CAPITAL ACCUMULATION
IN A BEQUEST ECONOMY

Michel Strawczynski

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Research Department, Bank of Israel, POB 780, 91007 Jerusalem, Israel

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ABSTRACT

An augmented life cycle model is used in order to characterize capital accumulation in an overlapping generations economy with altruistic agents. Precautionary savings serve as an instrument to face two different types of idiosyncratic income uncertainty: i) individual's income uncertainty in the future and ii) future generation's income uncertainty. By performing a calibrated simulation of the model, it is shown that precautionary savings aimed at providing bequests can account for a vast portion of capital accumulation. Empirical evidence on income uncertainty calls for further tests on the impact of future generation's income uncertainty on precautionary savings.

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The main purpose of this paper is to characterize capital accumulation in an overlapping generations economy with income uncertainty. Two basic frameworks are generally considered in the literature in order to characterize capital accumulation: (*i*) the "pure" life cycle model, according to which savings are motivated by individual's income or life uncertainty in future periods, and (*ii*) the "pure" bequest model, according to which savings are motivated by the desire to provide transfers to heirs, whose welfare increases individual's utility.

Yet it is possible to think of a bequest economy where the augmented life cycle theory[1] can be compared to the bequest theory. By introducing income uncertainty, it is possible to think about two alternative cases. In the first case — "life cycle economy" — savings are motivated by income uncertainty of the individual in the retirement period. In the second case — "bequest economy" — savings are motivated by income uncertainty of the future generation. This common framework will allow us to deal with the basic question postulated by the well-known controversy between Kotlikoff and Summers (1981) and Modigliani (1986): in which of these two economies we obtain a higher capital accumulation? What are the steady state interest rates and how do we characterize optimal government intervention in these two types of economy?.

The paper is organized as follows: in section 2 a brief description of the literature is presented. Section 3 describes the basic model, distinguishing between a life cycle economy and a bequest economy. Section 4 characterizes the steady state interest rate and introduces government intervention. Section 5 includes empirical evidence and the results of a calibrated simulation. Section 6 presents the conclusions and agenda for future research.

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[1] Modigliani (1986) analyzes an augmented life cycle model, in which life-cycle savings are complemented with a bequest motive.

2. Brief Survey of the Literature

Kessler and Masson (1988) explain the main controversy of the different approaches: in one hand we have Kotlikoff and Summers (1981) findings according to which 80 percent of capital accumulation is explained by intergenerational transfers, while on the other hand Modigliani (1988) estimates that they contribute only for 20 percent. According to these two extremes in one case we have an economy of bequeathers where savings are driven by the desire of providing wealth to heirs, and in the other we have life-cycle savers where capital accumulation is driven by permanent income considerations. We briefly survey some of the recent works according to these two explanations.

i) Life Cycle Explanation

Kessler and Masson (1988) and Hurd (1990) present detailed surveys of the life cycle explanation for capital accumulation. The simple life cycle hypothesis specify capital accumulation during active life, followed by decumulation of the elderly, leading to a hump-shaped age-wealth profile.

The fact that calibrated simulations of the "naive" life-cycle approach leaves a significant portion of capital accumulation unexplained, combined with findings on the importance of intergenerational transfers (Kotlikoff and Summers, 1981) and the lack of dissaving by the elderly (Hurd, 1990 and Weil, 1994), lead to the substitution of the simple life cycle model by the "augmented" life cycle model. In this augmented model capital accumulation is mainly driven by life cycle considerations like future period's income uncertainty and uncertainty about the date of demise, while some of it is related to the desire of providing intentional bequests to a heir.

Two recent contributions on augmented versions of the simple life cycle hypothesis are provided by Hubbard, Skinner and Zeldes (1993) and Carroll (1994). The first paper examines predictions of a life cycle simulation model with uncertainty regarding their length of life, earnings and medical expenses in a world of imperfect insurance and lending markets. This augmented life-cycle model matches different features of U.S. capital accumulation like wealth-age and consumption-age profiles by education group. Carroll (1994) provides mixed evidence on the life-cycle hypothesis: although future predictable changes in income do not seem to affect consumption, but income uncertainty has an important effect on consumer's consumption allocation.

ii) Bequest Explanation

The simple bequest explanation is based on the fact that a significant portion of individual's savings is driven by his welfare from transfers to relatives (lifetime transfers -"inter-vivos"- or bequest transfers).

There is a wide literature on the different motivations for bequests, surveyed by Kotlikoff (1988). Blinder (1988) provides a detailed discussion on the relevant issues in order to assess the importance of the bequest motive. Some of the topics stressed by this author are the definition of intergenerational transfers, the status of accumulated interest on past inheritances and the treatment of durables. Bernheim (1991) presents some recent evidence on the relevance of the bequest motive. This author shows that the provision of Social Security annuity benefits by the government crowds out private annuity purchases, and at the same time reinforces the acquisition of life insurance. These patterns are interpreted by the author as a clear sign that the typical household would choose to maintain a positive fraction of its resources in bequeathable forms, even if insurance markets were perfect.

In the next section we present a model where these two approaches can be studied simultaneously in a common framework. By doing so we emphasize the relevant parameters which help us to account for the different forces driving savings.

3. The Model

Assume an overlapping generations model with constant population[2]. Each individual has a single child, lives for two periods and receives income in both periods (i.e., the old receives some income also during the second period). The assumption of a positive income in the second period is crucial for the purpose of modeling income uncertainty, since both second period's and future generation's income uncertainties are relevant. For analytical convenience, we assume equal shares in total income:[3]

$$Y^0_{t+1} = \frac{1}{2}Y_{t+1} = Y^Y_{t+1} \tag{1}$$

where Y^Y_{t+1} and Y^0_{t+1} represent the income of the young and the old in the period t+1, respectively. Each agent solves a dynamic program according to the following equation:

$$V(Y^Y_t, W_t) = \underset{C^Y_t}{Max} \left\{ U(C^Y_t) + \delta E \underset{C^0_{t+1}}{Max} \left[U(C^0_{t+1}) + \alpha V(Y^Y_{t+1}, W_{t+1}) \right] \right\} \tag{2}$$

where V is the optimal utility obtained as a solution of the program, sub-indexes t and t+1 indicate the period, Y^Y and W represent income while young and inherited wealth respectively, U is the utility function, C^Y is the consumption while young, δ is the subjective discount rate, E is the expectation operator, C^0 is the consumption while old and α is the altruism coefficient. The maximization problem stated in equation 2 is solved backwards in

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[2] The model can be extended to the case of a constant population growth rate n, by imposing the condition r > n, where r is the real rate of return on capital (of course, this condition is met under the present assumption of n=0).

[3] In general it is empirically accepted that the mean income of the old is low (Hubbard, Skinner and Zeldes, 1993). Different evidence was provided by Hurd (1990), who shows that after correcting for different sources of income (mainly social security) and family size, the income of the elderly is roughly the same as the non-elderly. Reducing the share of the old in the present model, will reinforce the main result presented in the next section.

two steps: the first step solves for the optimal C^0_{t+1} given each state of nature of Y_{t+1}; the second step solves for the optimal C^Y_t, taking into account the expectation (over all states of nature) of the maximum obtained in the first step.

The budget constraint of the problem is given by the wealth equation, which evolves according to:

$$W_{t+1} = R\left(Y_t^Y + W_t - C_t^Y\right) + Y_{t+1}^0 - C_{t+1}^0 \tag{3}$$

where R is 1 plus a certain rate of return on capital.

We note that the problem as proposed here is artificial in the sense of Abel (1988), since the wealth equation is dynastic: the father chooses the hole allocation of resources (the artificial character of the problem will be further clarified later).

In order to obtain analytical expressions for the main variables, we will assume an absolute risk aversion utility function:

$$u = -e^{-\Omega C} \tag{4}$$

The analytical convenience of this function is based on the linearity of consumption and wealth equations in the optimal solution. This feature was introduced first by Hey (1980), and is stressed by Sheshinski (1988), Kimball and Mankiw (1989) and Caballero (1991).

As explained before, Y_{t+1} is produced both by the young and the old. In the general case, we will assume that income uncertainty is related to both sources of income. At this first stage, we assume a common variance (σ^2_0) for all sources of income uncertainty; i.e., uncertainty is related to the income produced in the period t+1:

$$Y_{t+1} = Y_t + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N\left(0, \sigma_0^2\right) \tag{5}$$

where ϵ is a purely uncorrelated idiosyncratic shock. Note that the assumption of an additive income shock may derive in an increasing variance of the income distribution[4]. The assumption of a normal distribution for the income shocks allows for a simple solution, but at the same time does not rule out a positive probability for a negative income, consumption and bequest[5]. Note also that in the solution to the dynamic program, corner solutions are ruled out by assumption[6]. Finally note that the uncertainty is resolved at the beginning of the second period, so that precautionary savings are optimal in the first period.

As remarked before, with a constant absolute risk aversion utility, the optimal consumption and wealth functions are linear (see appendix A)[7]:

$$C_t^{Y*} = -\ln(\delta R)^{\frac{2R}{(R+1)(R-1)\Omega}} \alpha^{\frac{1}{(R-1)\Omega}} + Y_t^Y + \frac{R-1}{R+1} W_t - \frac{\sigma_1^2 \Omega R}{(R+1)(R-1)} \tag{6}$$

$$C_{t+1}^{0*} = \ln(\delta R)^{\frac{R^2-2R-1}{\Omega(1+R)(R-1)}} \alpha^{-\frac{1}{\Omega(R-1)}} + Y_{t+1}^0 + \frac{R-1}{R+1} W_t - \frac{\sigma_1^2 \Omega(-R^2+2R+1)}{2(R+1)(R-1)} \tag{7}$$

$$W_{t+1}^* = \ln \alpha \delta R^{\frac{R+1}{\Omega(R-1)}} + \frac{\sigma_1^2 \Omega (R+1)}{2(R-1)} + W_t \tag{8}$$

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[4] This feature is common to all frameworks where income follows a Markov process (see Deaton and Paxson, 1991). An alternative assumption that avoids an increasing variance is to assume a memory-less income process ($Y_{t+1} = E(Y) + \epsilon_{t+1}$, where E(Y) is a constant), as in Caballero (1991).

[5] In Strawczynski (1994, p. 484) it is shown that for plausible values of the relevant parameters this probability is insignificant. For a closed solution see Weil (1993).

[6] For the effect of corner solutions on consumption see Strawczynski (1995).

[7] Note that σ^2_1 is equal to $\sigma^2_0/4$.

Some features of the solution are:

- The uncertainty of income in the optimal solution is entirely transferred to the consumption stream, while the wealth function does not depend on Y. This result means that consumption follows a random walk (Hall, 1978).
- The dynamic path of wealth at the individual level does not depend on ϵ, which means that the distribution of wealth is stationary.
- Precautionary savings are driven by risk aversion and the variance of the income[8] in the period $t+1$.[9]

The artificial character of the solution is seen by assuming different values for α, the coefficient of altruism. When α tends to zero, consumption streams of the father when young and old tend to $+\infty$ while bequests (W_{t+1}) tend to $-\infty$. This result means that children's consumption is sacrificed in favor of the father. When α tends to $+\infty$, consumption of the father tends to $-\infty$ and bequests to $+\infty$, which means that consumption of the father is sacrificed in favor of the children[10].

We distinguish now between different economies, according to the type of income uncertainty; in particular - a life cycle economy in which individuals save as a consequence of second period's income uncertainty, and a bequest economy in which individuals save as a consequence of child's income uncertainty.

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[8] Note that in the optimal solution precautionary savings are independent of income. This result is a consequence of the constant absolute risk aversion assumption.

[9] An additional risk cited often in life cycle models is the probability of demise before entering the second period. Taking into account this uncertainty, in addition to income uncertainty, would lead to a <u>lower</u> capital accumulation (see Strawczynski, 1993).

[10] As will be shown in section 4, a necessary condition for the existence of a steady state is that $\alpha=1$.

a. *A life-cycle economy*

A "pure" life-cycle economy maybe introduced by assuming that the only uncertain stream is second period's income; i.e., Y^{Y}_{t+1} is certain and Y^{0}_{t+1} is uncertain[11]:

$$Y^{Y}_{t+1} = Y_t, \qquad Y^{0}_{t+1} = Y_t + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N(0, \sigma^2_{LC}) \tag{9}$$

The new solution is linear as in equations 6,7 and 8, but now the forces driving precautionary savings are related to σ^2_{LC}, which is the variance of parent's second period income.

Clearly, if the relevant case includes both kinds of income uncertainty, a simulation of the "pure" life-cycle economy will be not enough to explain total capital accumulation.

b. *A pure bequest economy*

Assume now that uncertainty is related only to future generation's income (σ^2_{BE}):

$$Y^{0}_{t+1} = Y_t, \qquad Y^{Y}_{t+1} = Y_t + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N(0, \sigma^2_{BE}) \tag{9}$$

Again the new functions are linear as in equations 6,7 and 8, but now the relevant variance is the one of the bequest economy, σ^2_{BE}. This result shows that precautionary savings in the bequest economy are driven by the uncertainty of future generation's income. As in the previous section, a simulation of a "pure" bequest economy will be not enough in order to explain total capital accumulation.

[11] Examples that explicitly assume this feature are shown in Weil (1987) and Feldstein (1988).

The characterization of the steady state in these two different economies depends on the observed values of σ^2_{BE} and σ^2_{LC}. In section 5 we provide empirical evidence on this topic.

c. *A "mixed" case*

Finally we can think about a more realistic case, in which both incomes are uncertain:

$$\begin{aligned} Y^0_{t+1} &= Y^Y_t + \epsilon_1 \qquad \epsilon_1 \sim N\left(0\ ,\ \sigma^2_{LC}\right) \\ Y^Y_{t+1} &= Y^0_t + \epsilon_2 \qquad \epsilon_2 \sim N\left(0\ ,\ \sigma^2_{BE}\right) \end{aligned}$$

The analytical solution shows again linear functions, but now the forces driving savings are composed by both second period's income uncertainty and future generation's income uncertainty.

4. Characterization of the Steady State and government intervention

a. *Steady State*

It is possible to characterize the aggregate steady state by noting that: i) aggregation is immediate since consumption and wealth equations are linear, and ii) given a large number of individuals, at the aggregate level there is no uncertainty. Denoting aggregate variables by lower case letters, the conditions for a steady state in the life-cycle economy are: $c_t = c_{t+1}$ and $w_t = w_{t+1}$.

As shown in appendix B, a necessary condition for the existence of a steady state in this artificial problem is that $\alpha=1$. The intuition for this result is that once we find the values of time preference and interest rates which are consistent with a constant consumption stream for the individual when young and old, a value of α which is higher (lower) than one will cause a reallocation of resources toward (outward) wealth transferred to the child. Thus, the two necessary conditions for the existence of a steady state are: $\alpha = 1$ and

$$\ln R\delta = -\frac{\sigma_1^2\Omega^2}{2} .$$

Note that this steady state is dynamically inefficient, since $R < 1/\delta$, which represents the marginal productivity of capital according to the golden rule. This "low" interest rate means that capital is too large. The same result was emphasized by Aiyagari (1993), who obtains a similar result for a different type of precautionary savings.[12] Given that the level of capital is beyond the golden rule level, there is a place for government intervention.

b. *Government Intervention*

It can be shown that government intervention aimed at providing insurance may reduce precautionary savings and cause capital accumulation to reach the golden rule level.

For simplicity, we assume a memory-less income process where Y_{t+1} is equal to $\bar{Y} + \epsilon$. The government intervenes through the following tax-transfer system: $T = \theta Y$, $S = \theta\bar{Y}$; i.e., government is assuring to everyone the same level of income to the extent of the system, θ. According to this system, individuals above the mean income $\bar{Y}$ are transferring resources to those below the mean income. In this case, equation (8) becomes (see appendix C):

[12] In Aiyagari 's paper there is no need to assume the traditional $u''' > 0$ assumption for precautionary savings, given that private savings are driven by the desire of generating a buffer for protection against illiquidity, under bad realizations of the idiosyncratic risk.

$$W^*_{t+1} = \ln \alpha R \delta^{\frac{R+1}{\Omega(R-1)}} + \frac{\sigma_1^2 \Omega(R+1)}{2(R-1)} (1-\theta)^2 + W_t \qquad (8')$$

From this equation it becomes clear that as θ tends to 1, capital in the steady state approximates the golden rule capital level ($R=1/\delta$)[13]. In other words, an income tax-transfer system is optimal in this context, since it drives the economy to the optimal capital level.

5. Variance of Life Cycle Income vs. Variance of Intergenerational Links

Before turning to an empirical comparison of income variances, note that from the point of view of the father, the variance of second period's income is conditional on his first period income (which is determined by his own human capital and occupation), while the variance of future generation's income is an unconditional mean, since the human capital of the child and his occupation are revealed in the second period. In this sense we expect that the variance of future generation's income is higher. This difference could be moderated if there is an empirical correlation between the income of subsequent generations, in which case the variance of future generation's income is conditional on father's income.

In table 1 we show existing evidence as surveyed from different works on income uncertainty and social mobility.

[13] Clearly, we are explicitly ignoring the efficiency effects of tax rates, which discourages savings. An example that considers both insurance and efficiency considerations is provided by Varian (1980).

Table 1

Evidence on Income Uncertainty of Fathers and Sons

	Coefficient of Variation		Standard Deviation of Log Income		
	Father	Son	Father	Son	Sample,Survey
Jencks (1972)					
Unconditional[1]		0.65			37 mill.,census
Conditional[1]		0.63			
Behrman and Taubman (1985)[2]	0.81	1.12			3,768, Twin Sample
Barsky, Mankiw and Zeldes(1986)[3]	0.5-0.55	0.63			PSID and Censu
Solon, Corcoran, Gordon and Laren (1991)				0.59	1,854, PSID
Solon (1992)	0.68	0.67			322, PSID
Zimmerman (1992)[4]					
Earnings			0.418	0.502	192, NLS
Wages			0.412	0.406	188, NLS

Notes: i) Some of the figures were calculated using the estimates reported in the cited works. ii) PS! Panel Study of Income Dynamics; NLS- National Longitudinal Survey.

[1] Based on 1968 full-time year round annual earning of male workers. The unconditional coeffic of variation for all workers is 0.72.

[2] Based on 1980 yearly earnings (families with reported offspring).

[3] Father's estimate was calculated using Hall and Mishkin (1982) data. Son's estimates are accor(to Jencks (1972).

[4] Based on four years average of father's earnings (Table 6, p. 421).

The results suggest that both future earnings of the individual and earnings of the future generation are exposed to a great deal of uncertainty. The other important result is that future generation's income uncertainty is higher than individual's uncertainty as perceived from the first period.

In table 2 we show the results of a calibrated simulation in the frame of the "mixed" case, in which both sources of income uncertainty interact. For this purpose we use the measures of uncertainty as estimated by Barsky, Mankiw and Zeldes (1986) and the results of the life-cycle simulations by Hubbard, Skinner and Zeldes (1993).

Table 2- Capital Accumulation with income uncertainty

(percent of total capital accumulation)

Source of uncertainty	Free-Market	Government Intervention (θ=0.2)	Government Interv.(θ=0.4)
Second Period's Income Uncertainty	20		
Future Generation's Income Uncertainty	30		
Savings- compared to free market		82	68

Note: simulations assumed the following parameters: R = δ = 2.43 (3 percent yearly on a 30 years basis), α=1, σ_{BE}=0.6, σ_{LC}=0.5, as in Barsky, Mankiw and Zeldes (1986), W_t = 50 percent of total wealth, as in Hubbard, Skinner and Zeldes (1993), p. 17.

The first column in table 2 summarizes the result of Hubbard, Skinner and Zeldes (1993): life-cycle savings explain roughly 50 percent of savings, while the rest must be explained by an augmented model. In the present paper the additional feature that explains the remaining 50 percent is precautionary behavior. We see from the simulations that the most important part of precautionary savings (sixty percent of total precautionary savings) is explained by the desire of generating a buffer in order to deal with child's income uncertainty.

The other result reported in table 2 refers to government intervention: a tax-transfer system aimed at providing insurance reduces savings to 82 percent and 68 percent of the savings obtained in the free-market case (for $\theta=0.2$ and $\theta=0.4$, respectively).

6. Summary and conclusions

The augmented life cycle model has become the basic framework for capital accumulation studies. It is surprising to note that given this fact, we can hardly find models that consider future generation's income uncertainty as a possible source of precautionary savings. In this paper we characterize the fact that once we accept the altruistic model as the basic framework, there are good theoretical and empirical reasons to believe that this kind of uncertainty could account for a vast part of capital accumulation. In particular, empirical findings of income uncertainty suggest that income variance of sons is higher than fathers'. Further future research should concentrate on direct empirical tests of precautionary behavior given future generation's income uncertainty. One possibility for such a test would be to compare the bequests received by sons with professions characterized by different degrees of income uncertainty (after controlling for mean income differences). This line of research could help to clarify the well-known puzzle that the elderly do not dissave, in contradiction to the standard life-cycle model.

Appendix A: Consumption and Wealth Equations*

The functional equation is given in equation (1) of the text:

$$V(Y_t^Y, W_t) = \underset{C_t^Y}{Max}\ U(C_t^Y) + \delta E\{U(C_{t+1}^{0*}) + \alpha V(Y_{t+1}^Y, W_{t+1})\} \qquad \text{A.1}$$

where C_{t+1}^{0*} is the optimal solution to the problem in the second period of life, after uncertainty has been resolved. The control variable is C_t^Y.

The budget constraint is**:

$$W_{t+1} = R\left(Y_t^Y + W_t - C_t^Y\right) + Y_{t+1}^0 - C_{t+1}^0 \qquad \text{A.2}$$

First step: Calculate C_{t+1}^{0*} from F.O.C. A.3:

$$U'\left(C_{t+1}^{0*}\right) = \alpha\ V_w \qquad \text{A.3}$$

where V_w is the derivative of V on its second argument. In order to solve A.3, a guess of V must be used in its general form:

$$V = -A\ e^{-\Omega(k+nW+mY)} \qquad \text{A.4}$$

* A more detailed mathematical appendix is available from the author.

** In the solution we will use the assumption $Y^Y = Y^0$.

$$C_{t+1}^{0*} = -\frac{\ln \alpha A n}{\Omega(1+n)} + \frac{k}{(1+n)} + \frac{nR}{(1+n)}\left(Y_t^Y + W_t - C_t\right) + \left(\frac{n+m}{1+n}\right)Y_{t+1}^Y \qquad \text{A.5}$$

Second step: Calculate C_t^{Y*} from F.O.C. A.6:

$$U'\left(C_t^{Y*}\right) = \delta E U'\left(C_{t+1}^{0*}\right) \qquad \text{A.6}$$

Given that in the right hand side C_{t+1}^{0*} is an optimal solution, by the envelope theorem we use the budget constraint in order to calculate the derivative of the right hand side in A.6. The expected value is calculated by using the property that if aX is distributed $N(0,\sigma^2)$, the expected value of X is $a^2\sigma^2/2$. The solution is:

$$C_t^{Y*} = \frac{1+n}{1+n+nR}\left\{\frac{1}{\Omega}\left[-\ln\delta R(\alpha A n)^{\frac{1}{1+n}}\right]\right\} + \frac{1}{1+n+nR}\left\{k + (n+m+nR)Y_t^Y + nRW_t - \frac{1}{2}\frac{(n+m)^2}{(1+n)}\sigma_1^2\Omega\right\} \qquad \text{A.7}$$

Third Step: Calculate undetermined coefficients A, m, n and k.

Using F.O.C. A.3 and A.6, we get:

$$V(Y_t, W_t) = U\left(C_t^{Y*}\right)\left[1 + \frac{1}{R} + \frac{1}{nR}\right] \qquad \text{A.8}$$

Using the general form of V given in A.4 and equation A.7, the coefficients are:

$$
\begin{aligned}
A &= \frac{R+1}{R-1}\,\alpha^{\frac{1}{R-1}}(\delta R)^{\frac{2R}{(1+R)(R-1)}} \\
m &= 1 \\
n &= \frac{R-1}{1+R} \\
k &= -\frac{R\sigma_1^2\Omega}{(R-1)(R+1)}
\end{aligned}
\qquad \text{A.9}
$$

Fourth Step: Plug the coefficients calculated in A.9 into A.7, A.5 and A. 2 in order to achieve the equations shown in the text for C_t, C_{t+1} and W_{t+1}.

Appendix B: Characterization of the Steady State

The conditions for a steady state are:

$$B.1)\ c_{t+1}^{0*} - c_t^{Y*} = 0$$

$$B.2)\ w_{t+1}^{*} - w_t = 0$$

Using equations 6 and 7 of the text:

$$B.3)\ c_{t+1}^{0*} - c_t^{Y*} = \ln(\delta R)^{\frac{R^2-2R-1}{\Omega(R+1)(R-1)}}\alpha^{\frac{-1}{\Omega(R-1)}} + y_{t+1}^{0} + \frac{R-1}{R+1}w_t - \frac{\sigma_1^2\Omega(-R^2+2R+1)}{2(R+1)(R-1)} +$$

$$\ln(\delta R)^{\frac{2R}{(R+1)(R-1)\Omega}}\alpha^{\frac{1}{\Omega(R-1)}} - y_t^{Y} - \frac{R-1}{R+1}w_t + \frac{\sigma_1^2\Omega R}{(R+1)(R-1)}$$

$$= \ln(\delta R)^{\frac{1}{\Omega}} + (y_{t+1}^{0} - y_t^{Y}) + \frac{\sigma_1^2\Omega}{2}$$

Using equation 8 of the text:

$$B.4)\ w_{t+1}^{*} - w_t = \ln(\alpha\delta R)^{\frac{R+1}{\Omega(R-1)}} + \frac{\sigma_1^2\Omega(R+1)}{2(R-1)}$$

Equalizing equations B.3 and B.4 to zero allows for the characterization of the steady state. The simultaneous fulfillment of these equalities occurs if:

$$B.5)\ \alpha = 1 \ \wedge\ \ln \delta R = \frac{-\sigma_1^2\Omega^2}{2},$$

as shown in section 4.

Appendix C: Government Intervention

Private budget constraint is, after government intervention:

$$W_{t+1} = R(Y_t + W_t - C_t) + (1-\theta)Y_{t+1} + \theta\bar{Y} - C_t$$

Solving again the problem allows for new parameters, k' and n' instead of k and n as obtained in appendix A:

$$k' = \frac{-R\sigma_1^2\Omega}{2(R-1)(R+1)}(1-\theta)^2$$

$$n' = \frac{R-1}{1+R}(1-\theta)$$

Plugging back the new parameters in the consumption and wealth equations, allows us to get equation 8' of the text.

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