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Monetary Studies

Normality, Modal Risk Level, and Exchange-Rate Jumps

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Yoel Hecht* and Helena Pompushko**

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* Research Unit of the Banking Supervision Department, Bank of Israel, yhecht@boi.gov.il.

** Monetary Department, Bank of Israel, helenap@boi.gov.il

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Abstract

This article presents three indexes that may be used to examine the expected exchange rate as reflected in trading in exchange-rate options. With these indexes one may examine, on a daily basis, whether the expectations of exchange-rate change were determined in a normal-distribution environment (hereinafter: the "N-Index"), the modal risk level in the forex market (hereinafter: the "R-Index"), and the expected direction and intensity of exchange-rate change in the event of an exchange-rate jump (hereinafter: the "J-Index").

We applied the indexes to daily trading in NIS/dollar exchange-rate options on the Tel Aviv Stock Exchange. By analyzing the indexes for the October 2002–June 2004 period, we found that even though the NIS appreciated perceptibly against the dollar (about 10 percent in the first half of 2003), the Israeli public continued to associate the exchange-rate risk with depreciation: When the N-Index reflected an abnormal market environment and the R-Index reflected a high modal risk level, the J-Index reflected expectations of an exchange-rate jump only in the direction of depreciation.

One of the possible reasons for the decrease in the forex sector's contribution to financing activity earnings in the Israeli banking system in 2003 (Supervisor of Banks, 2004) may have been the rather severe misalignment between the expected behavior of the exchange rate and its de facto behavior.

Key words: exchange rate, options, stochastic process, bi-lognormal distribution, expectations, normality index, modal risk level index, exchange-rate jump index

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A. Introduction

The information implicit in the prices of sophisticated financial assets, especially options, is of crucial interest to central banks as an input in the shaping of monetary policy and to commercial banks in the management of foreign-exchange exposures. This information reflects the market's expectations about the future price of the underlying asset upon the expiration of the options, since the income to be obtained upon expiration depends on the price of the underlying asset at the time. Thus, option prices depend on market expectations about the price of the underlying asset, the standard deviation (Std) of it, the possibility of an abrupt change ("jump") in the price of the underlying asset, and additional parameters that are distribution indicative. For example, options on the NIS/dollar exchange rate at various exercise prices may be indicative not only of the average expected rate and its expected volatility but also of the probabilities of various exchange-rate changes (Hecht and Stein, 2004).

The distribution, as observed by the markets, is clearly reflected in option prices at various exercise prices. This is because option prices are highly sensitive to expected developments in financial markets. The distribution of the future exchange rate depends on the stochastic process that is typical of the exchange rate. Various models that have attempted to monitor the stochastic process statistically have undergone an evolutionary process, from the Random Walk to the mixed-diffusion-jump process with conditional heteroscedasticity. Below we describe the models in the order of their evolution.

1. Random Walk

The basic process used to describe exchange-rate behavior statistically is the Random Walk, the continuous formulation of which is Brownian motion (Ross, 1996). Krugman (1979) used this process to analyze an exchange-rate management regime that relies partly on a target zone. Black and Scholes (1973) invoked it for the pricing of options generally and Garman and Kohlhagen (1983) employed it for the pricing of exchange-rate options specifically.

One of the premises of the basic process is that exchange-rate changes are distributed normally. It was found, however, that this premise does not always obtain and the distribution of changes in the prices of financial assets at large, and of those in forex specifically, are better described by distribution with "fat tails" than by normal distribution (Boothe and Glassman [1986]). Such findings led to studies that proposed alternative exchange-rate distributions and more refined processes than the Random Walk.

2. Mixed-Diffusion-Jump

The mixed-diffusion-jump, a combination of pure diffusion and discontinuous jumps, is a more advanced development of Brownian motion. The process is defined by five parameters: the expectation and variance of the diffusion process, the frequency of the jumps, and the expectation and diffusion of the jumps (Booth and Akgiray [1988]).

Booth and Akgiray (1988) presented this process and showed that it is superior to a mix of normal distributions in a model of exchange-rate changes in the pound sterling, the Swiss franc, and the East German mark against the dollar is concerned. Booth and Akgiray performed an empirical examination of exchange rates between October 1976 and September 1985, when a "dirty" float regime was in effect. They noted that monetary policy does have an effect on the formulation of exchange rates but that exchange rates do not necessarily respond to this effect in the same way. Their findings are consistent with the idea that the stochastic exchange-rate process may change over time. In their estimation, markets respond to information in consideration of policy targets such as money supply and interest rates, and that structural changes may be related to inflation.

3. Autoregressive Conditional Heteroscedasticity

Autoregressive conditional heteroscedasticity is a process that describes the development of exchange rate variance. Hecht (2000), basing himself on the Random Walk model, compared several processes that describe the distribution of NIS/dollar exchange-rate variance. The models examined were ARCH, GARCH, TARCH, and EGARCH on daily exchange rates between December 1991 and June 1999. The findings show that the model best suited to describing the development of NIS/dollar exchange-rate variance is TARCH.

Johnston and Scott (2000) examined the extent to which the GARCH model contributed to our understanding of the stochastic exchange-rate process and asked whether more advanced GARCH processes were in fact promising. Their findings show that the GARCH process is not promising and that there is a better formulation that uses a standardization of the data by means of expectation and variance.

4. *Mean Reversion, Conditional Heteroscedasticity, and Jump*

Jiang (1998) presented an integration of the diffusion-jump process with autoregressive conditional heteroscedasticity and added an element of mean reversion. Arguing that there is some difficulty in estimating the model, he presented an estimation of a parametric model formed from observed data by means of indirect induction on the basis of simulations.

His results indicate that jumps are an important element in the exchange-rate dynamic even when conditional heteroscedasticity (ARCH) and mean reversion are taken into account. Models that take conditional heteroscedasticity into account, however, tend to overestimate the frequency of the jumps and underestimate their size.

The general parametric model estimated by Jiang follows:

(5) $dS_t / S_t = (\alpha_t(\beta) - \lambda\mu_0)dt + \sigma_t(\beta)dW_t + (Y_t(\beta) - 1)dq_t(\lambda)$

where:

S_t = asset price during period t.

α_t = expected yield in the immediate term

σ_t^2 = the immediate-term variance of the asset yield provided that a poissonian jump does not take place

W_t = the standard Gauss-Wiener process or the ordinary Brownian process

$dq_t(\lambda)$ = an iid poissonian process

λ = the parameter of the poissonian process

$Y_t(\beta) - 1$ = random size of the jump when $Y_t \geq 0$

μ_0 = expectations of jump size, i.e., $E[Y_t - 1]$

$dq_t(\lambda), dW_t$, statistically independent

$\theta = (\beta, \mu_0, \lambda) \in \Theta$ = the parametric space that defines the function coefficients, jump size, and the intensity of the poissonian process

Jiang phrased (5) in an alternative way:

$ds_t = \mu_t(\beta)dt + \sigma_t(\beta)dW_t + \ln Y_t(\beta)dq_t(\lambda)$

where:

$s_t = \ln S_t$

$$\mu_t = \alpha_t + \lambda\mu_0 - \tfrac{1}{2}\sigma_t^2$$

The jump diffusion process described by Equation (6) is a Markov process with one discontinuous parameter and one continuous one.

5. Summing Up the Processes

In sum, the evolution of the stochastic processes may be presented in the following way:

(1) Black and Scholes Non-Jump Model (1973):

$$ds_t = \mu dt + \sigma dW_t$$

(2) Merton's Jump Model (1976)

$$ds_t = (\mu - \lambda\mu_0)dt + \sigma dW_t + \ln Y_t dq_t(\lambda)$$

(3) Conditional Heteroscedasticity and Jump (in Jiang 1998)

$$ds_t = (\mu - \lambda\mu_0)dt + (\sigma + \sigma_t s_t)dW_t + \ln Y_t dq_t(\lambda)$$

(4) Mean-Reversion, Conditional Heteroscedasticity, and Jump (in Jiang (1998))

$$ds_t = (\mu - \beta s_t - \lambda\mu_0)dt + (\sigma + \sigma_t s_t)dW_t + \ln Y_t dq_t(\lambda)$$

where,

$$\ln Y_t \approx iid\ N(\mu_0, \nu^2)$$

For simplicity's sake, this study assumes, in accordance with the model in Merton (1976), that the changes in the underlying asset are continuous, random, and accompanied by jumps, e.g., as in Ball and Torous (1983, 1985), Bates (1991) and Beber and Brandt (2004) . Ball and Torous (1983) applied a model of jumps that invokes this premise to forty-seven shares listed on the NYSE over 500 trading days and found that 78 percent of the shares showed price jumps at a significance level of 1 percent.

Ball and Torous (1985) examined and compared two models for options pricing: the Black and Scholes model—assuming that the changes in the underlying asset develop in a continuous Random Walk pattern, meaning that the distribution function is lognormal—and that of Merton (1976), which assumes that the underlying-asset changes behave in a random-walk-jumps manner. Ball and Torous used the Bernoulli version of the jump-diffusion model, in which jump size is not a stochastic variable. In this case, the largest possible number of jumps during the life of the option is one. Accordingly, one may describe the distribution function by mixing two lognormal distributions. Ball and Torous found that the difference between the two models in the shape of the distribution of changes in shares commonly traded on the NYSE is not substantial. They noted, however, that the Merton model is more suitable for other assets such as forex, in which price jumps are rare but large. Due to the paucity of the daily data, our study presumes that the exchange rate jumps only once at the most.

Beber and Brandt (2004) examined the effect of regular and ordinary macroeconomic announcements on the beliefs and preferences of players in the American bond market by comparing short-term distributions before and after the announcements. Using the standard diffusion-jump model, they found that the announcements reduced the implicit uncertainty at the second moment of the distribution irrespective of what the announcements had to say.

Distribution within a bi-lognormal framework is easy to estimate and elicits a variety of parameters that provide information about the possible progression of the underlying asset. Furthermore, as Aguilar and Hördahl (1991) note, this estimation method is flexible: it may elicit a wide spectrum of types of distributions, including the lognormal distribution as a private case. Within the framework of the bi-lognormal distribution, four parameters (moments) that are typical of the expectations may be calculated: expectation, standard deviation, kurtosis, and skewness. Accordingly, the distribution that may be elicited within the bi-lognormal framework is more realistic than a lognormal distribution.

This study is organized in the following way: Sections B and C describe the methodology and the data. These sections, quoted from Hecht and Stein (2004), are added because of their centrality in this article. Section D presents the index that we use to examine normality of the foreign-exchange market. Section E presents an index for modal risk level in the forex market. Section F presents an index of aberrant exchange-rate change (jumps). Section G presents additional findings about the forex market and compares the VIX® index with the implicit standard deviation as calculated using the method in this study. Section H summarizes the study.

B. Methodology

1. General background

The working hypothesis in this study is that the price of an option exercisable at a predetermined time is equal to the discounted value, at risk-free interest, of the sum of possible payments multiplied by the probability of their occurrence. One may write a general pricing formula on the basis of this working hypothesis. The formula for a call option is shown in Equation 1 and that for a put option is given in Equation 2:

$$(1) \qquad c(S,t) = e^{-it} \int_{X}^{\infty} q(S_T)(S_T - X)dS_T$$

$$(2) \qquad p(S,t) = e^{-it} \int_{0}^{X} q(S_T)(X - S_T)dS_T$$

where:

$c(S,t)$ = call option value

$p(S,t)$ = put option value

S_T = expected spot price at time T

X = exercise price of the option

$q(S_T)$ = general density function of S_T

i = domestic interest rate.

Theoretically, any density function, $q(S_T)$, may be adapted to the pricing formula, provided that one may extract from it the parameters that dictate its shape. We accomplish this by aligning the theoretical price with the actual one. We assume that the density function $q(S_T)$ is composed of a mix of two lognormal distributions. Therefore, we may write the pricing formula of the options as follows:

$$(3) \qquad c(S,t) = e^{-it} \int_{X}^{\infty} \left[\theta f(\mu_1, \sigma_1; S_T) + (1-\theta) f(\mu_2, \sigma_2; S_T)\right](S_T - X)dS_T$$

$$(4) \qquad p(S,t) = e^{-it} \int_{0}^{X} \left[\theta f(\mu_1, \sigma_1; S_T) + (1-\theta) f(\mu_2, \sigma_2; S_T)\right](X - S_T)dS_T$$

where:

$\theta \in [0,1]$ = coefficient

μ_1, σ_1 = expectation and standard deviation of normal distribution 1

μ_2, σ_2 = expectation and standard deviation of normal distribution 2

The two-distribution assumption helps us to process the data more usefully than a single lognormal distribution assumption would because the former contains a wider variety of parameters. It allows us to examine, among other things, several indicators that reflect expectations of exchange-rate change, the level of market uncertainty about its expectations, the likelihood of a steep currency depreciation ("jump"), the existence of leptokurtosis in the distribution, and the extent of skewness of the distribution.

On the basis of Equations (3) and (4), this study applies the Garman-Kohlhagen (1983) formula (hereinafter: G&K) to the pricing of exchange-rate options. G&K adapted the Black and Scholes formula to the forex market and retained the premise that the expected exchange-rate changes are normally distributed. Accordingly, the values of a call option and a put option are dictated by a combination of two normal distributions and the weight of each is determined as follows:

(5) $$c(S,t) = e^{-it}\left[\theta\left[Se^{\mu_1 t}N(d_1) - XN(d_2)\right] + (1-\theta)\left[Se^{\mu_2 t}N(d_3) - XN(d_4)\right]\right]$$

(6) $$p(S,t) = e^{-it}\left[\theta\left[Se^{\mu_1 t}N(-d_2) - XN(-d_1)\right] + (1-\theta)\left[Se^{\mu_2 t}N(-d_4) - XN(-d_3)\right]\right]$$

where:

$$d_1 = \frac{\ln(X/S) + (\mu_1 + \frac{1}{2}\sigma_1^2)t}{\sigma_1\sqrt{t}}$$

$$d_2 = d_1 - \sigma_1\sqrt{t}$$

$$d_3 = \frac{\ln(X/S) + (\mu_2 + \frac{1}{2}\sigma_2^2)t}{\sigma_2\sqrt{t}}$$

$$d_4 = d_3 - \sigma_2\sqrt{t}$$

$c(S_i,t)$ = value of call option

$p(S_i,t)$ = value of put option

N(d) = cumulative distribution of d by standard normal distribution

i = domestic interest rate

$\theta \in [0,1]$ = coefficient

μ_1, σ_1 = expectation and standard deviation of normal distribution 1

μ_2, σ_2 = expectation and standard deviation of normal distribution 2

X = exercise price of the option

S = spot price

The following formula uses θ to weight two G&K formulas, each of which contains one expectation and standard deviation that dictate the shape of its distribution. According to this model, both formulas together elicit one binormal distribution that is described by five parameters—two expectations (μ_1, μ_2), two standard deviations (σ_1, σ_2), and one weight (θ).

On the basis of these variables, one may calculate the expectation of the entire distribution of exchange-rate changes, μ_e, by calculating a weighted mean according to the θ of both estimated expectations:

(7) $$\mu_e = \theta\mu_1 + (1-\theta)\mu_2$$

Similarly, one may also, theoretically, calculate the standard deviation of the exchange-rate changes, σ_e, by producing a weighted average according to the θ of both estimated standard deviations and the covariance:

(8) $$\sigma^2{}_e = \theta^2\sigma^2{}_1 + (1-\theta)^2\sigma^2{}_2 + 2\theta(1-\theta)\sigma^2{}_{1,2}$$

The meaning of the two expectations and standard deviations may be the description of two different states. Say, for example, that only one change in the market is expected within a month, ceteris paribus, a change in the domestic interest rate, which affects the exchange rate. Let us also assume that no one knows whether the change will or will not take place. Expectation and Standard Deviation 1 (State 1) describe a change in the interest rate; Expectation and Standard Deviation 2 (State 2) describe no change in the interest rate. According to the model, each state is assigned a weight that describes the likelihood of its falling into one of the two possibilities.

One may generalize the example for a more complex case by choosing to describe an assortment of expected changes that are aggregated into two separate possibilities. For example, one may augment the example of expectations of interest-rate change by adding expectations of a reform in taxation of the Israeli capital market. This example, broader than the previous one, elicits four states.[2] One may, however, describe the four states by means of two groups that are differentiated by their effect on exchange-rate development; each group has one expectation and one standard deviation that are appropriate for all natural states in the group. Reality is obviously more complex and the number of possible states is infinite. Therefore, the meaning of the two expectations and standard deviations should be expanded to represent two groups into which many states are aggregated.

The table below gives a condensed presentation of the variables in the model.

[2] The states are: an interest rate increase with no change in taxation; an interest rate increase with a change in taxation; an interest rate cut with no change in taxation; and an interest rate cut with a change in taxation.

Table 1: Variables in the Model

Variable	Calculation Method	Possible range*	Explanation
μ_1	Estimation	$\Re$	Average expectations of exchange-rate change in State 1
μ_2	Estimation	$\Re$	Average expectations of exchange-rate change in State 2
σ_1	Estimation	$\sigma_1 > 0$	Std of exchange rate in State 1
σ_2	Estimation	$\sigma_2 > 0$	Std of exchange rate in State 2
θ	Estimation	$0 \le \theta \le 1$	Weighting of states
μ_e	$\theta\mu_1 + (1-\theta)\mu_2$	$\mu_1 \le \mu_e \le \mu_2$	Average expectations of exchange-rate change
$\sigma^2{}_e$	$\sigma^2_e = \theta^2\sigma^2_1 + (1-\theta)^2\sigma^2_2 + 2\theta(1-\theta)\sigma^2_{12}$	$\sigma_1 \le \sigma_e \le \sigma_2$	Estimated implicit market exchange-rate risk
Skewness (coefficient)	$\frac{E[(x-\mu_e)^3]}{\sigma_e^3}$	In normal distribution, value = 0	Extent of distribution asymmetry
Leptokurtosis (degree of excess)	$\frac{E[(x-\mu_e)^4]}{\sigma_e^4} - 3$	In normal distribution, value = 0	Kurtosis

* The estimation method was devised without the constraint that the five parameters must fall within a possible range.

Source: *Economic Quarterly*, 2004:1, p. 44 (Hebrew).

2. Estimation

The method of estimating the double-log-normal distribution is based on the loss function whose aim is to minimize the squared deviations between the prices of options as assessed and priced by investors and the estimates obtained from the pricing equation. The idea behind the method is to give a general description of the options market by means of a limited number of variables (Hecht and Stein, 2004). In practical terms, we are trying to find a set of variables which will minimize the following objective function:

$$(9) \qquad \underset{\{\mu_1,\mu_2,\sigma_1,\sigma_2,\theta\}}{Min} \sum_{i=1}^{N} \left[\frac{\hat{c}_i - c_i}{c_i} \right]^2 + \left[\frac{\hat{p}_i - p_i}{p_i} \right]^2$$

Where:

c_i – the price of a call option.

p_i – the price of a put option.

$\hat{c}_i$ - the price of a call option, estimated by equation 5.

$\hat{p}_i$ - the price of a put option, estimated by equation 6.

The objective function is minimized by using the Gauss-Newton method, which is based on changes in the gradient of the objective function. Use of this method guarantees finding the global minimum by a complete search of the parameter space.

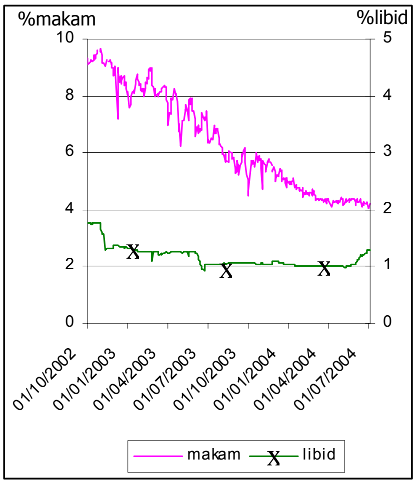
In the estimation there is no a priori constraint requiring the five parameters to be within any possible range. Nevertheless, the values of the parameters obtained via this estimation method are within reasonable bounds. Even so, the implied distribution on the basis of a double log normal assumption sometimes has occasional spikes or a fat tail, which seems to be a drawback in the estimation method. This happens when, statistically, the estimated distribution is characterized as a one-log-normal distribution and not as a double-log-normal distribution. Examining the significance of the weight's parameter (θ) helps to overcome this drawback: If the weight's parameter (θ) is significantly different from zero then the estimated distribution should be based on only a one log-normal distribution.

C. Description of the Data

NIS/dollar options traded on the Tel Aviv Stock Exchange are typically series that are differentiated by dates of maturity. In each series, both call and put options are traded at several constant exercise prices. Since the exercise dates of the series are monthly, at any point in time there are three series of options for the coming three months and an additional series to the end of the next quarter. An additional series of options was issued in 2002 and its expiration date was set at the end of the calendar year. The determining exercise price of the options is the last price published by the Bank of Israel before the exercise date, provided that the exercise date is a trading day on the stock exchange. Options traded on the exchange are "shelf products" that have homogeneous characteristics, unlike commercial bank options and Bank of Israel options that are not typified by series. For the purposes of the model, which estimates the prices of NIS/dollar options of identical maturities but at different exercise prices, we used data pertaining to options traded on the Tel Aviv Stock Exchange.

Figure 1: NIS/Dollar Exchange Rate

Figure 2: Makam (Treasury Bill) Yield and $LIBID, 1 month Rate



To estimate the option prices, we sampled, in addition to the prices that were known at the time of the sampling, the NIS/dollar exchange rate (Figure 1) and the

NIS interest rate for the term of option life—the yield on Treasury bills redeemed at approximately the time of option expiration (Figure 2). Notably, the dollar interest rate (Figure 2) was not sampled here because it is endogenous in the model.

The option prices are sensitive to the price of the underlying asset (the NIS/dollar exchange rate), market interest rates, and players' expectations about the future behavior of the underlying asset. Therefore, a change in one of these factors at any time during the trading day is expected to affect the prices of the options, provided that time for at least one transaction remains. Furthermore, when option prices are sampled on the basis of transactions actually made, e.g., the closing price on the stock exchange, it is not necessarily the case that transactions actually took place in all options that were simultaneously listed at a given point in time. One option, or even several options at different exercise prices, may have been traded earlier in the trading day on the basis of information that had become irrelevant by closing time. An alternative way to sample option prices solves the problem of temporal uniformity: calculation of average bid and ask prices, as shown in the books of the stock exchange at a predetermined point in time, and sampling of the dollar exchange rate and the NIS interest rates at the same point in time.

In view of these characteristics of the data, we sampled, at a specific time in the afternoon of each trading day, the best bid and ask prices for each option, as recorded in the books of the stock exchange, along with the rest of the data—the known NIS/dollar exchange rate and the Treasury bill yield at the time. The number of options (put and call) at different exercise prices for which bids exist is not constant throughout the trading day. On average, there are twenty options at different exercise prices—all liquid options that exist in the market.

Most of the trading in options on the stock exchange is concentrated around options whose exercise prices are close to the representative exchange rate and are of short maturity. Therefore, one may obtain sufficiently extensive information by sampling relatively short terms options — up to fifty days. At this maturity, the trading volume is greater and spans a larger number of exercise prices; at the longest maturities, in contrast, only three options at different exercise prices are traded and such trading usually takes place around the representative NIS/dollar exchange rate.

These data gave us a basis for estimating the expected distribution of the NIS/dollar exchange rate, on each trading day, with the help of the bi-lognormal distribution function.

The data included trading days between October 2002 and May 2004—410 days in all—and 9,818 series of options (see Appendix A).

To perform the estimation, we used only observations for which the implicit standard deviation ranged from 2 percent to 20 percent. We treated other observations as aberrant and deleted them. Notably, the deletion hardly affected the nature of the estimated distribution (lognormal or bi-lognormal) but did affect the estimates of the parameters.

There are four interrelated approaches to the distribution of the future exchange rate (Hecht and Stein, 2004):

(1) The actual distribution of the future exchange rate is the only distribution that describes the behavior of the future exchange rate. This distribution cannot be identified.

(2) The subjective distribution of each player in the market is the distribution by which the players price transactions in derivatives. This distribution does not necessarily correspond to that of the actual future exchange rate; it may include subjective elements related to decision-making under risk conditions, as Kahneman and Tversky's Prospect Theory (1979) and Levy and Levy (2002) notes. Furthermore, since various market players observe different "actual" distributions, the distribution observed by the various players is a weighting of these distributions. Because players are strongly influenced by the distributions that they observe, however, these distributions may be of greater interest than the actual distribution.

(3) A distribution based on a theoretical assumption about exchange-rate behavior is a function (or a set of functions) that describes the exchange-rate distribution. The distribution may be derived from a stochastic theoretical process and/or from a theoretical process of market equilibration. This distribution may not describe reality with exactitude but is usually more convenient to use in analyzing the behavior of the exchange rate. Thus, for example, Krugman (1979) assumes a theoretical exchange-rate process and, on this basis, presents a regime of exchange-rate flexibility within a range.

(4) Estimated exchange-rate distribution—average of subjective distributions is the result of empirical examination of the market data by means of various techniques. The result pertains to a specific period in time, a predetermined frequency, specific currencies, etc.

The various approaches are interrelated. The actual distribution of the exchange rate (1) affects the distribution observed by the players (2). However, changes in players' expectations may affect the actual distribution (1). The methods that players use to shape the distributions that they observe (2) are usually an estimated distribution of the exchange rate (4) and a distribution based on a theoretical premise (3).

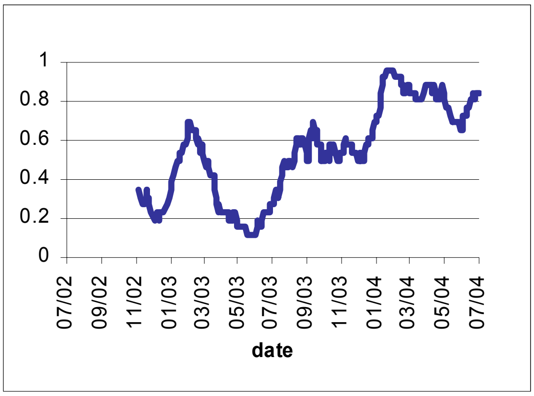
We may improve the estimated distribution by examining these interrelations. One well-known phenomenon is the "volatility smile": the estimated distribution indicates that the farther the exercise price is from the money, the larger the implicit standard deviation of the options is. The "smile" phenomenon is elicited by using the Black and Scholes formula, which bases the exchange-rate distribution on the theoretical assumption (3) of a lognormal distribution. The "smile" effect clashes with the theoretical premise of our model. By implication, the exchange-rate distribution that the players observe (2) is not lognormal. Consequently, various ways of improving the theoretical distribution have been proposed.

A regime of exchange-rate flexibility within a trading band, such as Israel's, is expected to affect the exchange-rate distribution as observed by the market players. The closer the exchange rate is to the boundaries of the range, the stronger the effect on the distribution is expected to be. Campa, Chang, and Refalo (1998) examined the credibility of Brazil's exchange-rate trading band during 1994–1997 on the basis of options and found that credibility increased after 1996. Our study did not examine the effect of the band on exchange-rate distribution. Israel's trading band evidently had little effect on the distribution as long as the NIS/currency-basket exchange rate was relatively far from the bounds of the band. In late 1996, during the first half of 1997, and in the first half of 1998, however, the rate verged on the lower bound. Although the market data during the sampling period of this study showed a negligible likelihood of returning to the vicinity of the lower bound, this may change in the future. In such a state, the bi-lognormal distribution will reflect the distribution of the exchange rate more credibly and the exchange rate will be more indicative of the credibility of Israeli's exchange-rate regime in investors' eyes. When the exchange rate stays within the band but approaches one of its bounds, the bi-lognormal distribution will be indicative both of the probability of breaching the bound in an exchange rate "jump" and the size of the jump as an indication that the exchange-rate policy lacks credibility.

D. The Forex-Market Normality Index ("N-Index")

Options pricing in financial markets is based on the accepted assumption that exchange rate changes distributes normally. The N-index quantifies ex-post the percentage of days, in a trading month, that trading did in fact reflect the normality assumption. Therefore, the N-index allows us to determine the economic environment in which the expectations of exchange-rate change in the forex market took form. The frequency of the index is daily (Figure 3).

Figure 3: Forex-Market Normality Index



The index is composed of three phases:

(1) Examining the daily distribution of exchange-rate changes as to whether it is normal or binormal.

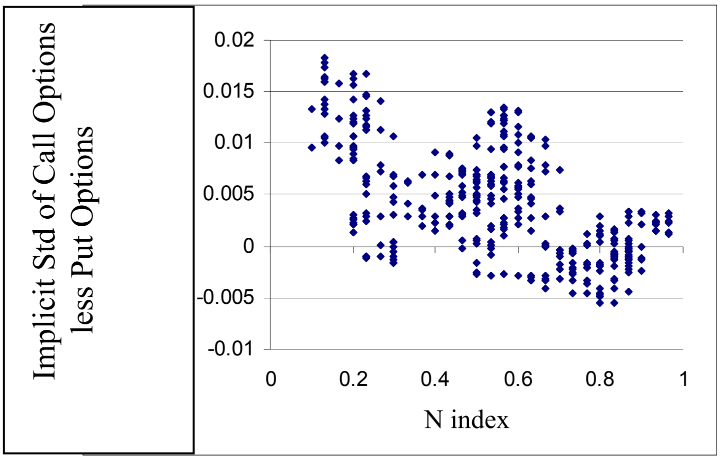
(2) Assigning the value of 1 to days on which the distribution was found to be normal and 0 for days when it was found to be binormal.

(3) Calculating (in percent) how many days in the most recent month were normal.

Phase 1 is performed by means of four tests that allow us to determine whether, at 10 percent statistical significance, θ, $1-\theta$, Std_1, and Std_2 are other than zero (see Appendix B). If one of these is not significantly different from 0, the exchange-rate change is normal. If all are different from 0, the distribution of the exchange-rate change is composed of two normal distributions and that of the exchange rate itself is bi-lognormal. Notably, the statistical test is incomplete because it allows only two possibilities of exchange-rate distribution even though additional possibilities may exist in other time ranges. This is why we call our test of normality an "index" and not a "test." The values that this index may receive range from 0 percent to 100 percent.

Analysis of the index during the recent period shows that it changed perceptibly, climbing from 10 percent normal distribution days in the middle of 2003 to almost 100 percent in early 2004. These changes are consistent with stylized facts that are familiar from forex-market and Nis/dollar exchange-rate developments during that time: an increase in nonresident capital inflows in the first half of 2003 and a halt to the inflow and reversal of the exchange-rate trend in late June (Supervisor of Banks, 2004).

Figure 4: Implicit Std of Call Options less Put Options Versus the N-Index



We examined the N-Index in an additional direction by calculating the implicit standard deviation in call options and put options separately. At times of normal exchange-rate distribution, we expected the separately calculated standard deviations to be equal and, therefore, expected the difference between the calculations to be closer to 0. We posited the difference against the N-Index (Figure 4) and found, as expected, a negative correspondence between the two series. That is, when the distribution was normal—when the N-Index was high—the implicit standard deviations of call options and put options were similar and, therefore, the difference between them was small, and vice versa.

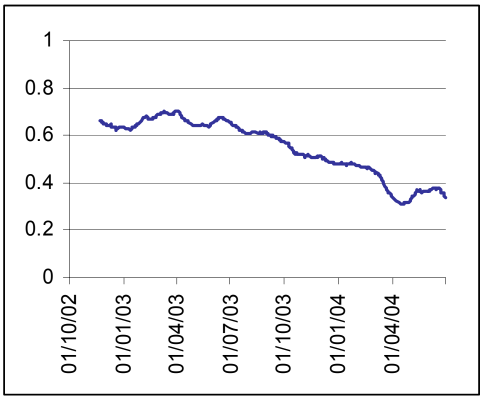
Notably, we did not expect to find, in "abnormal" times, that the difference between the implicit standard deviations of call options and put options would go in one direction most of the time. The direction obtained during "abnormal" times was positive. The explanation lies in a phenomenon typical of the Israeli economy: the public associates exchange-rate risk with depreciation (Hecht, 2000). Therefore, when risk increases, the market assigns in the right tail of the distribution a higher price to call options than to put options in the same distance in the left tail, as reflected in a higher implicit standard deviation. This correspondence was expressed in the intensity of the pro-depreciation skew of the distribution during most of the sample period.

The depreciation expectations calculated by the method used in this study correspond to the financial behavior of Israel's private sector. In the first half of 2000, the business sector had a net asset surplus in forex. In the event of currency appreciation, however, this sector is liable to sustain a considerable loss.

E. Index of Modal Risk in the Forex Market ("R-Index")

The index of modal risk in the forex market examines the level of the modal distribution. The higher this parameter is, the lower the risk level according to this index. The index frequency is daily (Figure 5).

Figure 5: Modal Risk Index



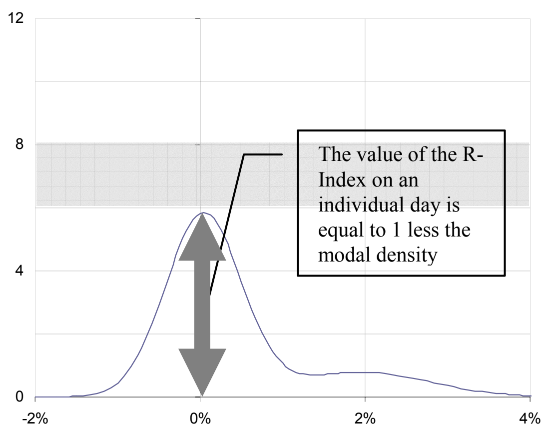
The index is computed in three phases:

(1) Calculating the daily density of the modal of the distribution (Figure 6).

(2) Calculating 1 less the result of the previous phase (so that the higher the index, the higher the risk represented by the index value obtained).

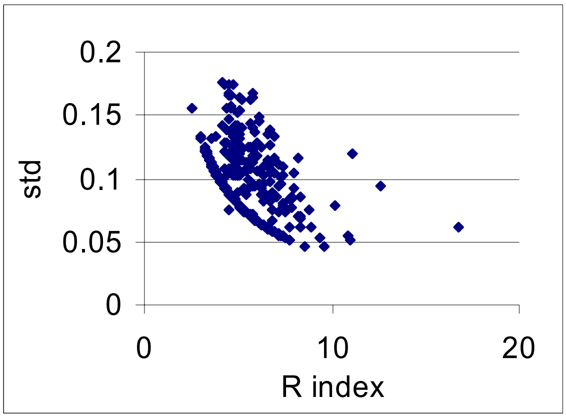
(3) Averaging the results obtained during the previous month of trading.

Figure 6: Exchange-Rate Distribution and Modal Risk Index



Since the implicit standard deviation is often used to examine the level of risk in the market, the index shown here would seem to have nothing additional to offer. Indeed, as Figure 7 shows, when the distribution of exchange-range changes is normal, there is a one-to-one correlation between the index and the modal risk level in the forex market and the implicit standard deviation (the gray line in the figure). Thus, in the normal distribution the index and the implicit standard deviation are indeed interchangeable. When the distribution is not normal, however, this interchangeability does not exist and the implicit standard deviation alone cannot serve as an indicator of market risk.

Figure 7: Modal Risk Index Versus Implicit Standard Deviation

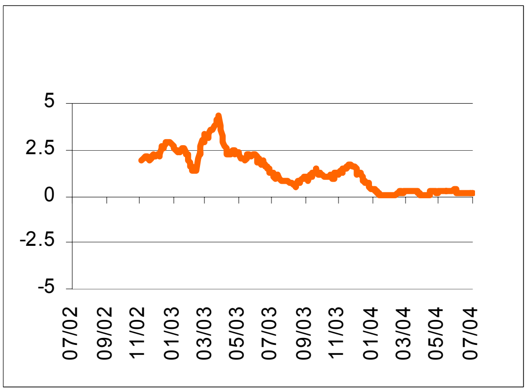


We believe that to get a clearer picture of changes in market risks, both the implicit standard deviation and the modal risk-level index should be examined. If both decline, we may presume that market risk has declined. If the modal risk-level index declines but the standard deviation rises, the conclusions will be less clear and an additional index that examines exceptional exchange-rate changes should be used. This index is presented below.

F. Exchange-Rate Jump Index ("J-Index")

The J-Index examines the direction and intensity of expected exchange-rate changes according to capital-market indicators. The frequency of the index is daily (Figure 8).

Figure 8: Exchange-Rate Jump Index



The index is built in three phases.

(1) Calculating the differential between the two estimated expectancies ($\mu_2 - \mu_1$) in order to obtain the direction and the intensity of exchange-rate change.

(2) Multiplying the difference obtained by θ, which estimates the probability of an exchange-rate jump.

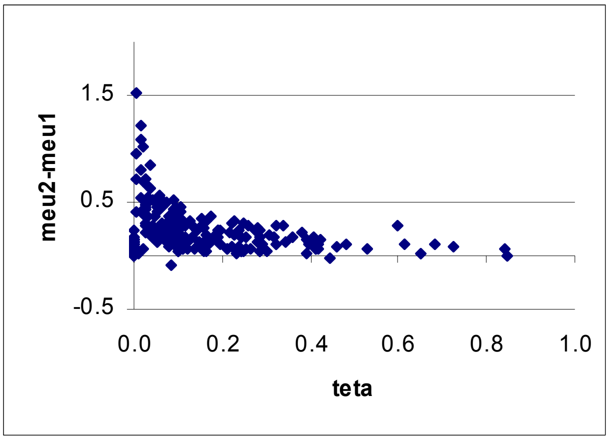
(3) Averaging the results obtained during the most recent month of trading.

To calculate this index, we also use days on which normal distributions were obtained. In such a case, the result is 0 because $\theta = 0$. This means that no jump is expected and that, therefore, the direction and intensity are equal to 0. The values that this index can receive are infinite. Most of the time, however, those in the sample were close to 0.

Figure 9 shows the relationship between the differential of the expectancies and θ. As may be seen, there is a negative correlation between the differential of the expectancies and θ: the higher the probability of an exchange-rate jump, the smaller the size of the jump, and vice versa. By multiplying the two values, we express the intensity and the direction of the jump and also the likelihood of its occurrence. Again, what we present here is an index, not a test.

Analysis of the index in the recent period shows a protracted decline from the middle of 2003, indicating that expectations of exchange-rate jumps leveled off. The analysis also shows, however, that the direction of the expected exchange-rate jump was almost always positive except for a few days of low-intensity expectations of appreciation. This means that the Israeli capital market has not yet internalized the idea that the exchange rate may also head in the direction of a significant nominal appreciation, as actually occurred in early 2003.

Figure 9: Differential of Expectancies and θ Value



G. Additional Findings

Further reinforcement of the claim that the capital market has not yet internalized the possibility of protracted appreciation may be seen in the extent of "tilt" in the distribution, as reflected in skewness (Figure 10). When the skewness parameter is positive, it indicates that the distribution carries a "tail" that points in the direction of depreciation. When it is negative, the "tail" points toward appreciation.

The index was positive during almost all of the sample period. Even the exchange rate was narrowly distributed, it was skewed toward depreciation. One may learn about the narrowness of the distribution from the extent of its kurtosis, as measured by the degree of excess (Figure 11 and Table 1). The value obtained is always greater than 0 and the narrower the distribution, the greater the value is.

Figure 10: The Skew Parameter (Monthly Running Average)

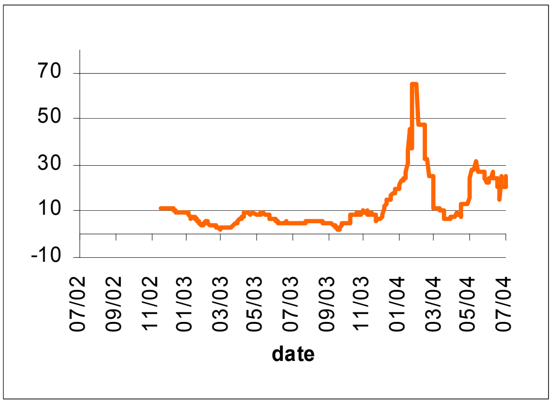


Figure 11: Degree of Excess (Monthly Running Average)

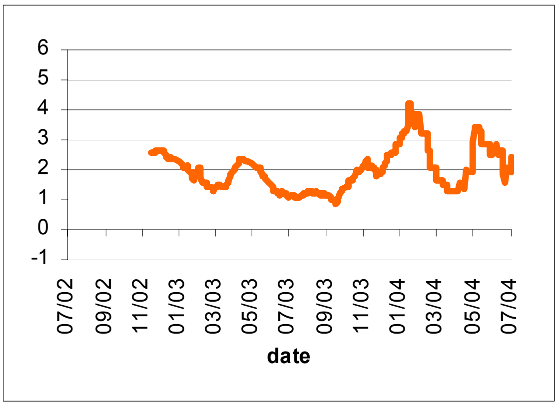


Figure 12: Degree of Excess /Skew

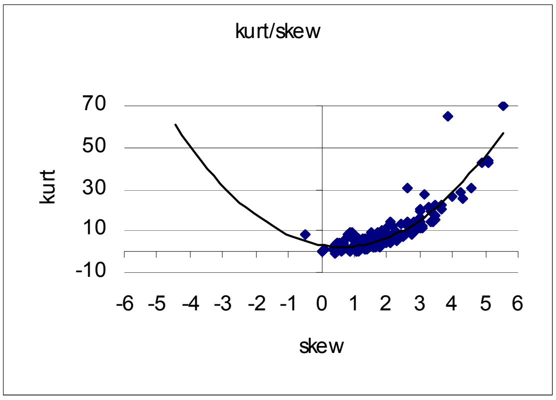


Figure 13: Expectation

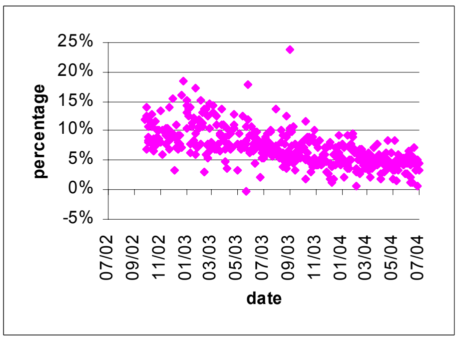
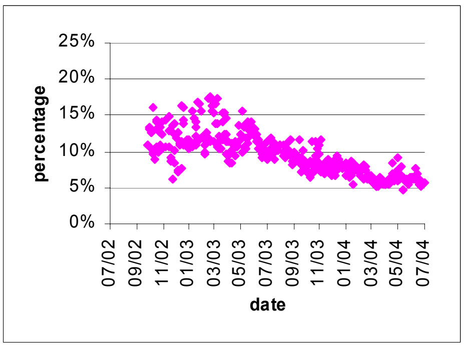


Figure 14: Implicit Standard Deviation



In principle, no correlation is expected between skewness and kurtosis; it may be negative or positive (the black theoretical line in Figure 12). Figure 12 shows, however, that the capital market did elicit a significant correlation: the narrower the distribution (the less the kurtosis), the more skewed it was. This indicates that the capital market has not yet internalized the fact that the exchange rate can also move toward a significant nominal appreciation, such as the one that occurred in early 2003. The direction of the expected exchange-rate jump was almost always positive—except for a few days when it did turn toward appreciation but at low intensity.

Figures 13 and 14 show the expectation and standard deviation of the expected exchange rate. Both parameters declined during the period reviewed.

During 1993, the Chicago Board Option Exchange (CBOE®) introduced the VIX® index, a nonparametric method to calculate the implicit standard deviation of options. The index became a yardstick for the volatility of the American stock market (CBOE [2003]). Since volatility sometimes reflects exceptional financial changes, the VIX® index has evolved into an "investor-fear index" as well. With this in mind, we applied the VIX® index to the forex options data. Afterwards we compared the VIX® index results with the implicit standard deviation of options as calculated on the basis of the methods shown in this study (Figure 15). As Figure 15 and Table 2 show, the behavior of the VIX® index is perceptibly different from the implicit standard deviation as calculated in this study. We believe that our method is preferable to the VIX® index as an "investor fear index" for the Israeli forex market from two standpoints. Firstly, from an economic standpoint, the index in this study provides a better reflection of periods of calm and turmoil in the Israeli forex market. Secondly, form a statistical standpoint, the loss function of the estimation elicited a lower value for the standard deviation in this study (3.7) than for the VIX® index (63.3).

Figure 15: VIX® Index Versus Implicit Standard Deviation (Std) as Calculated in this paper

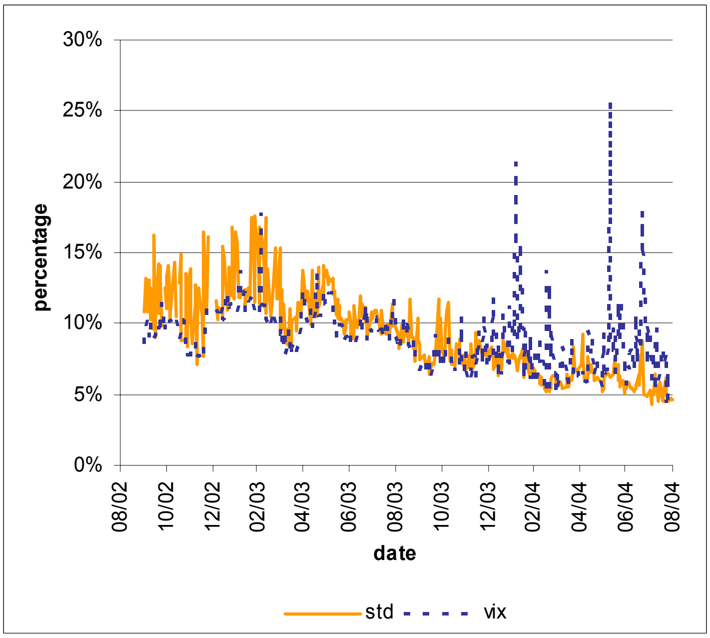


Table 2: VIX® Index vs. Implicit Standard Deviation as Calculated in this Study (Std), average by periods, percentage

Period	VIX®	Std
1oct02 - 31dec02	9.3	11.7
1jan03-30jun03	10.6	12.2
1jul03-31dec03	8.4	8.9
1jan04-30jun04	6.7	8.1
1jul04-31aug04	7.8	5.4
Total	**8.9**	**9.1**

H. Conclusion

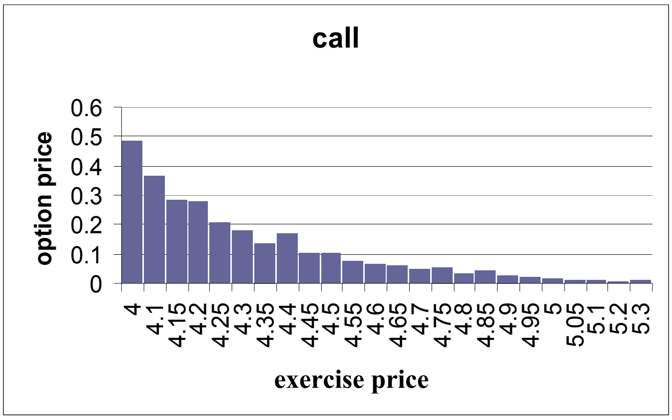
The exchange rate is a crucial parameter in economic thought due to its effects on economic and financial developments in global economies. This explains why economists are ardently interested in understanding and analyzing various indicators of its behavior. This study examined expectations of exchange-rate changes as reflected in daily trading on the Tel Aviv Stock Exchange. These expectations are effectively reflected in trading in exchange-rate options because the income attained upon the expiration of the options depends on the price of the underlying asset. This study describes these expectations as the estimated distribution of the exchange rate. The distribution estimated in this study is of exchange-rate changes on the assumption of bi-lognormal distribution of the rate itself. This premise is consistent with the stochastic process proposed by Merton (1976), according to which changes in the underlying asset are continuous, random, and accompanied by jumps. Due to the small number of observations per day, we assumed in this study that the exchange rate is expected to undergo no more than one jump during the process.

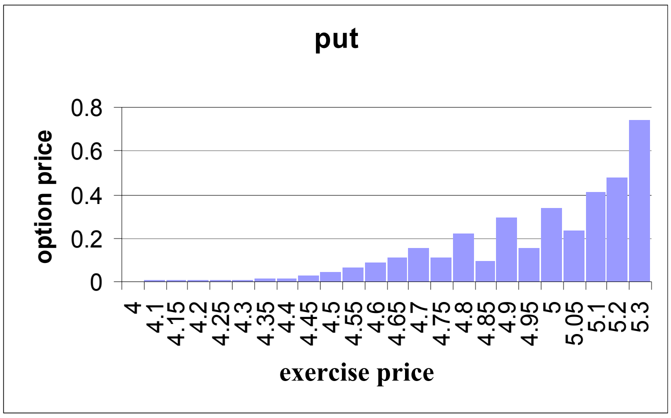
Using data on options trading on the Tel Aviv Stock Exchange, we estimated 410 daily distributions between October 2002 and June 2004 and found that the direction of the expected jump in the NIS/dollar exchange rate was almost always positive—except for a few days when it pointed toward appreciation but at a low intensity. We adduced from this that the Israeli capital market has still not internalized the awareness that the exchange rate can also move in the direction of significant nominal appreciation, as indeed happened in early 2003.

One of the factors behind the decline in the contribution of the forex sector to Israeli banks' financing activity earnings in 2003 (Supervisor of Banks [2004]) may have been the rather severe misalignment between the expectations of a currency depreciation and the appreciation that actually occurred.

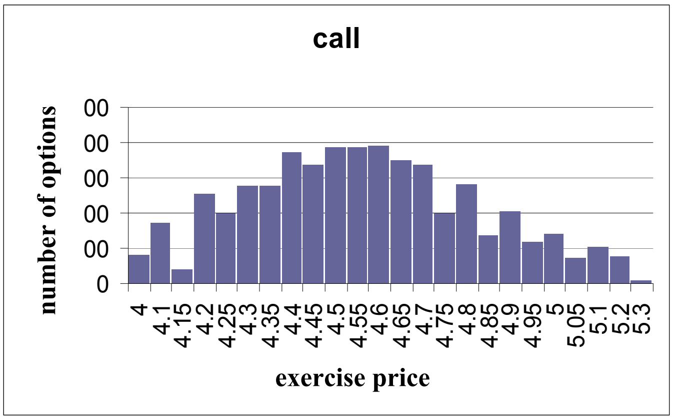
Appendix A

Figures 16a, 16b: Average Price by Exercise Price





Figures 17a, 17b: Number of Options by Exercise Price



Appendix B

The daily examination of the distribution of exchange-rate changes (normal or binormal) was performed on the basis of the values shown on the following screens, generated by the SAS8.0 program.

Figures I a,b

August 31, 2004: Normal Distribution of Exchange-Rate Changes

Phase a: Refutation of the Binormal Distribution Hypothesis

SAS: Output-Untitled

File Edit View Tools Solutions Help

date=31/08/2004

The MODEL Procedure

Nonlinear OLS Parameter Estimates (Not Converged)

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
teta	-0.11783	56.3215	-0.00	0.9984
MEU1	0.218397	15.6476	0.01	0.9891
std1	0.04254	1.6866	0.03	0.9803
MEU2	0.075272	5.5815	0.01	0.9895
std2	0.049156	0.2818	0.17	0.8647

t- tests showing that at 10% significance θ, 1-θ, Std_1, Std_2 the hypothesis that they are other than 0 is not refuted.

Phase b: Acceptance of Normal Distribution Hypothesis and Estimation of Distribution

SAS: Output-Untitled

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The SAS System 2004

08:57 Thursday, September 2, 2004

date=31/08/2004

The MODEL Procedure

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
MEU1	0.042734	0.0163	2.62	0.0202
std1	0.046398	0.00286	16.24	<.0001

Number of Observations		Statistics for System	
Used	16	Objective	0.2287
Missing	0	Objective*N	3.6585

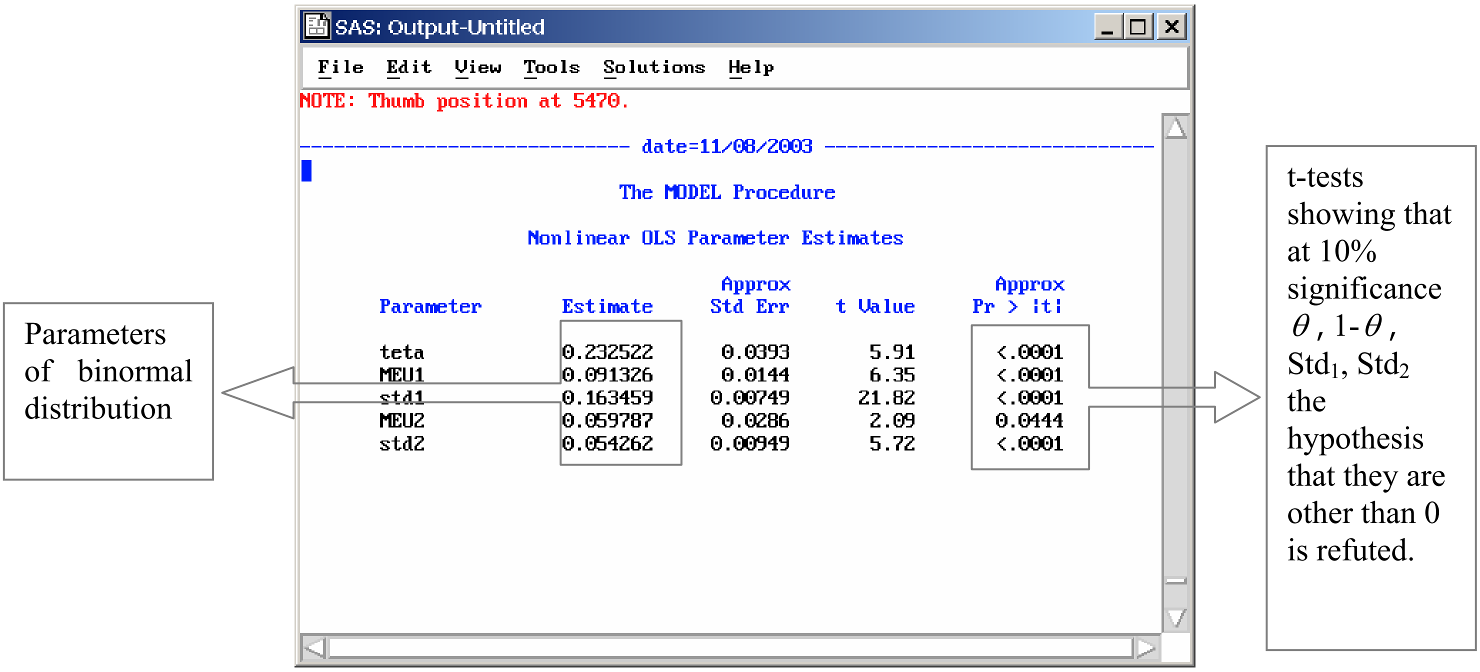
Parameters of normal distribution

t-tests showing that at 10% significance θ, 1-θ, std_1, Std_2 the hypothesis that they are other than 0 is refuted.

Parameters

Figure II

August 11, 2003: A case of Binormal Distribution of Exchange-Rate Changes


SAS: Output-Untitled

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NOTE: Thumb position at 5470.

---------------------------- date=11/08/2003 ----------------------------

The MODEL Procedure

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx Std Err	t Value	Approx Pr > |t|
teta	0.232522	0.0393	5.91	<.0001
MEU1	0.091326	0.0144	6.35	<.0001
std1	0.163459	0.00749	21.82	<.0001
MEU2	0.059787	0.0286	2.09	0.0444
std2	0.054262	0.00949	5.72	<.0001

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Bank of Israel – Monetary Department
בנק ישראל – המחלקה המוניטרית
POB 780 91007 Jerusalem, Israel
ת"ד 780 ירושלים 91007