources (as in Naor, 2012) but also time. Second, changes in the cost of human capital originate in a government subsidy rather than technology.

Several features of the model presented in Naor (2012) recur here: the patient agent works less than the impatient agent, has more human capital, enjoys more consumption, and owns all the assets and firms in the economy. The main new theoretical prediction is that a higher subsidy to human-capital accumulation leads to more income inequality. This is so because patient agents react by investing more in human-capital accumulation and working more to enjoy the fruits of their investment, while impatient agents, though investing more in human capital, choose to work less. The net effect of these behaviors on total production and firms' profits is positive.

These results are also supported by data showing that the subsidy to tertiary education is positively correlated with the Gini index, the share of tertiary-education graduates in the 25–64 age group, and employment.

Since time cannot be traded and is equally allocated across agents, adding time to the production function of human capital leads to more equality. Similarly, borrowing constraints limit steady-state debt and thus mitigate inequality.

Financing the subsidy to human-capital accumulation via taxes and the division of tax shares between the types of agents play important roles in income inequality. Shifting taxes from patient (rich) agents to impatient (poor) ones has a positive income effect on the patient agent, encouraging her to work less and accumulate less human capital. Higher taxes on impatient agents have an opposite income effect on them, leading to increase in labor supply and investment in human-capital accumulation. Thus, the patient agent's income drops and the impatient agent's income rises. Consequently, gross income inequality declines (although net inequality rises).

Another important element in the model is the borrowing constraint. Theoretically, impatient agents borrow as much as they can and reach their borrowing limit in the steady state. Thus, although a looser borrowing constraint ostensibly improves their condition, in the steady state the opposite actually occurs: they amass more debt and have to make higher interest payments. Therefore, relaxing the borrowing constraint has a negative effect on impatient agents' income in the steady state. Impatient agents choose to work more and accumulate more human capital in order to meet their interest payments. Concurrently, however, patient agents own more assets and thus choose to work less and accumulate less human capital. This mechanism mitigates steady-state inequality.

Appendix A: Partial Effects of the Model Parameters

Appendix A presents the analytical partial effect of the discount factor, *β*, on optimal human capital, *h*, and labor, *l*. As noted in the text, Equations (13) and (14) are simultaneous. Thus, to show the effect of the discount factor on human capital I rearrange Equation (14) as: $\Gamma(h,l,\beta)=0$ and Equation (13) as $\Omega(l,h)=0$.

$$(A.1) \qquad \Gamma(h,l,\beta) = h^{1-\gamma} - \frac{w(\beta\gamma l - h(1-\beta(1-\delta)))}{e(1-g)(1-\beta(1-\delta))} = 0$$

$$\Omega(l,h) = l - \frac{1-\delta h}{1+\varphi} - \frac{e(1-g)\delta h - (R-1)b + \tau}{wh^{\gamma}\left(1+\frac{1}{\varphi}\right)} = 0$$

$$\rightarrow \; l = \omega(h) = \frac{1-\delta h}{1+\varphi} + \frac{e(1-g)\delta h - (R-1)b + \tau}{wh^{\gamma}\left(1+\frac{1}{\varphi}\right)}$$

$$\Rightarrow \; \Gamma(h,\omega(h),\beta) = h^{1-\gamma} - \frac{w(\beta\gamma(\omega(h)) - h(1-\beta(1-\delta)))}{e(1-g)(1-\beta(1-\delta))}$$

$$(A.2) \qquad \frac{\partial h}{\partial \beta} = -\frac{\partial\Gamma/\partial\beta}{\partial\Gamma/\partial h}$$

$$\frac{\partial \Gamma}{\partial h} = \left.\frac{\partial \Gamma}{\partial h}\right|_{l} + \frac{\partial \Gamma}{\partial l}\frac{\partial l}{\partial h}$$

$$\frac{\partial l}{\partial h} = -\frac{\partial\Omega/\partial h}{\partial\Omega/\partial l}.$$

$$(A.3) \qquad \frac{\partial l}{\partial h} = \frac{\delta}{1+\varphi} - \frac{e(1-g)\delta(1-\gamma) + \frac{\gamma((R-1)b-\tau)}{h}}{w\left(1+\frac{1}{\varphi}\right)h^{\gamma}} \Rightarrow$$

$$(A.4) \qquad \begin{cases} if \quad (R-1)b > \dfrac{\delta w h^{\gamma+1}}{\gamma\varphi} + \tau - \dfrac{e(1-g)\delta(1-\gamma)h}{\gamma} & \dfrac{\partial h}{\partial l} < 0 \\ if \quad (R-1)b < \dfrac{\delta w h^{\gamma+1}}{\gamma\varphi} + \tau - \dfrac{e(1-g)\delta(1-\gamma)h}{\gamma} & \dfrac{\partial h}{\partial l} > 0 \end{cases}$$

Equation (A.4) shows that at a high level of assets, *b*, with respect to the marginal return to human capital and tax, human capital drops with labor; while at a low level of assets (debt), human capital rises with labor.

$$(A.5)\qquad \frac{\partial\Gamma}{\partial h} = (1-\gamma)h^{-\gamma} + \frac{w}{e(1-g)} - \left(\frac{\beta\gamma}{e(1-g)(1-\beta(1-\delta))}\right)\left(\frac{e(1-g)\delta(1-\gamma)+\gamma((R-1)b-\tau)}{\left(1+\frac{1}{\varphi}h^{\gamma}\right)}\right)$$

$$(A.6)\qquad \frac{\partial\Gamma}{\partial \beta} = \frac{(1-\beta(1-\delta))\left(wh\beta(1-\delta)^2\right)+wl\gamma}{e(1-g)(1-\beta(1-\delta))^2} = \frac{\left(wh\beta(1-\delta)^2\right)+wl\gamma}{e(1-g)(1-\beta(1-\delta))} \Rightarrow > 0$$

$$(A.7)\qquad \frac{\partial h}{\partial \beta} = -\frac{\dfrac{\left(wh\beta(1-\delta)^2\right)+wl\gamma}{e(1-g)(1-\beta(1-\delta))}}{(1-\gamma)h^{-\gamma} + \dfrac{w}{e(1-g)} - \left(\dfrac{\beta\gamma}{e(1-g)(1-\beta(1-\delta))}\right)\left(\dfrac{e(1-g)\delta(1-\gamma)+\gamma((R-1)b-\tau)}{\left(1+\frac{1}{\varphi}\right)h^{\gamma}}\right)} \Rightarrow$$

$$(A.8)\qquad \begin{cases} if\quad (R-1)b > h^{-\gamma}\left(\dfrac{(1-\gamma)e(1-g)}{\beta\gamma^2}\left((1-\beta(1-\delta)\left(1+\frac{1}{\varphi}\right)\right)-\delta\beta\gamma\right)+\tau+\dfrac{w(1-\beta(1-\delta))\left(1+\frac{1}{\varphi}\right)}{b\gamma^2} & \dfrac{\partial h}{\partial\beta}<0 \\ if\quad (R-1)b < h^{-\gamma}\left(\dfrac{(1-\gamma)e(1-g)}{\beta\gamma^2}\left((1-\beta(1-\delta)\left(1+\frac{1}{\varphi}\right)\right)-\delta\beta\gamma\right)+\tau+\dfrac{w(1-\beta(1-\delta))\left(1+\frac{1}{\varphi}\right)}{b\gamma^2} & \dfrac{\partial h}{\partial\beta}>0 \end{cases}$$

Equation (A.8) shows that at a high level of assets, *b*, a higher discount factor reduces human capital, while at a low level of assets (high debt), the discount factor has a positive effect on human capital. Note that the critical level differs from the one in Equation (A.4) above.

Appendix B: The Model with a Labor Income Tax

To show how a lump-sum tax affects the results of the model, I solve a simplified model with two alternative tax systems: a lump-sum tax and a labor-income tax. In this simplified model, there is only one agent and she makes the same decisions as in the model presented in the paper.

In the presence of a labor-income tax, the agent maximizes:

$$\text{(B. 1)} \quad \begin{aligned} &\max \sum_{t=0}^{\infty} \beta^t \left(\ln(c_t) + \varphi \ln(1 - l_t - s_t) + \ln(G_t)\right) \\ &s.t: c_t = (1-\tau) w_t l_t h_t^{\gamma} - e(1-g) s_t + \frac{1}{m}\pi_t \qquad \forall t = 0,1,2... \end{aligned},$$

where τ is the labor-income tax rate and the rest of the notation is as in the text: c_t is consumption, l_t is labor, s_t is investment in human capital so that human capital, h_t, follows $h_{t+1} = (1-\delta)h_t + s_t$. δ is the depreciation rate of human capital, φ is the relative importance of leisure as against consumption and β is discount rate. e is the cost of human-capital accumulation, g is the subsidy to human capital, m is population size, and π is firms' profits. G is government expenditure on uses other than the human-capital accumulation subsidy and its value is fixed, and τ is income tax. A higher subsidy to human capital, e, leads to higher income tax, τ.

The government maintains a balanced budget, so that:

$$m \cdot e \cdot g \cdot s_t + G_t = m \cdot \tau \cdot w_t \cdot l_t \cdot h_t^{\gamma}.$$

The first-order conditions are:

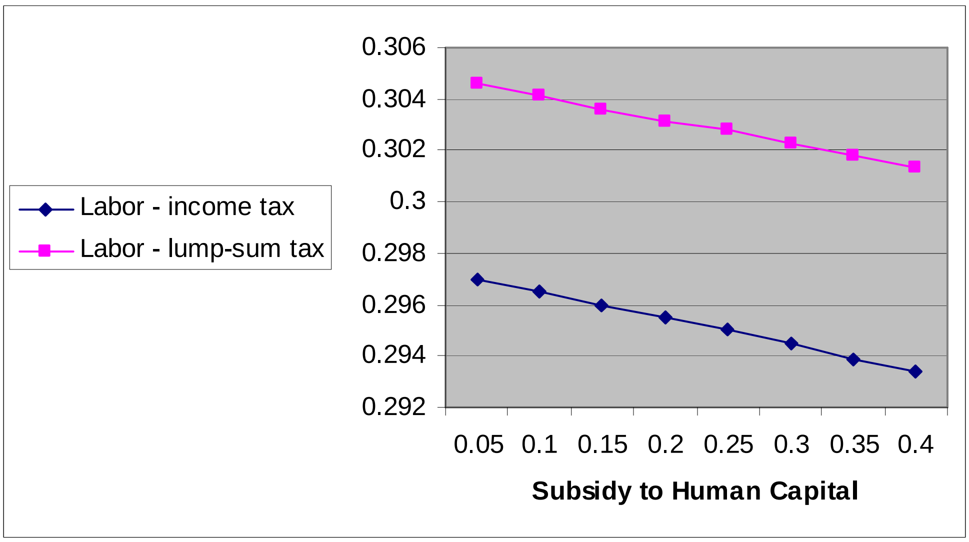
$$\text{(B. 2)} \quad \begin{aligned} &-\frac{1}{c_t} + \frac{\beta R}{c_{t+1}} = 0, \\ &-\frac{e(1-g)}{c_t} - \frac{\varphi}{1-l_t-s_t} + \beta\left(\frac{\gamma(1-\tau)w_t l_t h_t^{\gamma-1} - e(1-g)(1-\delta)}{c_{t+1}} + \frac{\varphi(1-\delta)}{1-l_{t+1}-s_{t+1}}\right) = 0, \\ &\frac{(1-\tau)w_t h_t^{\gamma}}{c_t} - \frac{1}{1-l_t-s_t} = 0. \end{aligned}$$

I simulated the model for the two different tax policies using the calibration elaborated in Section 3.2. The income-tax simulations are performed as follows. For each subsidy level, there is an interactive calculation until the government-budget constraint is satisfied: starting from an initial tax rate, the equilibrium is computed and the government budget balance is calculated. If, say, tax revenue is too low, the tax rate is increased successively until the government budget is balanced.

The results, shown in Figures B.1 and B.2, indicate that an increase in the subsidy to human capital reduces labor and raises human capital under both tax systems. Hence, the results for both tax systems are similar.

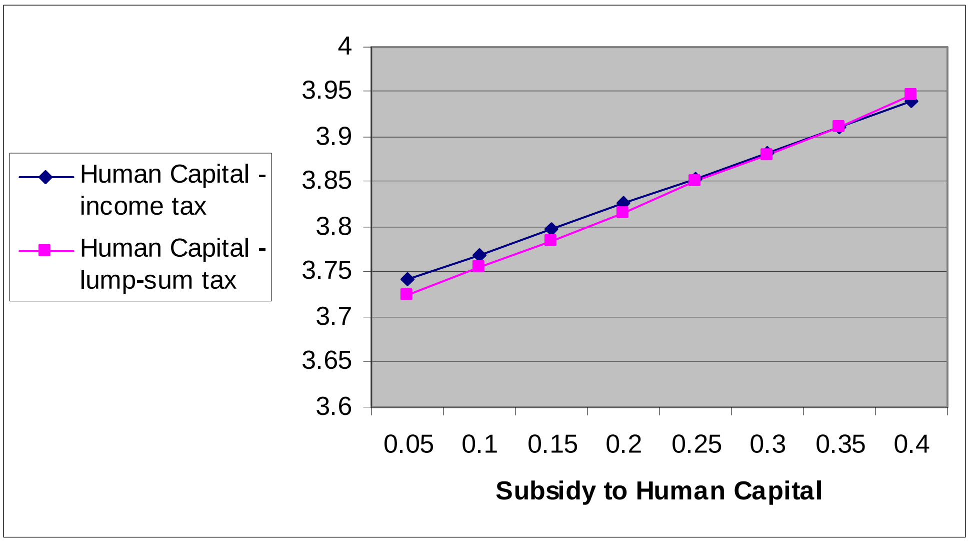
However, labor is lower with an income tax than with a lump-sum tax. This is so because an income tax directly reduces the (net) return to human capital, which, in turn, reduces the return to labor. Note that labor supply reacts to the subsidy to human capital somewhat more strongly under an income-tax regime than under a lump-sum tax (elasticities of –3.36% and –3.28%, respectively).

Figure B.1: Labor



Furthermore, human capital reacts more strongly to the subsidy to human capital under a lump-sum tax regime than under an income tax. Hence, for low levels of subsidy to human capital, an income tax delivers higher human capital than a lump-sum tax, while higher levels of subsidy lead to lower human capital under an income-tax regime. This result matches the elasticity of labor to the subsidy under both tax policies.

Figure B.2: Human Capital



Note that although an income-tax regime yields higher human capital under most levels of subsidy to human capital, it lowers the pre-tax income. Moreover, tax payments are higher under an income tax (since more human capital is being subsidized). Thus, an income-tax regime leads to much lower net income.

References

1. Becker G. S. (1975), *Human Capital*, The University of Chicago Press.
2. Ben-Porath, Y. (1967), "The Production of Human Capital and the Life Cycle of Earnings", *The Journal of Political Economy*, 75(4) 352-365.
3. Bils, M. and Klenow P. J. (2000), "Does Schooling Cause Growth?", *American Economic Review*, 90(5), 1160-1183.
4. Crook J. and Hochguertel S. (2006), "Household Debt and Credit Constraints: Comparative Micro Evidence from Four OECD Countries", unpublished manuscript.
5. Dynarski S. (2002), "The Behavioral and Distributional Implication of Aid to College", *American Economic Review*, 92(2), 279-285.
6. Education at a Glance (2009), OECD publications.
7. Fernandez R. and Rogerson R. (1995), "On the Political Economy of Education Subsidy", *Review of Economic Studies*, 62(2), 249-262.
8. Forster M. and Pearson M. (2002), "Income Distribution and Poverty in the OECD Area: Trends and Driving Forces", *OECD Economic Studies*, 34.
9. Friedman M. (1955), "The Role of Government in Education", *Solo Robert A., ed., Economics and Public Interest.* New Brunswick: Rutgers University Press.
10. Galor O. and Tsiddon D. (1997), "The distribution of Human Capital and Economic Growth", *Journal of Economic Growth*, 2(1), 93-124.
11. Galor O. and Zeira J. (1993), "Income Distribution and Macroeconomics", *Review of Economic Studies*, 60(1), 35-52.

12. Girouard N., Kennedy M. and Andre C. (2006), "Has the Rise in Debt Made Household More Vulnerable?", *OECD Economic Department Working Paper*, No. 535, OECD Publishing.
13. Greenwood J., Hercowitz Z. and Krusell P. (1997), "Long-Run Implications of Investment-Specific Technological Change", *American Economic Review*, 87(3), 342-362.
14. Haley W. J. (1976), "Estimation of the Earning Profile from Optimal Human Capital Accumulation", *Econometrica*, 44(6), 1223-1238.
15. Heckman J. J., Lochner L. and Taber C. (1999), "Human Capital Formation and General Equilibrium Treatment Effects: A Study of Tax and Tuition Policy", *Fiscal Studies*, 20(1), 25-40.
16. Human Development Report, (2009), UNDP.
17. Iacoviello M. (2005), "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle", *American Economic Review*, 95(3), 739-764.
18. Lawrance E. C. (1991), "Poverty and the Level of Time preference: Evidence from Panel Data", *The Journal of Political Economy*, 99(1) 54-77.
19. Mincer J. (1958), "Investment in Human Capital and Personal Income Distribution", *The Journal of Political Economy*, 66(4), 281-302.
20. Mincer J. and Polachek S. (1974), "Family Investments in Human Capital: Earnings of Women", *The Journal of Political Economy*, 82(2), S76-S108.
21. Naor Z. (2012), "Heterogeneous Discount Factor, Human Capital Accumulation and Inequality", unpublished manuscript.
22. Trostel P. A. (1996), "Should Education Be Subsidized?", *Public Finance Review*, 24(3), 3-24.