Research Department **Bank of Israel**

Asymmetry in Monetary Policy: An Asymmetric Objective Function and a New-Keynesian Model

by

WEITZMAN NAGAR [1]

Discussion Paper No. 2007.02
March 2007

[1] I would like to thank Michael Beenstock who guided me in preparing this paper, and Joseph Zeira. Thanks also go to participants in the Bank of Israel's Research Department seminar and the Israel Economic Association convention in May 2006, for their comments. Particular thanks to Joseph Djivre and Michel Strawczynski.
Bank of Israel, Research Department. http://www.boi.gov.il wezmngar@boi.gov.il

מחלקת המחקר, בנק ישראל ת"ד 780, ירושלים 91007
Research Department, Bank of Israel, POB 780, Jerusalem 91007, Israel

Abstract

This paper presents a theoretical examination of the possibilities of asymmetric monetary policy on inflation, in the light of the fact that inflation in Israel has been below the target for several years. The economic model used is the New-Keynesian one, which has featured very widely in the theoretical literature since the beginning of the decade. The model has two equations, one of aggregate demand and the other of aggregate supply, and both include "pure" rational expectations, without inertia from the past. The innovation in this study is the combination of the New-Keynesian model with the asymmetric objective Linex function, instead of the commonly used quadratic function. Unlike the quadratic function, which incorporates certainty equivalence, the Linex function, by including the variance, also relates to uncertainty that accompanies the variables in the objective function. The Linex function nevertheless does include the quadratic case as a special one, so that the results of the current study can be compared with those of other studies that used the same model but with the quadratic objective function. After developing the theoretical model to cover the case of policy under discretion, the paper present simulations, and several interesting results emerge: (a) in equilibrium (without shocks), adopting an asymmetric policy has no effect on the real variables, i.e., real expected monetary interest rate and the output gap, but only on the nominal variables, i.e., interest rate and inflation. In other words, in equilibrium, an asymmetric policy is neutral. (b) In the convergence process following a shock, an asymmetric policy has an effect only if the shock was a supply shock, not if it was a demand shock (apart from the effect on nominal interest rate). Hence identifying the source of the shock—supply or demand—assumes even greater importance. (c) The higher the degree of inflation aversion of the policy, the lower will be inflation; the real interest rate will also be higher, although nominal interest rate will be lower. (d) Discretionary policy may have a deflationary bias, a finding which contradicts the generally accepted view that discretionary policy results in an inflationary bias. Other results are given in the summary.

A. BACKGROUND

Since 1992, Israel's monetary policy has been directed at achieving target inflation rates. Illustration no. 1, which presents actual and target inflation rates, shows that the Bank of Israel has been off target almost ever since such targets were applied (13 years) – from 1998 onwards being below target (excluding 2002 when it was above target), and previously above target.[1]

A number of reasons contribute to missing the target, including: the target is set for a calendar year rather than a moving year, the lag impact of monetary policy, surprises due to unexpected shocks, and the choice of the specific index for the inflation target. Another cause lies in the policy's inherent preferences (behavioral function): where the policy is asymmetric – its reaction to an upward deviation from the target is not the same as its reaction to a downward deviation.

Illustration no.1 presents the expected inflation for a year ahead and the changes in Bank of Israel interest rates. The "eye econometrics", particularly since 1999, may indicate an asymmetric policy: interest rates were lowered gradually when expectations were below the inflation target, and they rose more rapidly when expectations deviated from the target's upper limit. We may therefore assume that the Bank of Israel has a greater risk aversion to a positive deviation from the target than to a negative deviation.

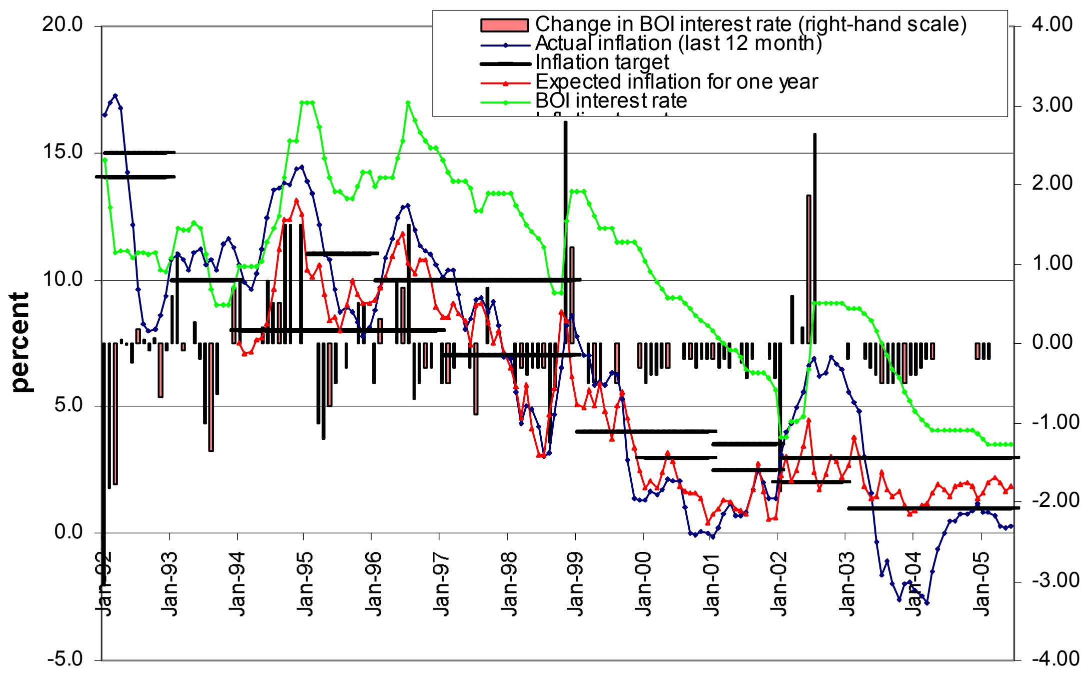
It is this phenomenon that has motivated this study. The study attempts to perform a theoretical analysis on the question of asymmetry in monetary policy with respect to deviations from the inflation target, within the context of a central bank's optimal policy behavior. Does the issue as it emerges from Illustration no. 1 – a sharp increase in the interest rate due to shock followed by gradual reductions, in fact reflect asymmetry? This may be an empirical question which is not under review here, but the paper presents a model for its empirical review as well as for reviewing its theoretical feasibility. The model combines a New-Keynesian economy with an asymmetric objective function of the Linex type in which the asymmetric index is built-in as part of the function, and also includes the symmetric (quadratic) state as a specific case. The asymmetric index a is contiguous – receives positive or negative values or tends to zero. A combined model of this type has not yet to appear in the literature. The paper

[1] The inflation target until 2002 was set for the calendar year, and from 2003 on, the target is set at a rate of 1-3%.

analyzes the case of policy under discretion and demonstrates the results using simulations.

The paper is formulated as follows: in Section B we will review the relevant literature. In Section C we present the Linex function. Section D presents the New-Keynesian model, in which the equations include the expected output gap and expected inflation – as it appears in Clarida, Gali & Gertler (1999), hereinafter CGG)[2] – combined with a Linex objective function. Part 6 in Section E derives the central bank's optimal conduct in the case of monetary policy without a commitment to an inflation target and analyses various results, and part 7 presents simulations that demonstrate and analyze the results. The paper concludes with Section F, and is followed by six appendices – the first expands the New-Keynesian supply curve issue, and the others are technical.

Illustration no. 1: Inflation targets, actual inflation, one year inflation expectations and Bank of Israel interest rates, 1992-2005, percent



B. Review of the Literature

The issue of asymmetry in monetary policy has preoccupied researchers from both the theoretical and the empirical perspectives. At the theoretical level, the literature addresses two issues: (a) the presence and direction of the asymmetry, an issue that emerged while addressing the question of the reliability of monetary policy and the

---

[2] This framework is also presented by Woodford (2003), although it also includes the natural interest rate.

setting of targets. (b) the type of the objective function applied by the central bank to test the asymmetry. Most of the theoretical work was undertaken within the context of the quadratic objective function. The empirical review was discussed in several papers to be presented below.

1. Results of the asymmetric policy

The issue of asymmetric monetary policy emerged while addressing the impact of the reliability of monetary policy, an issue which occupied scholars following the inflation of the 1970s and 1980s, and the development of the inflation expectations theory in the early 1970s. One school of thought in the literature[3] claims that adopting a discretionary policy as opposed to a commitment policy leads to a persistent *inflation bias* where a time-consistency result is obtained. This is for two reasons: the central bank's desire to achieve higher than natural output and a certain indecisiveness in the policy applied to reduce inflation, due to the high price tag inevitably associated with such policy; this approach affects price and wage agreements that are established, in part, based on the public's inflation expectations. To remedy this inflation bias, Roggof (1985) proposed selecting central bank governors with a conservative approach to inflation.

Another school of thought that analyzed policy from the practical perspective argued the opposite – a *deflation bias* in the case of commitment or the adoption of decision rules (for example of the Taylor type).[4] This is due to the fact that central bank governors are judged by their success in combating inflation, and they therefore refute monetary policy's ability to moderate cyclical unemployment, even in the short term, and adopt an inflexible inflation target (or one that is less sensitive to the real activity). Evidence to support this contention was put forward by Mishkin and Posen (1997) with regard to Canada and England; other evidence emerges from central bank annual reports, including those of the European Central Bank (ECB).

To complete the picture, we must mention that the literature also includes a school of thought that believes that the banks have a stronger aversion to depression than to

---

[3] The pioneering work on this subject was that of Kydland & Prescot (1977), Barro & Gordon (1983), and later papers that proposed ways of reducing the inflation bias, for example Rogoff (1985), Walsh (1995), Persson & Taballini (1997), and Svensson (1997). For a description see also Clarida, Gali & Gertler (1999) (hereinafter – CGG).

[4] As far as I am aware, this argument was pioneered by Stanley Fischer (1994), and he was joined by others such as Laxton & Rose (1995), and Mishkin & Posen (1997). See also Nobel & Peel (1998) (hereinafter NP), who cites central bank annual reports.

an over-expansion of activity, leading them to adopt a flexible inflation target. Either way, all the aforementioned approaches present asymmetric preferences.

Little work has been done at the empirical-econometric level: Clarida & Gertler (1997) (hereinafter CG) reviewed Germany's monetary policy (before the European monetary union) within the context of a Taylor-rule policy decision, and found evidence of asymmetry – deflationary bias. Dolado et al (2000) reviewed this question by comparing four countries – the US, Germany, France and Spain – and found evidence of deflationary bias at various intensities. (Germany showed the strongest.) When reviewing the question with regard to asymmetry relative to output, they found symmetry in the three European states, while the US showed a more aggressive response vis-à-vis depression.[5] This may be due to the effect of historical traumas – hyperinflation in the case of Germany and the Great Depression in the case of the US in the 1920s. These papers used a quadratic objective function which is widespread in the literature (see discussion below). Ruge-Murcia (2001) (hereinafter – RM) reviewed this question with regard to Canada, Sweden and England using the Linex function and supply equations only (Lucas type), and he too found evidence of deflationary bias asymmetry. In another paper, RM (2001) reviewed asymmetry with respect to unemployment, also using a Linex function, and found asymmetry indicating that in the US and France, there is a greater aversion to higher-than-natural rates of unemployment than to inflation, whereas in Canada, Japan, Italy and England the assumption of symmetric behavior cannot be rejected. Using a non-quadratic model, Cukierman & Muscatelli (2003) conducted an empirical review of the question of asymmetry regarding the US, England, Germany and Japan and found asymmetry with respect to inflation (excessive aversion to inflation) during the process of building the central bank' credibility, but asymmetry vis-à-vis output after the credibility had been achieved (see also below).

The second theoretical issue is the form of the central bank's objective function used to test the asymmetry. This issue will be addressed below.

---

[5] One could ask whether the results for the US are contradictory. The review was performed such that, with respect to the deviation from the inflation target the question was examined by distinguishing the central bank response (interest rate) to deviations above and below the target, while no distinction for output deviation of this kind was made. As for the second question, regarding deviations from the output target, a distinction was made in the response of the interest rate between observations of a positive deviation from the output target against negative deviation, while no distinction was made for deviations from the inflation target. The results for the US are therefore not contradictory.

2. Quadratic objective function

The widespread method in the literature is the use of a quadratic function, for example to minimize the following objective loss function for two targets [6]:

$$MinL = \frac{1}{2}[(\pi_t - \pi^*)^2 + \alpha(y_t - y^n)^2]$$

where the first expression represents the actual deviation from the inflation target, and the second expression represents a real factor; in this instance, the deviation of output from the economy's natural or potential growth rate. α is the relative weight that the central bank attributes to deviations from the output target. If $\alpha = 0$, this means that the inflation target is inflexible and it does not take the business cycles into consideration. When $\alpha > 0$, the inflation target is flexible and takes the business cycles into account. It is worth mentioning that attempts to formulate the problem of policy by including a utility function of a representative agent were also made, but these models were unsuccessful due to the considerable gaps between the agents which make it impossible to analyze social welfare. Thus, for example, CGG (1999): if one particular group of workers suffers from a recession more than another (for example metal workers compared with professors), and there are incomplete insurance and credit markets, a representative agent will be unable to reflect the impact of business cycles on welfare.[7] Adopting the quadratic function therefore represents a practical and intuitive approach whereby the goal of monetary policy is to minimize the quadratic deviations from the target(s); Woodford (1998, 1999) even presents formal justification for this approach: he contends that it has quadratic approximation for utility-based welfare function as accepted in other cases.[8]

In addition to the above, there are additional reasons for the widespread use of the quadratic function: the first is that the function is contiguous and differentiated (in contrast, for example, to an absolute value function). The second reason is that it is symmetric, that is – that identical deviations above or below the target receive the same values in the quadratic function. The third reason is that it is convenient and intuitive, as it tests deviations from the target in the same way that volatility is tested

[6] The goal of economic policy is to increase the level of welfare. The role played by monetary policy in achieving this goal (as it has been generally accepted since the 1990s) is to protect price stability as the prime target, with some concern for the real side, either explicitly or implicitly, usually by stabilizing output around potential output. (See for example, Svensson 2003a). This approach reflects the prevailing view that this is the best way the monetary policy can contributes to increase welfare.

[7] CGG (1999) bring several examples of this approach, including: King & Wolman (1995), Carlstrom & Fuerst (1995),), Ireland (1996a), Aiyagari & Braun (1997 and Rotemberg & Woodford (1997).

[8] See also CGG (1999), p. 1668.

(variance or standard deviation). Svensson (2003)[9] regards this function as extremely convenient and intuitive, as when the target is achieved, deflation is also perceived to be undesirable due to concern of a liquidity trap and deflationary spiral (as occurred in Japan in the 1990s), and due to the fact that in other functions the substitution relations (between inflation and output) are more complex, both with respect to implementation by the decision makers and with respect to transparency for the public.

3. The disadvantages of quadratic objective function

3.1 Adopting the principle of certainty equivalence

One of the important limitations of the use of quadratic function is that it ignores the uncertainty surrounding inflation – an important factor for individuals in their business and financial planning, accounting for considerable part of the cost of inflation, as well as for central banks that typically have a natural aversion to uncertainty which is perceived to be an important factor in determining the interest rate.[10]

The quadratic function adopts the principle of certainty equivalence attributed to Tinbergen (1952) and Theil (1958) (hereinafter TT), who analyzed objective functions and policy instruments. According to these findings, the combination of quadratic objective function with a set of linear constraints, and assuming additive uncertainty[11], results in the creation of certainty equivalence when making interest rate decisions, implying that the higher moments (such as variance) of the target variables' distribution are unimportant.[12] Thus, if the disturbance is normally distributed (*iid*) so that its expectancy equals zero $(E_t u_t = 0)$, the distribution is not affected by any moment. Svensson & Woodford (2000) provide general proof in this direction, confirming that in the quadratic objective function used by central banks, the optimum

[9] Svensson (2003) also examines different objective functions that combine, in different ways, quadratic functions and a function with an absolute value.

[10] Chadha & Schellekens (1999) (hereinafter CS) present various quotations from central banks reports - Sweden, England and ECB - that reflect the impact of uncertainty on their caution when making interest rate decisions. Such caution can also easily be found in various Bank of Israel publications.

[11] Additive uncertainty means uncertainty about the real state of the economy; this is reflected in the disturbances' residual factor. Examples: shocks in inflation, in the output gap, in the natural unemployment rate, in the natural interest rate, and in problems of data measurement. In contrast, multiplicative uncertainty is uncertainty with regard to the impact of the monetary instruments (for example elasticity of the interest rate) on inflation and output. See CS (1999), Al-Nawaihi & Stracca (2002) and also Alghalith & Ardeshir who refer to production function by firms, but the idea is similar.

[12] The version according to CS is as follows: if x is a control variable and y is a state variable, $y = f(x) + e$, e represents additive disturbances with zero mean, and f a determinist function. If the certainty equivalent is valid, the optimum value of x (for example, the value that minimizes y) is independent in any moment of the distribution function of e.

policy is not affected by the uncertainty of the state of the economy. We must also mention that Chadha & Schellekens (1999) (hereinafter CS) show that the principle of certainty equivalence in the case of *additive uncertainty and normal distribution of the disturbances* is also valid regarding a general group of convex loss function, and not only for quadratic function, even if the preferences are asymmetric – a result that contradicts our findings.

From an academic perspective, it would therefore appear that the use of the quadratic function means that the optimum policy takes into account the inflation expectation and ignores uncertainty. In contrast, in practice, central banks that adopt targets are naturally concerned about additive uncertainty (for example, regarding inflation or output), so that this function does not reflect the reality. Moreover, since in assuming certainty equivalence the degree of risk aversion is irrelevant, the gradual and cautious policy responses are also unimportant, so that the policy instrument can rapidly and completely neutralize any shock in the target variable (see also Al Nowaihi & Stracca (2002), hereinafter NS). In practice however, this is not the situation in the real world. Central banks frequently adopt gradual policies and are not indifferent to additive uncertainty (see also Blinder, 1998). Theoretical models as well as a practical approach adopt a smoothing of interest rates (which reflects gradual policy).

CGG (1999) also address the weakness of the quadratic objective function (p. 1668): "One limitation of this approach, however, is that the models that are currently available do not seem to capture what many would argue is a major cost of inflation, the uncertainty that its variability generates for lifetime planning and for business planning …".

3.2 Other criticism of the quadratic function

In addition to adopting the principle of certainty equivalence in the case of additive certainty, the quadratic function has been criticized due to its significance and application in conditions of multiplicative uncertainty. Criticism has also been leveled when the assumption concerning the normality of the disturbance factor is removed. We shall detail some of the criticisms here:

A. The direction and rate of the deviations is unimportant – the quadratic objective function implies that a 4% deviation *below* the target is as important as a 2% deviation *above* the target during four periods. This criticism is noted by CS (1999) who comments on William Brainard (1967) (cited there), claiming that deviations above and below the target should not be considered equally important.

B. Theil (1966), who together with Tinbergen (1952) was one of the first to formulate a framework for analyzing monetary policy in the 1950s, contended that there is no particular reason to assume that the loss function will always be quadratic; it is convenient for an initial estimate, but the use of a more general function produces complex results that cannot be managed, and for this reason the quadratic function is so popular (from CS 1999).

C. Blinder (1997) claims that the academic world tends to use the quadratic function for reasons of convenience and mathematic convention, without considering the implications which are not harmless; central banks and academics must devise a more serious approach to the type of the loss function (from CS 1999).

D. Goodhart (2001), based on the practical experience of central banks, contends: the meaning of this function is that as the deviation from 1 in absolute values increases, the faster the increase in the loss or correction, and the reverse is the case when the deviation is less than 1. It is unreasonable and incorrect that the reaction should be proportionately higher the greater the deviation from the target; it is unclear why, for example, the change in the interest rate will be one percent in respect of a one percent deviation from the inflation target, but four percent when the deviation is two percent (9% if the deviation is 3%, and so on). Goodhart contends that the reaction should always be in the same proportion to any deviation rate, and the function should therefore be an absolute value. In contrast, Svensson (2003a) contends that an absolute value that attributes the same degree of importance to each rate of deviation from the target lacks proportionality, and is therefore unreasonable. Thus, a deviation of 0.1% from the target is just as significant as a 2% deviation from the target.[13]

E. Quadratic function means that the central bank's response to a 1% deviation when the inflation rate is 15%, for example, is the same as the response to that deviation when the rate of inflation is 2%. NS (2002) contend that such behavior is unreasonable, and base their contention on risk aversion and economic psychology from the findings of Kahneman-Tversky: typical risk aversion shows that the loss of a 1% deviation when the inflation target is 2% is greater than the same deviation when the inflation target is 15%.

[13] The loss function derivative relative to the inflation gap is fixed, for example 1, and this is correct for any size of deviation from the target.

F. When multiplicative uncertainty is given (for example, when the flexibility of inflation and output to interest are unknown), there is a broad consensus that the principle of certainty equivalence is invalid, as the higher moments of the distribution are significant, and the quadratic function is therefore inappropriate. (See also Simon, Hall, et al (1999) regarding England.)

G. NS (2002) found that if the disturbances do not follow a normal distribution, the principle of certainty equivalence is invalid, and in this case an objective function that assumes quadratic preferences is not harmless.

4. Other functions

Criticism of the quadratic function resulted in several studies that reviewed non-quadratic behavior functions. Svensson (2003b) analyzes different forms of objective function – quadratic, absolute value and various combinations of the two. NS (2002) explain why we should depart from the widespread assumption of quadratic loss function, using economic psychology, particularly in the work of Kahneman-Tversky, who base themselves on the assumption of certainty equivalence, that is – addressing, rather than ignoring, inflation fluctuations (convex function with increasing risk aversion), and review a comparison with quadratic function by using simulation.[14]

The following papers reviewed asymmetric preferences solely from the theoretical perspective, using a Linex function, which include the quadratic case as a special case. Nobay & Peel (1998) compare optimum behavior in the case of inflation target under commitment versus under discretion in a model of Lucas-type supply function (which is not New-Keynesian), presenting results through the use of simulations. They find that deflationary bias is feasible in the discretion case. CS (1999) also analyze the question of the use of quadratic function compared with asymmetric alternatives at the theoretical level, using a Linex function, and making a distinction between additive risk and multiplicative risk (assuming normal distribution of the disturbances). The analysis is of a case where the aversion to positive deviation from the inflation target is greater than the aversion to negative deviation, and they also find that when using the alternatives, a lower rate of inflation and a higher interest rate is obtained compared to the quadratic function.

---

[14] The function analyzed there is of the $L = x^{\beta}$, where x signifies the deviation from the inflation target, and $2 \geq \beta \geq 0$; when $\beta = 2$, the function is quadratic. Their findings show that symmetric preferences that are characterized by quadratic function require an interest rate that differs from the substitutions.

Using a Linex function, Ruge-Murcia (2001) reviews the asymmetry regarding inflation in Canada, Sweden and England, this time using an empirical review. (As far as I am aware, this is the only paper that performed an empirical asymmetric review of inflation using a Linex function to date.) Since the quadratic function is a special case of the Linex function, where the asymmetric parameter is zero, the paper conducts an empirical review of the model's asymmetric coefficient. The results show that the central bank in these countries has a greater aversion to positive deviation than to negative deviation. Ruge-Murcia (2001) also used this function to analyze asymmetry with respect to the unemployment target, conducting an empirical review of the G7 countries.

The work of Cukierman & Muscatelli (2003) reviewed asymmetry regarding output and inflation together, unlike previous studies which reviewed asymmetry for only one of them. Their economic model is New-Keynesian with demand and supply equations, when they analyze the discretion case, as we did. Their objective function is similar to the quadratic function, but allows a partial third derivative for the output gap and the inflation gap, and this possibility represents the asymmetry.[15] According to this model, they conducted an empirical review of four countries (England, US, Germany and Japan) and found asymmetry: an aversion to inflation which is above target, while building credibility (disinflation process), and when the credibility has been achieved – reverse asymmetry: an aversion to a decline in output.

Following Ruge-Murcia's (2001) paper, we will be reviewing the preferences of monetary policy at the theoretical level using a Linex function. The main difference between our work and that of Ruge-Murcia lies in the structural model of the economy: in his work, the economy is formulated in a single equation – the Lucas-type aggregate supply (AS) equation, where real activity is reflected in terms of the unemployment rate. Moreover, he formulates the rational expectations for unemployment and inflation rates in standard neo-clasical terms, rather then New-Keynesian terms, in which the current variables that enter the equation are those that were expected in the previous period. (See below and appendix 1.)

In this paper, the economy is formulated using two equations. One is the aggregate demand equation (AD or IS) formulated in terms of the expected output gap and

[15] Regarding the output gap, they assume that a third, negative derivative represents a greater loss than a positive derivative of the same size. Regarding the inflation gap, they assume that a third positive derivative reflects a greater loss than a negative derivative of the same size.

expected inflation. This equation was developed from an optimization of utility functions for the individual (the work of McCallum & Nelsen (1999), and Woodford (2003)), which is based on substitution between consumption and savings that is dependent on future income and the interest rate (to be expanded further on). The second equation is an aggregate supply (AS) curve, or an expectations-augmented Phillips curve, where current inflation depends on the output gap and on inflation expectations today for the forthcoming period. This equation was also developed from the optimization of a business corporation in conditions of a competitive monopoly, implying that prices are not treated as given to the company, but are set by the company based on marginal cost. (See for example Fischer (1977) and Woodford (2003.)) Our paper reviews the asymmetry in the Linex function under discretion only; the case of commitment to inflation targets is not presented here.

Before we present the model, we will present the Linex function which is not well known.

C. DESCRIPTION OF THE LINEX FUNCTION

Varian (1974) proposed an asymmetric loss function (with respect to real-estate appraisal issues), in which the behavior is linear on one side of the zero and exponential on the other side (hence the name Linex – Linear Exponential). Other papers on the subject of optimal projection also adopted this function.[16] Nobay & Peel (1998) [17], and CS (1999) used Linex from the theoretical perspective when addressing a central bank's behavior function, without empirical tests. As noted, Ruge-Murcia (2001) used this function with respect to inflation and output, with an empirical test.

The Linex function is described as follows:

$$(1) L = (e^{ax} - ax - 1)/a^2$$

where e is the exponent (exp.), x, in our instance, describes the deviations from the inflation target, and a describes the degree of asymmetric preferences: in the case of deviation from the target the percentage deviation is important as well as its **direction**. Thus, where a is positive, there is a stronger aversion to a positive ($x > 0$) than to a negative deviation from the target, that is – there is less tolerance towards over-the-target inflation, and the reverse is the case where a is negative. Illustration no. 2

[16] Zellner (1986) and Cheristoferson & Diebold (1997).
[17] As far as I am aware, they were the first to use this function to analyze monetary policy.

describes the Linex function for $a = 1$. The horizontal axis describes the rate of deviation from the inflation target, and the vertical axis demonstrates the loss. The illustration shows that for a deviation of +1.5 from the target, the loss is approximately 6, whereas for a negative deviation at a similar rate (-1.5), the loss is only approximately 1. Moreover, in a positive deviation, behavior is exponential whereas in a negative deviation behavior is linear approximately. The reverse is the case where a is negative – the risk aversion in the case of a negative deviation is greater than for a positive deviation, and the curve will be described in the opposite form (Illustration 2.1). The set of preferences is therefore described by the asymmetry factor a, and it is built into the function.

Illustration no. 2: Asymmetric preferences in the Linex function

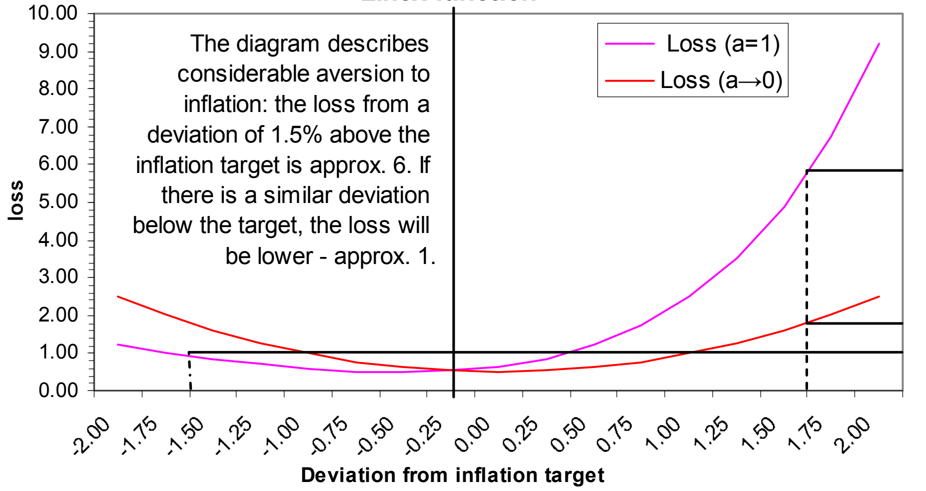


Illustration no. 2.1: Asymmetric preferences in the Linex function

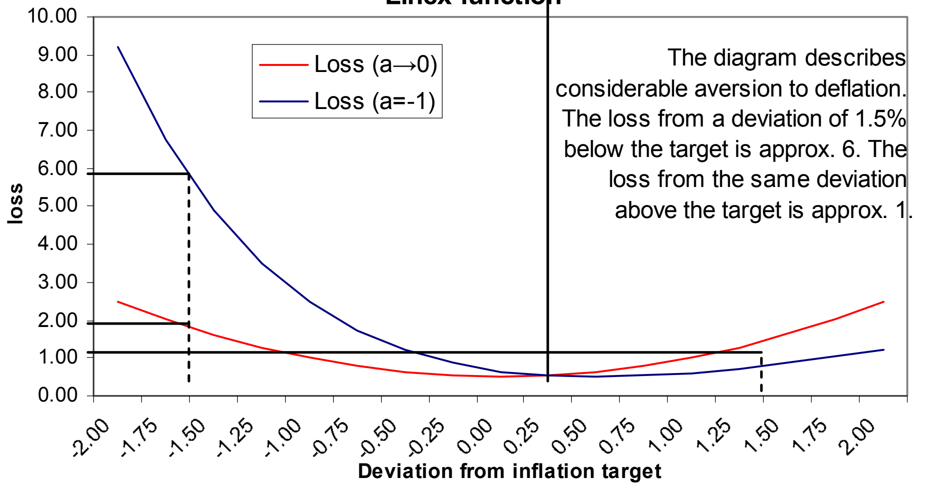
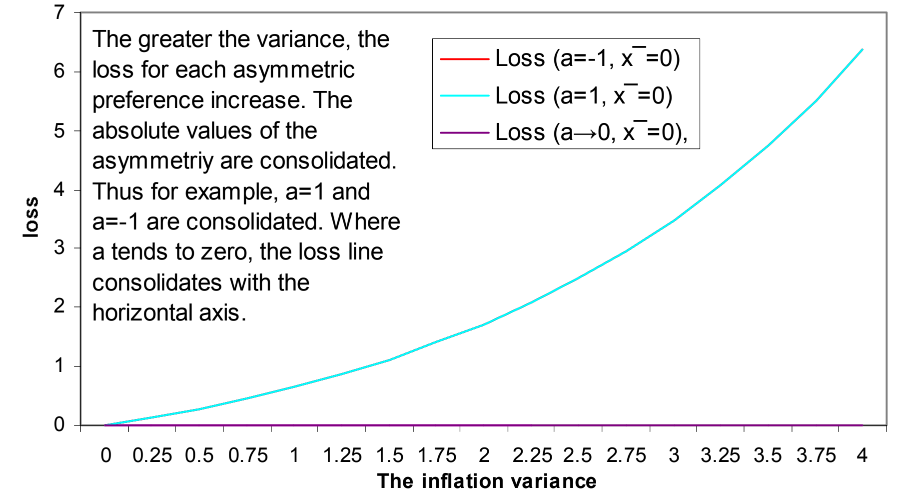


Illustration no. 2.2: The loss as a function of the inflation variance



For comparative purposes, illustration 2 also describes the quadratic function, which is an individual case within the Linex function, where $a \to 0$.[18] The quadratic function only takes into account the percentage deviation, and does not distinguish the direction of deviation; consequently, the loss for a deviation of 1.5% is identical in both directions. Where $a > 0$, the loss in the Linex function is greater than in the quadratic function if the deviation is positive, and less if the deviation is negative. In the central bank's set of preferences described by positive asymmetric behavior ($a > 0$), the central bank in response, attributes greater weight to a positive deviation of inflation from the target than to a negative deviation, and it can therefore be expected that the inflation rate that is eventually achieved will be below target rather than above the target, so that the loss will be smaller.

Another feature of the Linex function is its treatment of the uncertainty surrounding the target variable. If x is given by the process: $x = \bar{x} + \varepsilon$, where $\bar{x}$ is the expectation conditional in the process, and the distribution of the disturbance ε is conditionally normal with variance σ^2, Christoffersen & Diebold (1994) show that the predicted loss expectation of (1) is:

$$(2) E(L) = (e^{a(\bar{x}+\frac{a}{2}\sigma_\pi^2)} - ax - 1)/a^2$$

[18] By using L'Hopital rules and deriving the function according to a, we obtain a quadratic Linex function where $a \to 0$:

$$\lim_{a\to 0} L = (e^{ax} - ax - 1)/a^2 \Rightarrow \lim_{a\to 0}[(xe^{ax} - x)/2a] \Rightarrow \lim_{a\to 0}[(x^2 e^{ax})/2] = x^2/2.$$

(See also Nobay & Peel,1998), meaning that the optimum expectation of x is no longer given only by the average $\bar{x}$ (as in the case of a quadratic function), but by $\bar{x}+\frac{a}{2}\sigma_{\pi}^{2}$.

In other words: unlike the quadratic function, the Linex function takes uncertainty into account and it is costed by the loss function both by the **degree** of risk aversion, expressed by a, and the **amount** of risk σ^2. Where $a \rightarrow 0$ a quadratic function is obtained, and the degree (and thus also the amount) of uncertainty becomes irrelevant. Illustration 2.2 demonstrates the loss sensitivity (the vertical axis), at different levels of the target variable volatility σ_{x}^{2} (horizontal axis), and compares the asymmetry where a= -1, 0, 1. Where the variance is zero the quadratic and asymmetric preferences converge, and the loss is zero; where the variance is positive, the loss increases exponentially even if the deviation expectancy does not vary[19]. This result appears to be typical of the behavior of central banks: in countries which have a higher variance of the inflation rate, such as emerging economies (which maintain an inflation target), monetary policy will be less tolerant (deflationary bias) compared to stable, western economies.[20]

D. LINEX OBJECTIVE FUNCTION IN A NEW-KEYNESIAN MODEL

The basic model is a New-Keynesian dynamic model, with temporary nominal price rigidities. This model has become popular in recent years for analyzing monetary policy. The model consists of two equations: the aggregate demand (AD or IS) equation, and the aggregate supply (AS) equation – an expectations-augmented Phillips curve. The inflation and output gap variables are forward looking, and there is no inertia or lagged dependence of these variables. Two main policy types can be presented in this model: one is policy under discretion, and the other is policy in a

---

[19] Formally: where $a \rightarrow 0$, the variance is vanished in equation (2). For the asymmetric case, we will derive according to the variance:

$$\frac{\partial(EL)}{\partial(\sigma_{x}^{2})}=\frac{1}{2}e^{a(\bar{x}+\frac{a}{2}\sigma_{x}^{2})})>0\forall a.$$

The result is greater than 0 for every a. The second derivative is also greater than zero for every $a \neq 0$:

$$\frac{\partial^{2}(EL)}{\partial(\sigma_{x}^{2})^{2}}=\frac{a^{2}}{4}(e^{a(\bar{x}+\frac{a}{2}\sigma_{x}^{2})})>0\forall a \neq 0.$$

[20] Mishkin (1997), for example, contends on p. 78 that the impact of a change in the exchange rate on the prediction for inflation is stronger in emerging economies, where inflation and variance are high, and weaker in western countries.

regime of commitment. In this paper we will present only the discretionary case. In this model, as in CGG (1999), there is no money, and there is therefore no need for an equation with money (the money is endogenous to the interest rate).

5. Structure of the economy in the New-Keynesian model

5.1 The aggregate demand (AD) equation

The New-Keynesian model is the model presented in CGG's paper (1999). The AD (or IS) equation is as follows:

$$(3.1) x_t = E_t x_{t+1} - \varphi(i_t - E_t \pi_{t+1}) + g_t$$

where the output gap, x_t, is the difference between current output, y_t, and natural output, y^n (the output obtained if prices and wages are fully flexible), $x_t = y_t - y^n$; it is negatively affected by the real expected interest rate, which is the difference between the nominal monetary interest rate i_t and the inflation rate expected today for the forthcoming period, $E_t \pi_{t+1}$. We also assume that the coefficient is $\varphi > 0$. Monetary policy affects the output gap in the short term through the interest-rate instrument. The disturbance factor, g_t, indicates shock – government or private induced – on the aggregate demand side (a shock that causes movement of the AD curve). We should also note that the coefficient for the output gap in the forthcoming period, x_{t+1}, is 1, due to the fact that this is the result obtained from the optimization of the household in the savings-consumption model, where government expenditure is deducted from output (see for example, McCallum & Nelson 1999 [21] and Woodford 2003). In practice, this is an equation that indicates the household behavior equation, and it implies that choosing the quantity of consumption (demand) today depends on the real, expected interest rate, and on the expectations for consumption in the forthcoming period. This means that a higher level of anticipated output in the future increases demand today due to the smoothing out of consumption. (See also CGG 1999.) This can also be viewed as an income effect. The negative effect of the real rate of interest indicates the inter-temporal substitution of consumption. One of the important results of this optimization is that current output depends not only on the

[21] The accumulation of capital and investments is not addressed as part of this optimization, as detailed by McCallum & Nelson (1999), and as presented by CGG (1999) as well. Including them in the AD equation would add more details to the equation, but would not change its basic qualities: that aggregate demand responds negatively to an increase in the real, expected interest rate and positively to an expected increase in output.

current, real interest rate, but also on the output expected in future – a factor that does not appear in the traditional IS equations.

At this point we should note that the equation is described in terms of **deviations** from the inflation target: if, for example, the inflation target is 2%, and the expected inflation rate is 3%, then $E_t \pi_{t+1} = 1$. In a steady state there is no deviation from the inflation target, so that $E_t \pi_{t+1} = i_t = 0$. In addition, in a steady state the expected inflation should equal the inflation target, and the output gap during the current period as well as during the forthcoming period, should be zero; however, there is a natural real interest rate, r^n, which is usually positive, but does not appear in the equation. Thus, this equation should be interpreted such that the interest rate, i_t, is beyond the natural interest rate r^n and over and above the inflation target, where the central bank interest rate R_t is $R_t = i_t + r^n + \pi^*$.[22]

In this equation the output gap depends on the entire future path of the expected inflation and monetary interest rates. If we run forward one period we will receive:

$$(3.2) x_{t+1} = E_t x_{t+2} - \varphi(i_{t+1} - E_t \pi_{t+2}) + g_{t+1}$$

and through iteration of periods forward we will obtain the following infinite path (see Appendix 2):

$$(3.3) x_t = \sum_{j=0}^{\infty} \varphi(E_t i_{t+j} - E_t \pi_{t+j+1}) + \sum_{j=0}^{\infty} E_t g_{t+j}.$$

Equation 3.3 shows that the current output gap depends not only on the real, expected interest rate for one period ahead and on the current demand shock, but on the entire future paths of the expected inflation and monetary interest rate. Thus, even the monetary policy must take the entire future path into account, and not only the current interest rate and inflation for the forthcoming period. It is worth noting that although the expected output gap is vanished in equation (3.3), it apparently has an important effect on the dynamics of the economic system. (See also McCallum & Nelson (1999), CGG (1999) and Woodford (2003).)

The disturbances are also serially correlated to the past: $g_t = \mu g_{t-1} + \varepsilon_t$ where $1 > \mu > 0$ and only the current component, ε_t is unknown. This is an AR(1) process, implying that the disturbances cannot diverge after the first shock. There will be a

[22] The Woodford version (2003) of the New-Wicksellian model takes this into account and at this point in the equation places the natural interest rate r^n as well, so that the central bank interest rate i_t also includes the natural interest rate r^n.

gradual convergence of the disturbances until they dissipate entirely. However, if the appropriate policy is not adopted, these disturbances will cause inflation to diverge. Moreover – in this equation, the disturbances are the only variable with inertia. The endogenous variables have no inertia from the past and they are forward looking.

5.2 The aggregate supply (AS) equation

The aggregate supply equation:

$$(3.4)\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + \upsilon_t$$

is an equation that describes the inflationary process. This is a traditional expectation augmented Phillips curve that describes a positive relationship between the output gap and the inflation rate, but the expected inflation rate is also included in the equation;[23] this is in view of criticism leveled by Friedman (1968) and Phelps (1972), who contend that workers are only interested in real wages, not nominal wages. We assume that the coefficients are $\lambda > 0$ and $\beta \in (0,1]$, (where β is in fact a future discount factor).

The dispute in the literature concerns the expectation model for price adjustment. In this model, the expectations are New-Keynesian, implying that the current inflation rate is determined according to expectations **today** of inflation in the forthcoming period. The inflation anticipated today is obtained from an optimization of a firm in monopolistic competition, where at any point in time, the firm determines the price it can charge in an effort to maximize its profit, independently of the price set in the past, according to the Calvo (1983) model: for each period, each firm has a probability (of less than 1) to change the price of its products where another group of firms does not change the price. (See also CGG 1999 and Woodford 2003. For further information see also Appendix A.) We also wish to note that in empirical tests regarding the US and England, Sbordone (1998, 2002) found price adjustment behavior that justifies the Calvo model rather than the neo-classical model, whereby inflation during the current period is determined according to expectations in the previous period regarding the current period, that is – $E_{t-1}\pi_t$ (for example Sargent & Wallace 1975). This difference is material rather than semantic. According to neo-classical theory, fluctuations in the output gap must be unexpected, as only unexpected fluctuations in the nominal

[23] See Dornbusch & Fischer (1987), pp. 503. This curve is built assuming that the price increase rate equals the wage increase rate, and that there is a fixed markup regarding the other expenses. Nevertheless, the disturbance v_t also takes into account the price-changes of other production factors.

expenditure has nay impact on real activity. It follows that only an unexpected monetary policy response can have some form of impact on real activity; and all this only for one period, that is – no further than the period in which the rigid prices remain static. Any impact later on will be on prices, and not on real output. An unexpected monetary policy shock must therefore be purely transitory, and the AS curve tends to be inflexible (and therefore neo-classical). In contrast, New-Keynesian theory (Phelps 1978; Taylor 1979a, 1980) assumes that price adjustment occurs over a long period and as a result a continuous change develops in nominal expenditure which has an on-going effect on real activity. Price levels prior to the monetary shock may continue to affect real activity even after the new shock, even if the general price level has already changed since the shock. The model is thus reconciled with on-going fluctuations in real activity due to a monetary shock, and the supply curve tends to be more flexible or have a horizontal tendency (and therefore New-Keynesian).

In this equation (3.4) there is no inertia of inflation (dependence on past inflation) and the rate of inflation depends solely on current economic conditions and those predicted for the future. Firms set the (nominal) prices of products based on their future marginal price expectations; the variable x_t perceives changes in the marginal price resulting from changes in excess demand. In contrast, the shock υ_t is "cost push" that includes everything, except pressure from the demand side, that may have an unexpected impact on the marginal cost (for example, an increase in real wages over and above equilibrium, or an increase in the oil prices).

By forward iteration, we realize that the inflation rate depends on the entire future trajectory of the output gap and disturbance, and not only on the forthcoming period. If we run the equation (3.4) one period forward, we obtain the following:

$$(3.5)\pi_{t+1} = \lambda x_{t+1} + \beta E_t \pi_{t+2} + \upsilon_{t+1}$$

and by forward iteration of all the periods we obtain the infinite trajectory as follows (see Appendix 3):

$$(3.6)\pi_t = \sum_{i=0}^{\infty} (\lambda \beta^i E_t x_{t+i} + E_t \upsilon_{t+i}).$$

As Fuhrer & Moore (1995) show, the disturbance may create variance in the inflation that is created independently of changes in the excess demand. Nevertheless, the disturbances are correlated to the past and they obey $\upsilon_t = \rho\upsilon_{t-1} + \varsigma_t$ where $1 > \rho > 0$ and only the current component, ς_t, is unknown. This means that the disturbances do not diverge but dissipate gradually, and it is the graduality which in fact reflects price rigidity, resulting in the persistence of the inflation. The inertia in this equation therefore exists only in the disturbances.

The two equations of the model – equations 3.1 and 3.4 – together tell us that monetary policy, that determines the monetary interest rate, affects the real interest rate in the short term due to temporary price rigidities, and thus affects aggregate demand (i.e. – the output gap) which in turn affects inflation. The iterations also show that the rate of inflation depends on the entire future path of the output gap, which in turn is affected by the future path of monetary interest rates and the expected inflation rate (that is by the anticipated, real rate monetary interest rate). The emphasis here is that current output and inflation are determined by their entire future path, so that monetary policy must also look towards the entire future interest rate path. The importance of the credibility of monetary policy therefore follows, becoming a key issue. As we have already noted, the case of commitment is more credible than that of discretion, and the convention is that in the case of discretion, the policy suffers from inflationary bias.

Since the monetary policy instrument is the interest rate and not the money supply, in this situation defining money market equilibrium conditions is not critical (see CGG 1999).

5.3 The objective function

The central bank's problem is to choose an interest rate path in an effort to guide the target variables – inflation rate and the output gap; as a mathematical formulation – to reduce the loss function to a minimum, where the general formula is:

$$\frac{Min}{\{i_t\}} E_t \sum_{t=0}^{\infty} \beta^t L(\pi_t, x_t)$$

subject to the behavioral constraints in equations 3.1 and 3.4. This formulation contains two objectives, an output gap and an inflation gap relative to the target – a common formulation in the literature – but the transition from here to a formulation of one objective only is simple and does not entail the loss of generality. The endogenous variables – current and anticipated – are defined in terms of deviation from the target.

The new concept presented in this article is the integration of the Linex objective function with the New-Keynesian model, instead of the quadratic function. We should recall that the Linex function includes the quadratic function as a special case. The Linex objective function is:

$$(3.7) E(L) = E\{\frac{1}{a^2}[e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} - a(\pi_t) - 1] + \frac{\alpha}{2}(x_t)^2\}$$

subject to the constraint of equation (3.4):

s.t.: $(3.4) \pi_t = \lambda x_t + \beta E_t \pi_{t+1} + \upsilon_t$

where x_t is the output gap. The inflation variance is expressed in the objective function specifically by the factor σ_π^2. α reflects the weight that policy attributed to the output gap target, and $\frac{1}{\alpha}$ is the weight attributed to the inflation target. $\alpha \in [0,1]$ [24]. Where $\alpha = 1$, the weight attributed by the policy to both targets – inflation and output – is the same, and where $\alpha = 0$ the policy only operates to achieve the inflation target (see also Svensson, 2003). The factor a is the index of the asymmetry. Where $a > 0$, the asymmetry is positive relative to the target, and in other words, the aversion to inflation is strong, and the reverse where $a < 0$. Where a tends to zero, the function becomes quadratic, that is – certainty equivalent exists, and there is therefore no reference to the inflation variance. The objective function in this formulation (loss function) places a Linex function relative to the inflation target and a quadratic function relative to the output target. The paper only reviews asymmetry relative to inflation and not to output; this is also in line with CG (1997), Dolado et al (2000), and RM (2001, 2002) in which one target variable was tested, and the second target variable remained fixed ("symmetric"). For simplicity's sake, the function relative to the output target will therefore be quadratic.[25]

I will now proceed to the main task at hand – a theoretical analysis and development of the model to obtain the path of the endogenous variables – inflation rate and output gap – and the interest rate path. As we are discussing policy, where the issue of credibility is critical, as contended by Kidland & Prescott (1977) as early as 1977 (they were awarded the Nobel prize for this work in 2004), a distinction must be made between the case of policy under discretion and commitment policy, as undertaken by CGG (1999) and Woodford (2003).

---

[24] Later we will see that where $\alpha = 0$, no defined solution from the technical perspective is obtained, so that in practice α ε (0,1). From the economic perspective $\alpha = 0$ may exist, implying a rigid inflation target. Where α tends to zero, a defined solution is obtained from the technical perspective as well.

[25] Positioning of a Linex objective function with two target variables is in the following form:

$$E(L) = E\{\frac{1}{a^2}[e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2} - a(\pi_t) - 1] + \frac{\alpha}{b^2}[e^{b(y_t + \frac{b}{2}\sigma_\pi^2)} - b(y_t) - 1]\}$$

E. MONETARY POLICY UNDER DISCRETION

There is a key difference when dealing with the problem of monetary policy between policy under discretion as described in this paper and commitment policy, which is not presented here. In the discretion case, the central bank believes that its actions during period *t* have no impact, in terms of the objective function, beyond the coming year $(T \geq t+1)$, or on the public's expectations for the forthcoming period ($E_t\pi_{t+1}$). Consequently, the expectations for inflation in the discretion case are exogenous and are given to policy when solving the optimization problem. **In each period,** the central bank chooses the target variables and the interest rate (x_t, π_t, i_t) in an effort to optimize the objective function *L* (minimum loss). In contrast, in the case of commitment, the policy affects expectations so that the expectations are endogenous and the optimization is for the **entire future path**. It should be emphasized that in both cases, policy responds to the changes in the state of the economy. The difference between them is in their impact on credibility as perceived by the public: the public knows which method the bank will choose, and builds its expectations accordingly. In the case of policy under discretion, the public knows that the central bank creates a new optimization for each period; the central bank therefore has no incentive to change the policy unexpectedly, even though it can do so, and thus the policy is typically "time consistent". In contrast, in the case of commitment, the commitment to the future path motivates the policy to perform optimization for the entire future path and to converge towards equilibrium; it is this commitment that makes the policy credible, even though in this case too the policy may change in line with changes in the state of the economy.[26]

6. Optimization

We will now present the Lagrange multiplier where γ is the Lagrange constraint:

$$(3.8) E(L) = E\{\frac{1}{a^2}[e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)} - a(\pi_t) - 1] + \frac{\alpha}{2}(x_t)^2 + \gamma(\pi_t - \lambda x_t - \beta E_t\pi_{t+1} - \upsilon_t)\}$$

we will derive, according to the objective variables x_t, π_t. Policy under discretion has no effect on expectations, but the expectations are given to such policy where it performs optimization, so that in this sense they are exogenous to the policy (unlike

[26] See for example CGG (1999), p. 1671, Woodford (2003) pp. 471, 487 as well as Barro & Gordon (1983), pp. 595-597.

the case of commitment). The outcome of the optimization is that the public views the optimal behavior of the central bank from which the inflation rate during the current period is also derived, and determines its expectations for the future accordingly in a rational manner.

First order conditions:

deriving (3.8) with π_t yields:

$$\frac{\partial E(L)}{\partial \pi_t} = \frac{1}{a^2}(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} * a - a) + \gamma = 0$$

$$(3.9) \Rightarrow \frac{1}{a}(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} - 1) = -\gamma$$

deriving with x_t yield (3.10):

$$\frac{\partial E(L)}{\partial x_t} = \alpha x_t - \gamma\lambda = 0$$

$$(3.10) \Rightarrow \gamma = \frac{\alpha}{\lambda} x_t$$

combining (3.10) with equation (3.9) gives (3.11):

$$(3.11) x_t = -\frac{\lambda}{\alpha a}(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} - 1).$$

Equation 3.11 expresses x_t as a function of π_t in the optimum. This equation tells us that where inflation is above target, excess demand must be reduced (by raising the interest rate), and the reverse when inflation is below target. The factor λ indicates the gain entailed in lowering the rate of inflation at the price of losing one unit of output, and factor α indicates the weight attributed to the output target. The optimization also depends on the inflationary uncertainty and the weight attributed to it, not only in the inflation expectation, and as the variance increases, excess demand must be lowered even more to achieve the inflation target, for each $a \neq 0$ (positive or negative). *In other words: the substitution ratio between inflation and the output gap now also takes into account inflationary uncertainty – a variable that is not present in the quadratic function.*

To compare this with the common quadratic function, let us see what happens to equation (3.11) where $a \to 0$. We will use L'Hopital's rule:

$$\lim_{a \to 0} x_t = -\frac{\lambda}{\alpha}(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} .(\pi_t + a\sigma_\pi^2) = -\frac{\lambda}{\alpha}\pi_t$$

and this is also the result obtained by CGG (1999).

Placing (3.11) in the supply equation (3.4) yields:

$$(3.12)\pi_t = -\frac{\lambda^2}{\alpha a}(e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)} - 1) + \beta E_t\pi_{t+1} + \upsilon_t .$$

The result is that inflation depends on expectations and disturbances, and in the non-quadratic case – on the inflation variance and asymmetry as well,[27] **and this is the inflation rate that policy chooses as the optimum.** To solve the equation (3.12), we must first see what $E_t\pi_{t+1}$ is.

6.1 The process of creating expectations for inflation $E_t\pi_{t+1}$

In the case of policy under discretion, the expectations for inflation are exogenous, and through optimization the central banks regards the expectations as given, as seen in equation (3.12); the optimization of the policy tells us that inflation will be as described on the right hand side of equation (3.12). If this is the case, from the private sector's perspective rational expectations mean that this process should also describe the expected inflation rate. Moreover, in the Markov process, the process of expectations for inflation is uniform for all periods.[28] We will therefore place $E_t\pi_{t+1} = \pi_t$ in equation (3.12):

$$(3.13)\pi_t = -\frac{\lambda^2}{\alpha a}(e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)} - 1) + \beta\pi_t + \upsilon_t$$

and after rearranging terms we obtain:

$$(3.14)1 - \frac{\alpha a}{\lambda^2}[\pi_t(1-\beta) - \upsilon_t] = e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)} .$$

Using logs gives:

$$(3.15)\frac{1}{a}\ln\{1 - \frac{\alpha a}{\lambda^2}[\pi_t(1-\beta) - \upsilon_t]\} = \pi_t + \frac{a}{2}\sigma_\pi^2 .$$

For the purpose of solving this equation we will use the following approximation:

$$(3.16)\ln(1 - \frac{\alpha a}{\lambda}\pi_t) \cong -\frac{\alpha a}{\lambda}\pi_t$$

This is correct as long as we limit the inflation gap to a few percent, even up to 20%. This is not the case of a disinflation process from tens of percent to 2%, as in such

---

[27] In the quadratic case, the following is obtained: $\pi_t^Q = -\frac{\lambda^2}{\alpha}\pi_t + \beta E_t\pi_{t+1} + \upsilon_t$

[28] See for example Woodford (2003), p. 471.

instance the approximation is incorrect. After placing approximation (3.16) in equation (3.15), we obtain:

$$(3.15')\pi_t[\frac{-\alpha(1-\beta)}{\lambda^2}]+\frac{\alpha}{\lambda^2}\upsilon_t=\pi_t+\frac{a}{2}\sigma_\pi^2$$

and after moving and organizing terms we obtain the inflation rate as a function of the exogenous variables – disturbances and variance – and the parameters as follows:

$$(3.17)\pi_t=\frac{\alpha}{\alpha(1-\beta)+\lambda^2}\upsilon_t-\frac{\lambda^2}{\alpha(1-\beta)+\lambda^2}{}^{a}\!/_{2}\sigma_\pi^2$$

where, in the quadratic case, the second term on the right-hand side of the equation fails.

Inflation during the following period will be:

$$(3.18')E_t\pi_{t+1}=\frac{\alpha}{\alpha(1-\beta)+\lambda^2}\upsilon_{t+1}-\frac{\lambda^2}{\alpha(1-\beta)+\lambda^2}{}^{a}\!/_{2}\sigma_\pi^2$$

and as we have already noted, the disturbances are correlated to the past and maintain $\upsilon_t=\rho\upsilon_{t-1}+\varsigma_t$, so that:

$$(3.18)E_t\pi_{t+1}=\frac{\alpha}{\alpha(1-\beta)+\lambda^2}\rho\upsilon_t-\frac{\lambda^2}{\alpha(1-\beta)+\lambda^2}{}^{a}\!/_{2}\sigma_\pi^2.$$

Equation (3.18) describes the process of generating inflation expectations. The greater the weight given to output policy, α, the higher the expectations for inflation will be. (We should also note that if β is closer to 1, then $\frac{\lambda^2}{\alpha(1-\beta)+\lambda^2}\cong 1$.)

In the quadratic case, the right-hand expression on the right-hand side of the equation falls, and we obtain:

$$(3.18^Q)E_t\pi_{t+1}=\frac{\alpha}{\alpha(1-\beta)+\lambda^2}\rho\upsilon_t=\rho\pi_t.$$

There are two expressions on the right hand side of equation (3.18): the first is the dynamic factor, which depends on the disturbances, and it converges to zero over time; the second expression is constant (assuming that the variance is fixed and independent of time); if $a\neq 0$, then compared with the quadratic function, there will be a fixed gap in the size of the constant term in equation (3.18) **throughout the convergence path, and in equilibrium.** To show this, Appendix 4 solves equation (3.12) as a long-term differential equation and shows that the following inflationary process takes place:

$$(3.19)\pi_t = \sum_{j=0}^{\infty} (\frac{\alpha}{\alpha+\lambda^2})^{j+1} \beta^j E_t \upsilon_{t+j} - \frac{\lambda^2}{\alpha(1-\beta)+\lambda^2} {a}/{2} \sigma_\pi^2$$

This means that the constant expression that appears in equation (3.18) also appears in equation (3.19), and the dynamic factor in equation (3.19) does not depend on the asymmetric factor, so that it is also correct for the quadratic function. This equation states that the dynamics of the inflation process depends only on disturbances. The inflation variance maintains a fixed gap between the quadratic function and the Linex function throughout the convergence path and at its end point, and it does not affect the dynamics of the convergence.[29] The size of the gap is determined by the asymmetric parameter and the variance: where a is positive, the expectations for inflation at equilibrium will be negative (that is – below the inflation target), and the reverse where a is negative. The greater a, the greater the deviation from the target, *but its size will remain fixed throughout the entire convergence path*.

6.2 Solution to equation (3.12)

Having found the process of generating the expectations for inflation in equation (3.18), we will now set about solving equation (3.12) so that it depends only on the exogenous factors – disturbance and variance and on the parameters. As the constant expression in equation (3.18) is always correct for each *t*, the dynamic will only reflect the dynamic expression in equation (3.18), and in fact equation (3.18^{Q}). That is – we will place $E_t\pi_{t+1} = \rho\pi_t$ in equation (3.12):

$$(3.20)\pi_t = -\frac{\lambda^2}{\alpha a}(e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)} - 1) + \beta\rho\pi_t + \upsilon_t$$

We will solve this equation in a manner similar to the solution used in equation (3.12), as in equations (3.13)-(3.17). The following solution, with respect to inflation as a

---

[29] I assume here that the variance is fixed. Nevertheless, if the variance is not fixed during the convergence process, than the variance will also be part of the dynamic process, and in this case the gap between the quadratic function and the Linex function will not be fixed. The solution in this interesting instance is not presented in this paper. Moreover, the exogenous assumption also entails certain problems: although the variance of the inflation depends on the exogenous disturbances and on other structural issues (such as wage agreements in the economy, monopolies and the minimum wage agreement), it can also be claimed that the variance also depends on the actual monetary policy (or the size of α) and the variance is therefore endogenous. Thus for example, the Taylor curve (Taylor 1979) (see also CGG 1999 and Svensson 2003) shows substitution between the inflation variance and the output gap variance, and the greater the emphasis in policy on achieving the output target (rather than the inflation target), so the inflation variance will be higher, and the variance in the output gap will be smaller, and the reverse. Nevertheless, I assume here, for simplicity's sake, that the inflation variance is exogenous.

function of the exogenous variables and parameters, is obtained for the optimal policy (see Appendix 5):

$$(3.21)\pi_t = \frac{\alpha}{\alpha(1-\beta\rho)+\lambda^2}\upsilon_t - \frac{\lambda^2}{\alpha(1-\beta)+\lambda^2}\frac{a}{2}\sigma_\pi^2$$

or:

$$(3.22)\pi_t = \alpha q \upsilon_t - \lambda^2 A \frac{a}{2}\sigma_\pi^2$$

where

$$(3.23)q = \frac{1}{\alpha(1-\beta\rho)+\lambda^2}$$

$$(3.23')A = \frac{1}{\alpha(1-\beta)+\lambda^2}.$$

The optimal policy described in equation (3.21) states that the dynamics of the inflation depends only on the disturbances, which appears in the dynamic expression on the right-hand side of the equation. The second expression on the right-hand side describes the inflationary uncertainty that I assume here as a fixed factor. In the quadratic case, only the first expression on the right-hand side of the equation is valid, and when the disturbances dissipate entirely, the deviations from the inflation rate will also equal zero. However, in the non-quadratic case, during the convergence process inflation will differ from the quadratic case by a fixed rate (which is determined by asymmetry and variance), and even in a state of equilibrium (where the disturbances dissipate entirely), inflation will be different from zero by a fixed rate.

The dynamic therefore depends only on the disturbances. In equation (3.12), before describing how inflation expectations are generated, we also obtained inflation as a function of disturbance (and inflationary expectations). It therefore seems that the inflation rate and the expected inflation rate (equation (3.18)) rely on disturbances. We should recall that this is an almost pure model of rational expectations (excluding the AR(1) process of disturbances). That is – there is no anchor for endogenous state variable that belongs to the previous period on which the inflation expectations could rely. It should be emphasized that in this model, the serial correlation in the disturbances is the cause of the persistence of the inflation and the output gap; this is contrary to a model with inertia, where there is no necessity to assume serial correlation of the disturbances, and where the expectations for inflation are also based on the past.[30]

[30] See for example CGG, p. 1692 and note 74, as well as Woodford (2003) pp. 486-487.

We can therefore only link the inflation rate, as well as the expectations for inflation, to the disturbances. Nevertheless, we should recall that the disturbances are exogenous, develop independently, and dissipate gradually in a Markovian process. If policy is optimal, **inflation will depend on disturbances, but not the reverse**.

In the case of policy under discretion, the central bank believes that its policy will have no effect beyond the period $T \geq t+1$, and any impact on expectations during the ensuing period $E_t \pi_{t+1}$. The Markovian process also depends only on disturbances, and does not depend on the previous inflation rate and the output gap. The question is what happens if policy is not optimal; for example, if a decision is made not to adopt the necessary interest rate, but a lower rate. (For example, not to increase the interest rate as necessary when there is an exogenous disturbance, in order to achieve output beyond the potential level, or due to political pressures that make it impossible to set a higher interest rate.) Clearly in this situation the policy generates inflation through demand (AD equation), and it is clear that the rational expectations, when the public sees the policy, should also adjust themselves upwards, unrelated to the disturbances. **The implication is that expectations should be linked to inflation as well, which is a result of policy, and not only to disturbances which have an independent process and are unrelated to policy.**

It therefore appears that by linking the expectation process to inflation, as in equation (3.18^{Q}), that describes a quadratic function but also appears in equation (3.18) for a Linex function, the expectations are determined using both of these together: by the disturbances[31] and by policy. If the expectations were not linked to inflation, there would be no rational expectations in the model, and policy could do as it pleased without any impact on the behavior of individuals – an illogical situation. Expectations are therefore given to policy when it optimizes, the public then sees it and adjusts it expectations accordingly. In this way we can overcome the difference between optimization in the theoretical model, which relies on exogenous disturbances, and real life where policy deviates from the optimum situation.

[31] Woodford (2003) also has a natural output gap, $x^* > 0$, which is not included in the CGG model. This means that for Woodford, the endogenous variables – the inflation rate and output gap – are dependent not only on disturbances but also on the fixed factor x^*, and there are two exogenous factors. In contrast, for CGG they depend solely on disturbances.

Since the inflation variance is always positive, the question remaining in equation (3.21) is with regard to the size of asymmetry a and its sign. If for example, the uncertainty is priced so that $a > 0$ (aversion to inflation), the optimal policy will tend to a rate of inflation that is below target.

Several results emerge from this analysis:

Result 1: *in the New-Keynesian model described above and in the Linex function – if we assume that variance is constant – the uncertainty does not affect the dynamics of the inflation rate and the convergence process, but only its level, compared with the quadratic case, during the convergence process and at equilibrium point.*

This means that between the different $a's$ (all other things being equal) there will be a fixed gap of inflationary expectations throughout the convergence path.

Result 2: *in the New-Keynesian model and in the case of policy under discretion, at equilibrium and with rational expectations, it is possible that inflation and it's expectations will be below the inflation target, if we take into account the uncertainty factor, as in the Linex function. This result contradicts the convention in the literature concerning an inflationary bias in policy under discretion, a convention that is based on the widespread use of quadratic objective function.*

The important conclusion that emerges here is that in the analytical sense, policy under discretion and a Linex function with positive asymmetry a > 0 produce optimum situation with deflation, contrary to the convention whereby the outcome of policy under discretion is an inflationary bias. In other words: the contention with respect to deflationary bias states, in fact, that a < 0 always, but in optimization a > 0 can also occur. The finding concerning inflationary bias in policy under discretion appears among many authors under use of a quadratic objective function,[32] so that this may be the result of using the quadratic function. We should also note that Nobay & Peel (1998) reached the same result, as our own, of the possibility of deflationary bias, in a model of supply function only with a Linex objective function. The possible result of deflationary bias in this instance can be explained by the fact that the Linex function adds another constraint to policy – that of inflation variance. Consequently, policy under discretion has less freedom to "run wild" than in the quadratic case, and this comes closer to a set of rules where policy has much less freedom.

Result 3: *In the New-Keynesian model that has been described and in the case of*

[32] For example in CGG (1999), Woodford (2003), Barro and Gordon (1983) as well as in Kydland and Prescott (1977).

policy under discretion, if there are differences in the inflation variance in different economies, then if the central bank governors have the same preferences, the rate of inflation at equilibrium will also be different even though the inflation target is the same. It follows that the nominal interest rates will also be different.

This result is due to differences in the inflationary uncertainty, which in turn result in gaps in the risk premium reflected in the factor a. In other words: in countries with a tradition of inflation and high inflation variance, the inflation rate at equilibrium, if $a > 0$, will be lower than in countries that do not have a tradition of inflation.

6.3 Target variables in terms of the exogenous variables

As we have shown regarding the target variable, π_t, I will now express the behavior of the output gap and inflation (x_t, i_t) in terms of the exogenous variables, which are the disturbances υ_t and inflation variance σ_π^2, and the parameters. Equation (3.11) which was obtained from the optimization can be arranged as follows:

$$1-\frac{\alpha a}{\lambda}x_t = e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)}$$

taking logs from both sides gives:

$$\ln(1-\frac{\alpha a}{\lambda}x_t) = a(\pi_t+\frac{a}{2}\sigma_\pi^2)$$

by isolating π_t we obtain equation (3.24):

$$(3.24) \Rightarrow \pi_t = \frac{1}{a}\ln(1-\frac{\alpha a}{\lambda}x_t) - \frac{a}{2}\sigma_\pi^2.$$

We will now define x_t only in terms of the exogenous variables. Placing the inflation equation (3.22) in equation (3.24) yields (3.25). As a reminder:

$$(3.22)\pi_t = \alpha q \upsilon_t - \lambda^2 A \, {}^{a}\!/_{2}\, \sigma_\pi^2$$

$$(3.25) \Rightarrow (\frac{1}{a})[\ln(1-\frac{\alpha a}{\lambda}x_t)] - \frac{a}{2}\sigma_\pi^2 = \alpha q \upsilon_t - \lambda^2 A \frac{a}{2}\sigma_\pi^2.$$

We will use the approximation of (3.16). We should recall that in the real economy the output gap is defined in terms of annual (or quarterly) growth rates, which are single percentages, so that this approximation applies, particularly regarding the output gap which is usually even lower than the rate of growth. After arranging the equation we obtain:

$$\Rightarrow -\frac{\alpha}{\lambda}x_t = \alpha q \upsilon_t + \frac{a}{2}\sigma_\pi^2(1-\lambda^2 A),$$

and we can now isolate x_t and obtain:

$$(3.26) x_t = -\lambda q \upsilon_t - \frac{\lambda}{\alpha}\frac{a}{2}\sigma_\pi^2(1-\lambda^2 A).$$

In the same way we can also express the expected output gap in equation (3.11') in terms of the exogenous variables and the parameters (see Appendix 6):

$$(3.26') E_t x_{t+1} = -\lambda q \rho \upsilon_t - \frac{\lambda}{\alpha}\frac{a}{2}\sigma_\pi^2(1-\lambda^2 A).$$

Equation (3.26) defines the endogenous variable x_t in terms of the exogenous variables – the disturbances υ_t and the inflation variance σ_π^2 – and the parameters. The second expression in the right-hand side of the equation depends on the asymmetric factor that determines the gap of the equilibrium between different $a's$. In a state of equilibrium, where $\upsilon_t = 0$, then if the function is quadratic, the output gap is also zero (the result obtained by CGG). However, even when the function is not quadratic ($a \neq 0$), the second expression depends on the discount factor β as well, and where $\beta = 1$ the factor inside the parentheses is zero ($A\lambda^2 = 1$) and the asymmetric function is then no longer important. Consequently, the closer that β is to 1, the less important the asymmetry, and at equilibrium there will be no considerable differences in the output gap between the different $a's$. (The discount factor is usually extremely close to 1; for example, in quarterly terms $\beta = 0.99$.) In this situation, the expected real interest rates in equilibrium will also be similar for the different $a's$ (AD equation). Since in the inflation equations (3.22) and inflation expectation equation (3.18) different $a's$ also determine different levels of equilibrium, the nominal interest rate will also be different for the different $a's$, so that it creates similar real rates of interest. We will see this later on in the simulations.

Result 4: *the closer the discount factor β is to 1, the smaller the impact of the asymmetry factor on the output gap at equilibrium; where $\beta = 1$ the asymmetry factor is no longer important with respect to the output gap at equilibrium and it will be zero at each a.*

The implication of this result, in the present model and in a state of equilibrium, is that if an asymmetric policy is adopted that creates a deviation from the inflation target, it has no real cost such as harming output, even though the nominal interest rate will be different from the interest rate in symmetric policy. In other words: asymmetric policy in a state of equilibrium is neutral from the perspective of the real variables.

6.4 The optimal response rule

We will now wish to find the rule of the optimal response. Let us start with equation (3.1), which describes the output gap. As a reminder:

$$(3.1) x_t = E_t x_{t+1} - \varphi(i_t - E_t \pi_{t+1}) + g_t .$$

Let us isolate i_t from equation (3.1):

$$\Rightarrow i_t = [E_t x_{t+1} - x_t + \varphi E_t \pi_{t+1} + g_t]\frac{1}{\varphi} .$$

From the optimum conditions (3.11), it emerges that –

$$(3.11') E_t x_{t+1} = -\frac{\lambda}{\alpha a}(e^{a(E_t \pi_{t+1} + \frac{a}{2}\sigma_\pi^2)} - 1)$$

substituting the optimum condition (3.11) for x_t and for $E_t x_{t+1}$ gives:

$$(3.27) i_t = -\frac{\lambda}{\alpha a \varphi}(e^{a(E_t \pi_{t+1} + \frac{a}{2}\sigma_\pi^2)} - 1) - (\frac{-\lambda}{\alpha a \varphi})(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} - 1) + E_t \pi_{t+1} + g_t(\frac{1}{\varphi})$$

and after organizing we obtain:

$$(3.27') i_t = \frac{\lambda}{\alpha a \varphi}[(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} - 1) - (e^{a(E_t \pi_{t+1} + \frac{a}{2}\sigma_\pi^2)} - 1)] + E_t \pi_{t+1} + g_t(\frac{1}{\varphi}) .$$

The expression (3.27') is complicated. Here too we will use the following approximation feature:

$$(3.28) e^{z1} - e^{z2} \cong e^{z1-z2}$$

where the $z's$ are small enough. In our case, the percentage inflation and output are measured in single percentages and decimal equations, so that for example, a 2% inflation target is 0.02, and the approximation in equation (3.28) has no significant deviation: even where z_1 = 20% and z_2 = 10%, the deviation is only one percent. (Nevertheless, we should also emphasize that this approximation is not correct in a process of disinflation, where at its starting point, the inflation is tens of percent.)

We can now arrange equation (3.27'):

$$(3.27'') i_t = \frac{\lambda}{\alpha a \varphi}(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2) - a(E_t \pi_{t+1} + \frac{a}{2}\sigma_\pi^2)}) + E_t \pi_{t+1} + g_t(\frac{1}{\varphi})$$

the variances are also canceled, and after further arrangement we obtain:

$$(3.28) i_t = \frac{\lambda}{\alpha a\varphi}(e^{a(\pi_t - E_t\pi_{t+1})}) + E_t\pi_{t+1} + g_t(\frac{1}{\varphi}).$$

Regarding the dynamics of equations (3.17) and (3.18), we have shown that $E_t\pi_{t+1} = \rho\pi_t$. Substituting this in equation (3.28), we obtain:

$$(3.29) i_t = \frac{\lambda}{\alpha a\varphi}(e^{aE_t\pi_{t+1}(\frac{1-\rho}{\rho})}) + E_t\pi_{t+1} + g_t(\frac{1}{\varphi})$$

It is worth noting that the optimum response of the interest rate in equation (3.29) also contains the variance embodied in the expected inflation equation (3.18).

Deriving equation (3.29) according to the expected inflation gives the interest response coefficient to a change in the expected inflation:

$$\frac{\partial i_t}{\partial E_t\pi_{t+1}} = \frac{\lambda}{\alpha a\varphi}(e^{aE_t\pi_{t+1}(\frac{1-\rho}{\rho})}).a\frac{(1-\rho)}{\rho} + 1$$

$$\Rightarrow (3.30)\psi_\pi^l = 1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho}(e^{aE_t\pi_{t+1}(\frac{1-\rho}{\rho})}) > 1$$

where the parameter ψ_π^l indicates the change in the inflation rate as a response to a change in the expected inflation, in the Linex case. Equation (3.30) immediately shows us that the response in the quadratic function (*a→0*) is:

$$(3.31)\psi_\pi^Q = 1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho} > 1$$

(and where Q marked ψ_π^Q is for a quadratic function). This is precisely the result obtained by CGG for a quadratic function[33]. Equation (3.30) shows that the interest rate response coefficient to the change in the expected inflation is always greater than 1, and this even though the parameter a can also be negative. As Woodford proves, this condition is critical to reach a determinacy in a model such as ours.[34]

Equation (3.30) shows that at the optimum, the interest rate's response to an increase in inflation expectations is *greater than 1 for each a, so that determinacy is obtained*. This means that at the optimum, if inflation increases, there must be a

---

[33] In the case of a quadratic equation, equation (3.29) will be as follows. We will use L'Hopital's rule:

$$\lim_{a\to 0} i_t = \frac{\lambda}{\alpha\varphi}[e^{aE_t\pi_{t+1}(\frac{1}{\rho}-1)}.E_t\pi_{t+1}(\frac{1}{\rho}-1)] + E_t\pi_{t+1} + g_t(\frac{1}{\varphi})$$

$$(3.29)^Q \Rightarrow i_t = E_t\pi_{t+1}[1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho}] + g_t(\frac{1}{\varphi})$$

[34] For Woodford (2003), the objective function is quadratic. Here the result is that the interest rate response is greater than 1 for any asymmetry size, and includes the quadratic case.

change in the expected, real monetary interest rate (defined as the nominal interest rate less the expected inflation rate for the following period) in order to reach convergence. The asymmetric factor a only determines the extent of the interest rate response, but not the essence of convergency, even where a is negative which expresses a reduced aversion to inflation. This is the fifth result.

Result 5: *the interest response coefficient to a change in expected inflation is greater than 1 for each a (and for each $\rho < 1$), so that a determinacy is obtained.*

The interest rate equation (3.29) is a function of the inflation expectations. As we noted in equation (3.18), at equilibrium there will be negative expectations for a positive a and positive expectations for a negative a. It follows that the interest rate will be lower the more positive a is. This is not a trivial result; we would expect the interest rate to be higher for inflation to be lower. However, the inflation equation is not a function of the interest rate, but the interest rate is a function of inflation; in other words: the causality in the model is from inflation to the interest rate, and not the reverse. We therefore also obtain the result that a lower rate of inflation results in a lower nominal interest rate, and this is the sixth result.

Result 6: *the more positive that a is, the lower the rate of inflation (and expected inflation) will be, but the nominal interest rate will also be lower.*

As the inflation expectations are a function of the disturbances υ_t, which behave as AR(1), and as the duration of the convergence is the same for each asymmetry, the result is that the asymmetry factor affects the dynamics of the interest rate throughout the convergence path with respect to a supply shock. This means that between different $a's$ there will no fixed interest gap throughout the convergence path (but only at the points of equilibrium). We must recall that **Result 1** showed that the asymmetry factor does not affect the dynamics of inflation (equation 3.22) and of the expected inflation (equation 3.18), and there will be a fixed gap in the level of inflation (and expectations) throughout the convergence path. The result of both of these – the absence of impact of the asymmetry on the dynamics of inflation, on the one hand, and the impact of the asymmetry on the interest rate dynamics, on the other, tell us that the asymmetry factor also affects the dynamics of the real monetary interest rate. This is the seventh result.

Result 7: *the asymmetry factor affects the dynamics of the nominal and real interest rates throughout the convergence path that follows a supply shock; this means* *that between different a's there will be varying gaps of interests throughout the convergence process. In other words: the asymmetry factor also affects the dynamics of the output gap during the convergence process as it depends on the real, expected, monetary interest rate (equation AD, 3.1).*

The development of the model this far concludes the formal description of the optimal monetary policy – optimization of the behavior of the target variables x_t, π_t, and the policy response instrument, i_t.

Table 1 below summarizes the optimal equations of the target and interest rate variables, and compares the results of the Linex objective functions with the quadratic function results.

We should also mention that the expected output gap equation (3.11'), expressed in Table 1 through the expected inflation rate, can be expressed through exogenous variables and parameters, as appear in equation (3.26).

We will now discuss some of the results that emerge from development of the model, while presenting simulations.

7. Simulations, discussion and emphasis on some of the results

The purpose of the simulation is to receive an initial intuition of the model's results and to compare the common quadratic function where the asymmetry factor is $a \rightarrow 0$, with the Linex function. The simulation applies the optimum equations presented in Table 1. Table 2 presents the assumptions regarding the model's parameters.

The starting point in the simulation is a state of equilibrium for the symmetric case, where the inflation equals the target inflation rate. The simulation examines deviations from the inflation target, so that for simplicity's sake we can assume an inflation target of zero percent. We will then apply a shock as a result of which the inflation rate and the output gap depart from their path or equilibrium; monetary policy activates the interest rate instrument aiming to restores the target variables to equilibrium. Illustration no. 3 demonstrates the divergence of the inflation where monetary policy **does not** activate the interest rate instrument after the supply shock.

Table 1: Comparison of the optimal results of the target and interest rate variables between the Linex objective function and the quadratic function

	Linex function	Quadratic function
x_t	$(3.26) \Rightarrow x_t = -\lambda q \upsilon_t - \frac{\lambda}{\alpha}\frac{a}{2}\sigma_\pi^2(1 - A\lambda^2)$	$(3.26)^Q \Rightarrow x_t = -\lambda q \upsilon_t$
	$(3.11')E_t x_{t+1} = -\frac{\lambda}{\alpha a}(e^{a(E_t\pi_{t+1} + \frac{a}{2}\sigma_\pi^2)} - 1)$	$E_t x_{t+1} = -\frac{\lambda}{\alpha}.E_t\pi_{t+1}$
	$(3.23')A = \frac{1}{\alpha(1-\beta) + \lambda^2}$	$(3.23)q = \frac{1}{\alpha(1-\beta\rho) + \lambda^2}$
π_t	$(3.22)\pi_t = \alpha q \upsilon_t - A\lambda^2 \frac{a}{2}\sigma_\pi^2$	$(3.22)^Q \pi_t = \alpha q \upsilon_t$
	$(3.18)E_t\pi_{t+1} = A\alpha\rho\upsilon_t - A\lambda^2 \, {}^a\!/_2\, \sigma_\pi^2$	$(3.18)^Q E_t\pi_{t+1} = \rho\pi_t$
i_t	$(3.29)i_t = \frac{\lambda}{\alpha a \varphi}(e^{aE_t\pi_{t+1}(\frac{1-\rho}{\rho})}) + E_t\pi_{t+1} + g_t(\frac{1}{\varphi})$	$(3.29)^Q \Rightarrow i_t = \psi_\pi^Q.E_t\pi_{t+1} + g_t(\frac{1}{\varphi})$
	$\Rightarrow (3.30)\psi_\pi^l = 1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho}(e^{aE_t\pi_{t+1}(\frac{1-\rho}{\rho})}) > 1$	$(3.31)\psi_\pi^Q = 1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho} > 1$

Table 2: parameters in the simulation

$\alpha = 0.1$	$\beta = 0.99$	$\lambda = 0.4$	$\varphi = 0.7$	$\mu = 0.9$	$\rho = 0.9$	$\sigma_\pi^2 = 2.5$	$A = 6.211$	q=5.852

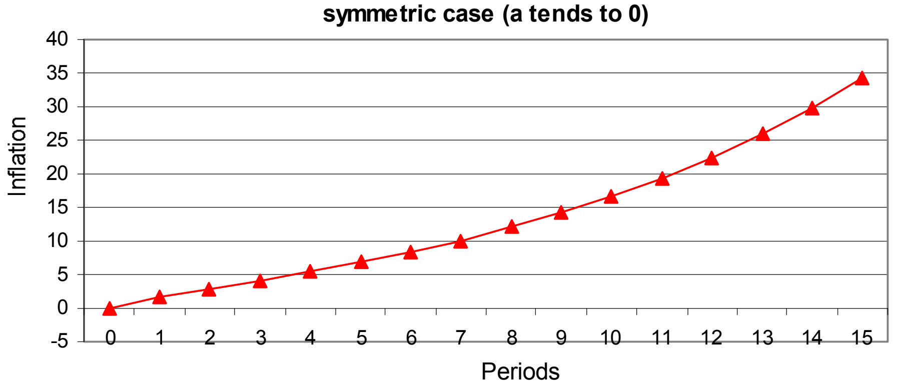


Illustration no. 3: Divergence of Inflation after a supply shock v_t of size 1, without any response by the interest rate, for the symmetric case (a tends to 0)

Simulation no. 1 is described in illustrations 4.1 (A-F) as a result of an (exogenous) cost push shock, υ_t, of size 1. The convergence paths to equilibrium are described for different rates of asymmetry a=1, 0.5, 0, -0.5, 1, where the horizontal axis describes the number of periods (for example quarter years). **Simulation no. 2** is described in Illustration 4.2 (A-F) as a result of an exogenous shock, g_t, of size 1.

These simulations emphasize and demonstrate some of the results that emerge from the model, and particularly the combination of inflation, interest rates and the output gap, as well as the real monetary interest rate, where the quadratic case may be used as a benchmark for comparative purposes. This combination shows the distinction between the point of equilibrium and the deviations from it when a shock is applied, as well as the differences between the types of shock – supply or demand.

Illustrations 4.1 (A) & (C), and 4.2 (A) & (C) demonstrate **Result no. 6**, whereby the more positive is a, the lower the inflation rate will be (negative deviation from the inflation target) and the nominal interest rate will also be lower. The added gain in achieving a lower level of inflation when a is higher is also a lower nominal rate of interest. The lower interest rate is caused by lower anticipated inflation (and lower inflation), which is influenced by the asymmetric factor. This outcome is obtained at the point of equilibrium and when departing from equilibrium when a shock occurs. In contrast, in real monetary interest rates (illustration 4.1 D) and output gap (illustration 4.1 C) there are differences in the effect of the asymmetry between the point of equilibrium and the departure from it, in the event of a supply shock, that is – they illustrate **Result no. 7**, whereby the asymmetry affects the dynamics of the nominal and real interest rates.

This illustration also shows that at equilibrium point, the asymmetry factor has no impact (or negligible differences) on the real monetary interest rate and the output gap, particularly where the discount factor β is closer to one (**Result no. 4)**. In contrast, when departing from equilibrium due to a supply shock, and during the convergence path to equilibrium, there are differences in the asymmetry factor's effect, so that where a is more positive, the real monetary interest rate too will be higher and the output gap will be more negative.

These results (4, 6, & 7) thus create a distinction between the model's real variables and the nominal variables. Regarding the nominal variables – inflation (and expected inflation) and interest rates – different asymmetry degrees creates different levels, and the higher a, the lower they will be at equilibrium as well as during the convergence process. In contrast, with the real variables – real monetary interest rate and the output gap - at equilibrium, different degrees of asymmetry have no effect and they are zero for every a (approximately); however, when the economy is no longer at equilibrium, and along the convergence path, *then in response to a supply shock* the degrees of asymmetry has an impact and the more positive *a*, the real monetary interest rate will be higher and the output gap will be lower. In other words: this result can be interpreted such that it is the real variables that reflect the asymmetry factor, or the central bank's (the governor's) set of preferences.

If at equilibrium the output gap is similar for each a, we can ask what is to be gained from lower inflation (as a is higher), that is, from the asymmetry? In other words: why should we care if inflation is higher, as long as the real activity is not affected? The reply to this is that other adverse effects of inflation, such as the impact on natural output, do not appear in the model. Regarding our model, there are no gains from asymmetric policy when the economy is in equilibrium, and the effect on the real variables is neutral.

The significance of Results 6-7 is that the more positive the asymmetry, the stronger the response to supply shock will be, and this is reflected in a higher real interest rate. Illustration 4.3 (A) shows the change in inflation, in the nominal interest rate, in the real interest rate and in the output gap for different *a*'s in response to a supply shock. The diagram also illustrates **Result 1**, whereby the asymmetry factor has no impact on the inflation dynamic; thus, in response to a supply shock, the change in the rate of inflation and in expected inflation relative to equilibrium point is the same for each degree of asymmetry. This means that the response of the variables – the nominal interest rate, real interest rate, output gap and expected output gap – to a supply shock will be different for each *a* preference, such that the change in the target variable, inflation and the expected inflation that feed it will be the same for each *a* preference.

Result 8: *in the event of a supply shock - cost push,* υ_t *- the optimization points to a gradual convergence of inflation and the output gap, not at once, as shown in illustrations 4.1 (B & C).*

This result differs from the case of aggregate demand shock (below). The gradual convergence is achieved due to a substitution ratio between inflation and the output gap:[35] The monetary policy response to a supply-side shock works to lower inflation

[35] In the AS-AD curve, where the vertical axis is inflation (and the interest rate) and the horizontal axis is output, a cost push shock (increase) causes AS to move upwards and left, thus creating a negative output gap and inflation. Raising the interest rate in an effort to reduce inflation causes AD to move left and down, resulting in a greater negative output gap. The optimal convergence is therefore gradual.

but also at the price of creating a negative output gap; the optimization therefore points to a gradual convergence of inflation. This means that some increase in inflation is possible with the shock and during the convergence in the next periods. As CGG explain, and can be seen in equations (3.26) and (3.22) – an immediate convergence will only take place in two instances: if there is no shock, υ_t, in the supply equation, or the inflation target is inflexible (that is $\alpha = 0$).[36]

However, the corner case of $\alpha = 0$ raises some reservations regarding CGG's conclusion as in this instance there is no defined solution; thus for example, the interest rate and the output gap (equations (3.29) and (3.26) ,respectively), also tend to infinity and to minus infinity, respectively, and this is improbable. Woodford (2003) also shows that the condition $\psi_\pi > 1$ for obtaining determinacy is necessary, but insufficient; in fact this is the lower limit, but there is also an upper limit. We will therefore make do with a situation where $\alpha \to 0$.

Another result that emerges from the simulation is:

Result 9: *the degree of asymmetry does not affect the duration of the convergence and it will be the same for every a.*

This result is demonstrated in Illustrations 4.1 & 4.2. It is obtained even though the intensities of the interest rate and output gap response to a given supply shock are different for different *a*'s. This means that policy based on factor *a* cannot affect the duration of the inflation convergence to equilibrium point (illustration 4.1 C). This reinforces CGG's conclusion (Result no. 2), whereby policy affects the gap between inflation and the inflation target along the convergence path, but cannot affect the duration of the convergence.[37]

Result 10: *contrary to a supply shock, in the case of a demand shock g_t, the optimal response is immediate convergence, not gradual convergence, of inflation.*

As the optimal equations that appear in Table 1 show, this kind of shock only affects the interest rate, and not the output gap and inflation equations. This result is also presented in Simulation no. 2, illustration 4.2 (A-F), where the shock is g_t (demand shock), for example government demand. The illustration shows that the

[36] The implication is that aside from the case of an inflexible inflation target, any price increase as a result of a supply shock can be interpreted as a disinflationary process rather than one prices-level increase.

[37] CGG contend that in the model with inflationary inertia (where inflation also depends on the inflation rate during the preceding period), policy will also affect the duration of convergence. A model of this kind is not addressed in this paper.

interest rate responds to shock, but the output gap and inflation remain unchanged. This implies that a shock based on government demand is **immediately** offset, in optimum, by the private sector, so that the output gap and inflation rate remain unchanged – all this through the interest rate mechanism. This result is obtained due to the fact that this shock affects the inflation rate and the output gap in the same direction, there is no substitution effect between them, and the interest rate response therefore also affects them both in the same direction. (See also CGG.). There is therefore immediate "crowding in" or "crowding out" here.[38]

Result 11: *in the event of demand shock, asymmetric preference is in no way reflected in the intensity of the policy response and the dynamics of the convergence; the interest rate response will be the same for each asymmetric preference. The asymmetric preference is reflected and affects only in the point of equilibrium, and only the level of the nominal variables – the nominal interest rate, inflation and expected inflation – but not the real variables: the output gap and the real monetary interest rate.*

Since the inflation gap for different *a*'s is fixed in any situation (see Result no. 1 and Illustration 4.3 A), and the interest rate response (change in the rate of interest) is the same for each *a* (Illustration 4.3 B), it is therefore essential that the real interest rates are also the same for each *a* throughout the convergence process, as seen in Illustration 4.2 (D). This result shows that in the case of a demand shock, policy does not enjoy the privilege of asymmetric preference; the asymmetry has no impact on the intensity of the required response in the interest rate, and the level of the real variables – the output gap and the real interest rate. The impact of the asymmetry is only at the point of equilibrium, and only on the level of the nominal variables. Compared to supply shock, the impact of the asymmetric preference declines (and may even reach zero) in the case of a demand shock. *It follows that in the case of a demand shock, the damage caused by use of the quadratic function that assumes certainty equivalence is less than in the case of a supply shock.*

It follows that the required interest rate response in the case of a demand shock is stronger than for a supply shock, as in this case there can be no inflation, and policy must eliminate it immediately. Another important conclusion that emerges here is that in order to respond correctly, the source of the shock must be identified.

[38] In other words: if policy responds properly, a demand shock can be interpreted as one price-level increases rather than in a disinflationary process.

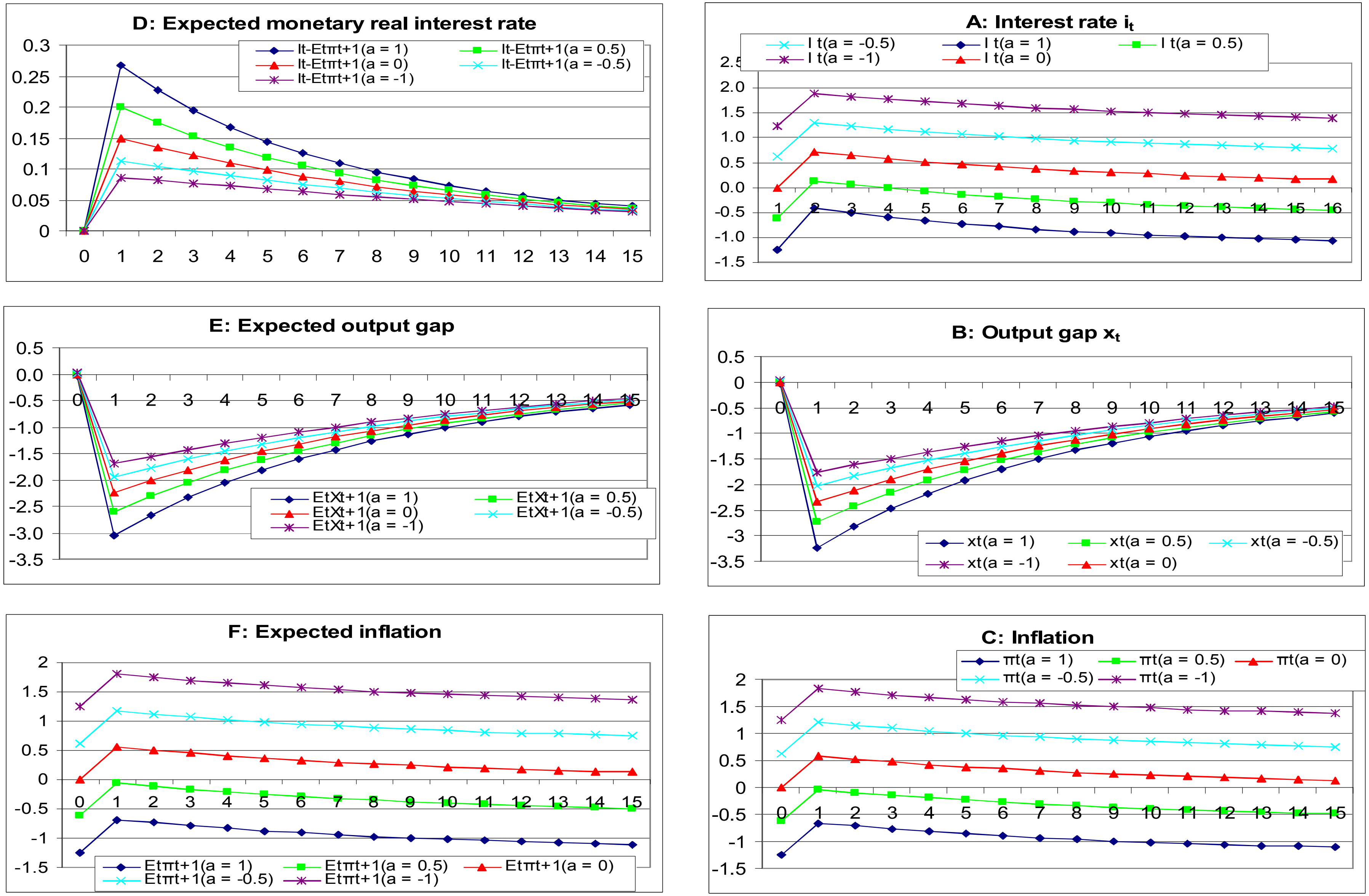
F. CONCLUSION

This article is a theoretical analysis of asymmetric monetary policy and its repercussions. The target variable under review is inflation rather than output. The model is New-Keynesian with demand equations (AD) and supply equations (AS), where the endogenous variables – the output and inflation gap – are forward looking, with no inertia from the past, and the disturbances that are distributed A(1), are the sole factor that cause the inflation to persist. The objective function is an asymmetric Linex type function, which also takes into account the inflationary uncertainty reflected in the inflation variance. This function contains various continuous levels of asymmetry as well as the quadratic (symmetric) case as a special case; in this manner the outcome of such asymmetric policy can thus be compared with that of symmetric policy, reflected in the quadratic objective function which is extremely common in the literature. The monetary policy analyzed here is that of policy under discretion where the inflation expectations are given to the policy when performing optimization - unlike the case of commitment which is not analyzed here. The paper presents the optimum of the target variables – inflation and the output gap – and the interest rate in terms of exogenous variables, which are the disturbances and inflation variance, and in terms of parameters include the asymmetric parameter. The paper also presents simulations to demonstrate the results obtained from the model.

When developing the model we find that the response coefficient of interest rates to changes in the expected inflation rate is greater than 1 for each degree of the asymmetry (positive or negative), so that determinacy exists. Several key results emerge from the paper: (A) at the point of equilibrium, asymmetric preference will only be reflected in nominal variables – the nominal interest and inflation rates – but not in real variables. This means that asymmetric policy at equilibrium is neutral. In this model, missing the inflation target when there is no shock is not necessarily evidence of a real price. (B) when a shock occurs, the optimization distinguishes between demand shock and supply shock inasmuch as it refers to real variables – the real expected interest rate and the output gap. In demand shock (unlike supply shock) the asymmetry has no impact on the real variables along the convergence to equilibrium. This means that the optimization does not give policy the privilege of asymmetric preference: each asymmetric preference has the same result. (From this perspective the use of the quadratic function is therefore harmless.) Asymmetric preference will be reflected only in the nominal variables – the nominal interest and inflation rates. In contrast, in the case of supply shock, different degrees of asymmetry require different degrees of response, and this has repercussions on the nominal and real variables throughout the convergence path; there is therefore greater importance in identifying the source of the shock (supply or demand). (C) for a given size of shock, a stronger interest rate response is required for a demand shock than for a supply shock. (D) the duration required for convergence to equilibrium will be identical for each asymmetric preference. (E) a demand shock requires immediate convergence of the inflation rate and output gap (but not of the interest rates), whereas a supply shock requires gradual convergence. The reason for results (B) to (E) is the substitution effect between inflation and output in supply shock that does not exist in demand shock. (F) the greater the asymmetry the lower the inflation will be, but the nominal interest rate will also be lower. This outcome is due to the inflationary expectations that also embody the asymmetric factor, so that they too are lower the stronger the aversion to inflation, but in turn generating a higher real interest rate. A strong policy aversion to inflation therefore leads to a low level of expectations of inflation – although they are also exogenous to the policy – and it is this variable that leads the system. (G) In policy under discretion a deflationary bias may exist at the optimum, and this contrary to the convention regarding inflationary bias. Other results appear in the main body of the paper.

The case of policy under discretion – Simulation 1

Illustration 4.1: Optimal convergence path for the interest rate, output gap, expected real and monetary inflation rates, expected output gap and expected inflation rate – after a supply shock (v_t) of size 1, for different *a*'s



The case of policy under discretion – Simulation 2

Illustration 4.2: Optimal convergence path for the interest rate, output gap, expected real monetary interest rate, expected output gap and expected inflation rate after a demand shock (g_t) of size 1 for different *a*'s

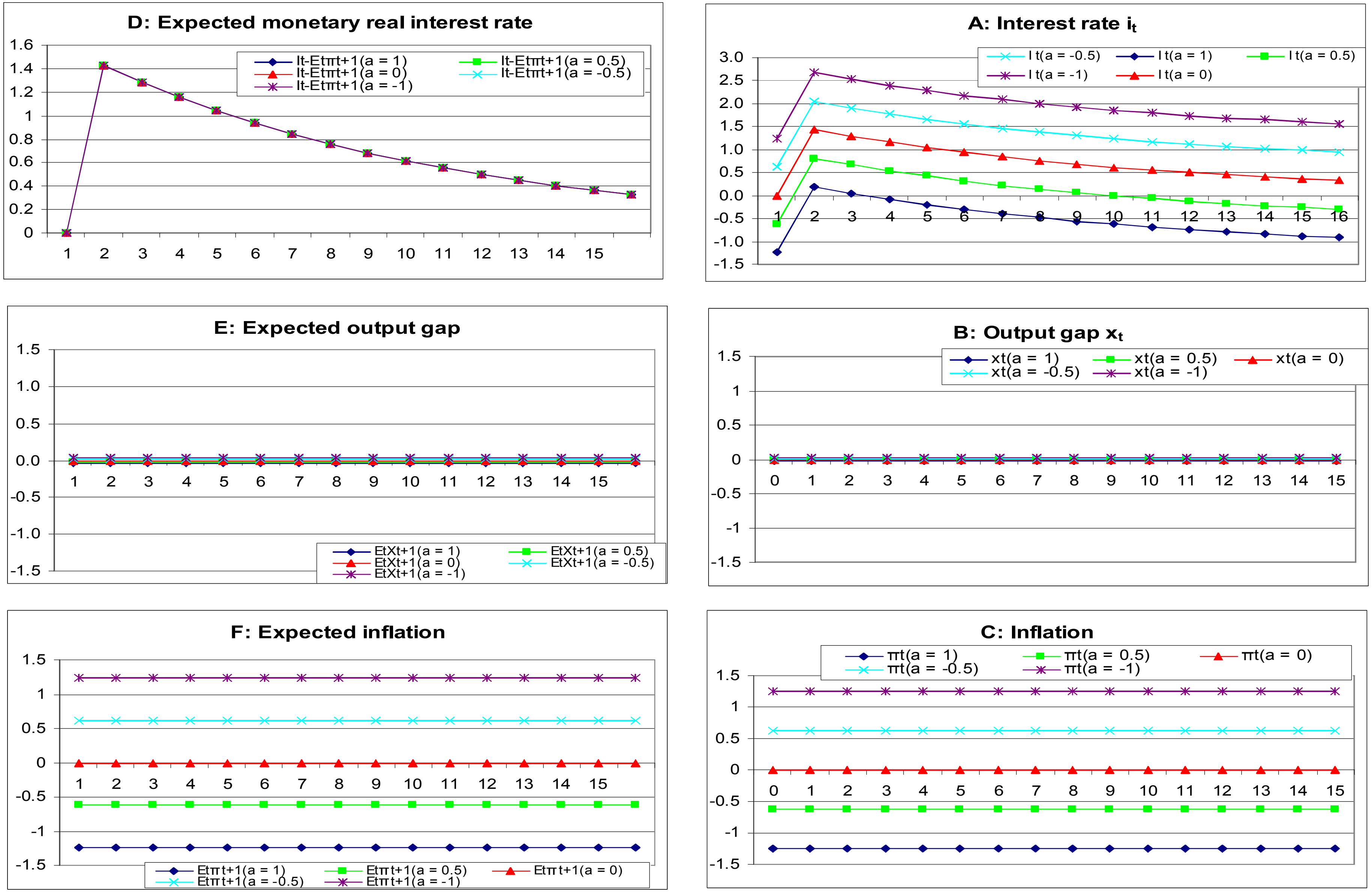


Illustration 4.3 A: Immediate response to a size 1 supply shock for different asymmetric preferences

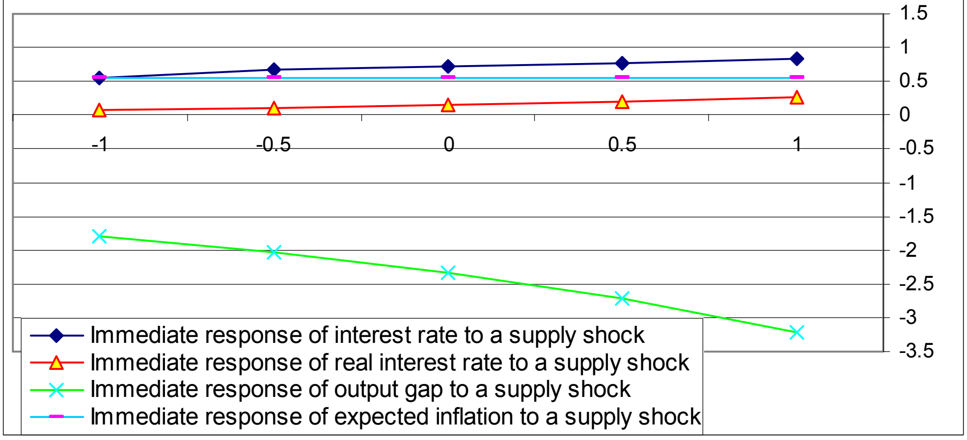


Illustration 4.3 B: Immediate response of the interest rate to a size 1 shock for different asymmetric preferences

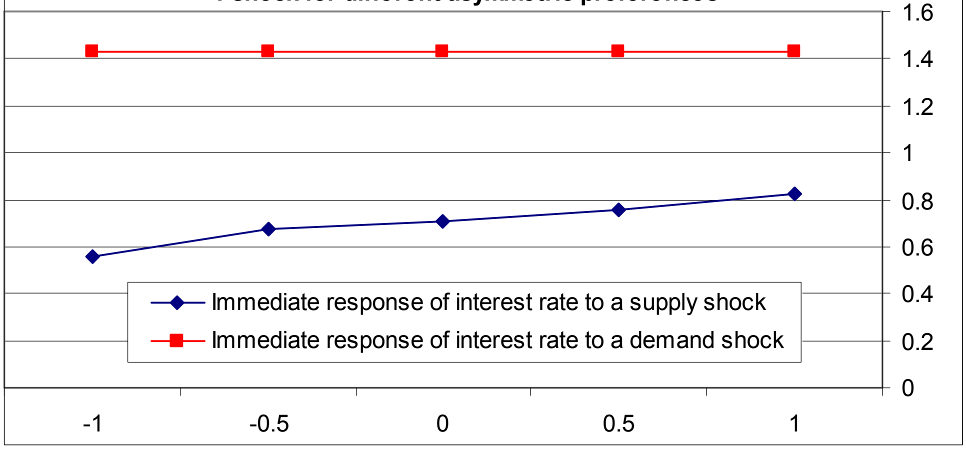
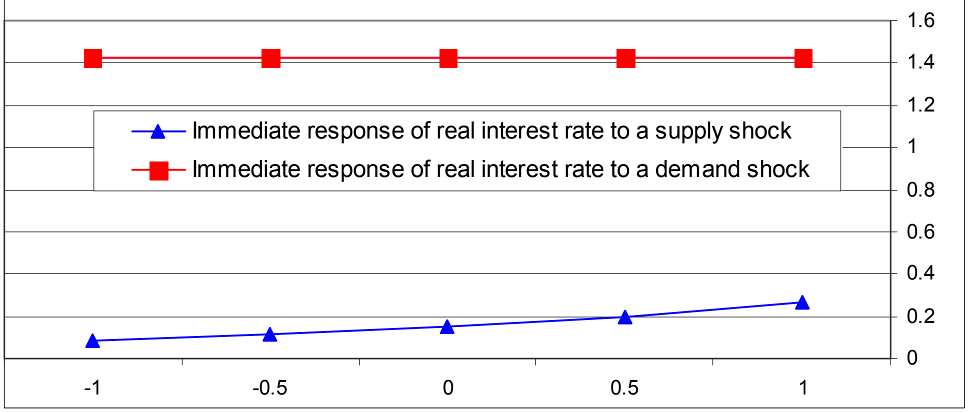


Illustration 4.3 C: Immediate response of the real expected interest rate to a size 1 shock for different asymmetric preferences



Appendices

Appendix 1: The supply curve (3.4)

An AS curve that is adjusted to inflation expectations (Fischer 1997; Taylor 1980) is described in similar fashion in chapters 13-14 of Dornbusch & Fischer's book (1987). This equation as it appears there is:

$$(A.1.1)\pi = \pi^e + \lambda(y - y^*)$$

where π^e is the expected inflation rate and it is added to the original AS equation that included only the second term – that is, the output gap. The reason for this addition is the contention by Friedman (1968) and Phelps (1972) that workers are interested in real wages and not in nominal wages. They therefore require compensation for expected inflation over and above the wage determined in the labor market that is reflected here in the output gap. Manufacturers will be willing to pay the compensation if they believe that their price will also be adjusted upwards, and in the final outcome, the result will be that real wages remain fixed. The assumption here is that wages increase precisely in step with the expected rate of inflation increases, so that the coefficient of the expected inflation rate is 1, as in our case.

Although the price of the product does not consist only of wages, but other factors as well, such as profit, raw materials and the like, it is assumed that there will be a markup in the price, so that in fact it is wages that determine the rate of inflation. This is also the result in the price adjustment model, and not only the wage adjustment model (see for example – Woodford 2003, Ch. 3).

The question remains as to how expected inflation is determined – not only whether it is determined adaptively or rationally, but also what is the rational method of determining neo-classical or New-Keynesian expectations.

Woodford (2003) devotes the entire third chapter of his book to the AS curve. He proves that the shape of the neo-classical aggregate supply curve (for example, as in Sargent & Wallace, 1975) is as follows (*ibid*, p. 159, theorem 3.2 and equation (1.32):

$$(A.1.2)\pi = \kappa(y_t - y_t^n) + E_{t-1}\pi_t$$

where $\kappa > 0$. This equation is obtained where some manufacturers set the product price one period ahead, based on the information available to them when setting the price (and accordingly undertake to supply any quantity required at this price from the time that the decision is made and up to period *t*); other manufacturers adopt a more flexible pricing policy. This equation is similar to equation (A.1.1), although here, which expectations are addressed is also mentioned: current inflation is determined by expectations from the previous period regarding the inflation rate in the current period. We should also note that this model is one of product price adjustment, rather than wage adjustment, but there is no difference in principle between them.

However, this form of (neo-classical) aggregate supply equation was sharply criticized in economic literature during the 1970s and '80s. This is due to the fact that the implication of the method of building the equation is that only *unexpected* fluctuations in nominal expenditure have any impact on real activity, and in other words: fluctuations in the output gap are essentially unexpected. It follows that only an **unexpected** response by monetary policy can affect real activity. Moreover, the impact will continue for one period only – only while the rigid prices remain fixed. Any impact beyond this will be on prices only, and not on real output. An unexpected shock in real activity generated by monetary policy is therefore by necessity, purely transitory after one period.

It further transpires that this theory is not easily reconciled with reality in the US and England: the VAR model of interest-induced shock shows that prices adjust themselves slowly and gradually, as does real activity. For this reason the impact of monetary shock is long-term – two and a half years or more. During this period several price adjustments occur, not just one, as other empirical findings confirm (see, for example, Blinder et al, 1998).

The New-Keynesian theory (Phelps 1978, Taylor 1979a, 1980) assumes that price adjustment is spread over a long period: as in the classical model, not all firms change their prices at once. However, here firms set their prices slowly and gradually, also taking into account the prices of other firms, and even though price changes may be frequent. Under a strategy of price adjustment by way of "strategic complements" (as opposed to "strategic substitute"): firms whose prices remain stable for the time being, force those firms that change their prices to set prices at a lower level; later on, the firms whose prices were stable will now adjust and increase their prices, raising them at a lower rate, in line with the firms who have already changed their prices more slowly, and so on. This process generates an on-going change in nominal expenditure which affects real activity; the price level before the monetary shock may continue to affect real activity even after a new shock, even if the general price level has already changed, at least once, after the shock. In this way, the model will coincide with on-going fluctuations in real activity, the result of a monetary shock.

Woodford (2003) also proves that the New-Keynesian aggregate supply curve (which is also a Phillips curve adjusted to rational expectations) has the following form (p. 187, equation (2.12), and proposition 3.5):

$$(A.1.3)\pi = \kappa(y_t - y_t^n) + \beta E_t \pi_{t+1}$$

where $\kappa > 0$. Here the assumption is that some product prices will remain fixed throughout the period, and all the products have the same probability of changing during the period, as in Calvo's model (1983), and the timing of the adjustment is independent of the past. Woodford also assumes that profit is discounted at an expected discount rate β, where 1> β > 0. Here too, if we ignore the discount factor (which does not have to be same as the discount coefficient in the utility function), the coefficient for expected inflation is 1. Woodford (2003) presents formal proof in Appendix B (p. 662).

The difference between the two theories is in the expectations for inflation: in the neo-classical model, the current inflation rate is determined by (rational) inflationary expectations that existed during the period preceding the current period. In contrast, in the New-Keynesian theory the current inflation rate is determined solely by the present expectations for inflation regarding the future. The difference seems to be small, but it is critical, as in the New-Keynesian model deviations in the output gap from those first expected are possible and cannot be discounted. Sbordone (1998, 2002) conducted an empirical review of the Calvo model for the period 1960-1997 in the US and found that this model predicts price development, contrary to the neo-classical model. (Surprisingly, Calvo's assumptions that were used only for the purpose of developing the model, were perceived as correct in reality, as shown by Sbordone.) Gali & Gertler (1998) also demonstrate empirical support for this type of equation.

As CGG note, since today's inflation rate is attributed to the output gap and to the expected rate of inflation, the general form is a traditional Phillips curve augmented to inflationary expectations; here too however, the main difference from a standard augmented Phillips equation is that the future inflation rate expected at present, $E_t \pi_{t+1}$, enters the equation and affects the current inflation rate π_t, contrary to the standard equation in which the current inflation rate that was expected in the previous period $E_{t-1}\pi_t$ enters the equation. This distinction is critical, as by forward iteration (Appendix 3) we realize that there is no inertia or dependence on inflation in the past, contrary to the original Phillips curve. The final result is therefore that the rate of inflation depends only on current and future conditions of the output gap and disturbances. Firms set their nominal prices on the basis of their expectations for the marginal price in the future: this is represented by the variable x_{t+i} which capture changes in the marginal price caused by changes in excess demand, and the disturbances are cost push which capture everything except expected changes in the marginal cost caused by changes in excess demand. The disturbances can create variance in the resulting rate of inflation independently of changes in excess demand.

****

APPENDIX 2:

In this appendix we will show the future infinite path of the aggregate demand equation (3.1), by forward iteration of the periods. We will begin with aggregate demand (AD) equation (3.1):

$$(3.1) x_t = E_t x_{t+1} - \varphi(i_t - E_t \pi_{t+1}) + g_t$$

We want to show that in this equation, the output gap depends on the entire future path of the monetary interest rate and expected inflation rate. If we run forward to period *t+1* and *t+2*, we obtain:

$$(3.2) E_t x_{t+1} = E_t x_{t+2} - \varphi(i_{t+1} - E_t \pi_{t+2}) + E_t g_{t+1}$$

$$(3.2') E_t x_{t+2} = E_t x_{t+3} - \varphi(E_t i_{t+2} - E_t \pi_{t+3}) + E_t g_{t+2}.$$

Let us place equation (3.2') inside equation (3.2):

$$E_t x_{t+1} = E_t [E_t x_{t+3} - \varphi(i_{t+2} - E_t \pi_{t+3}) + g_{t+2}] - \varphi(E_t i_{t+1} - E_t \pi_{t+2}) + E_t g_{t+1}$$

and after combining like terms and rearranging we obtain equation (3.2") as follows:

$$(3.2'') E_t x_{t+1} = E_t x_{t+3} - \varphi E_t (i_{t+1} + i_{t+2}) + \varphi E_t (\pi_{t+2} + \pi_{t+3}) + E_t g_{t+1} + E_t g_{t+2}.$$

In the same manner, we will place equation (3.2") inside equation (3.1):

$$(3.1') x_t = E_t [E_t x_{t+3} - \varphi E_t (i_{t+1} + i_{t+2}) + \varphi E_t (\pi_{t+2} + \pi_{t+3}) + E_t g_{t+2} + E_t g_{t+1}] - \varphi(i_t - E_t \pi_{t+1}) + g_t$$

and after combining like terms and rearranging, we obtain (3.1"):

$$(3.1'') x_t = E_t x_{t+3} - \varphi E_t (i_t + i_{t+1} + i_{t+2}) + \varphi E_t (\pi_{t+1} + \pi_{t+2} + \pi_{t+3}) + E_t (g_t + g_{t+1} + g_{t+2}].$$

A forward series is thus obtained from period *t* onwards of the expected interest rate, expected inflation, and the exogenous disturbances. Regarding the output gap, it is obtained only for the most recent period. Thus, if we continue the iteration to infinity, or up to N periods (where N is extremely large), we obtain x_{t+N}, and since the assumption is that the series does not diverge at infinity but converges, the output gap then also converges to zero and this factor will disappear. We then obtain equation (3.3):

$$(3.3) x_t = \sum_{j=0}^{\infty} \varphi(E_t i_{t+j} - E_t \pi_{t+j+1}) + \sum_{j=0}^{\infty} E_t g_{t+j} .$$

****

APPENDIX 3:

In this appendix we will see the future infinite path of the aggregate supply equation (3.4), by the forward iteration of periods. We will begin with equation (3.4), which is an aggregate supply (AS) equation:

$$(3.4) \pi_t = \lambda x_t + \beta E_t \pi_{t+1} + \upsilon_t .$$

We want to show that in this equation, the inflation rate depends not only on the forthcoming period but on the entire future path of the expected output gap and disturbances.

If we run equation (3.4) forward to period *t+1* and *t+2*, we obtain:

$$(3.5) E_t \pi_{t+1} = \lambda x_{t+1} + \beta E_t \pi_{t+2} + \upsilon_{t+1}$$

$$(3.5') E_t \pi_{t+2} = \lambda x_{t+2} + \beta E_t \pi_{t+3} + \upsilon_{t+2} .$$

We will place equation (3.5') inside equation (3.5) in order to obtain equation (3.5'') after combining like terms and arranging them as follows:

$$E_t \pi_{t+1} = \lambda E_t x_{t+1} + \beta E_t (\lambda x_{t+2} + \beta E_t \pi_{t+3} + \upsilon_{t+2}) + E_t \upsilon_{t+1}$$

$$(3.5'') E_t \pi_{t+1} = \lambda E_t x_{t+1} + \beta E_t \lambda x_{t+2} + \beta^2 E_t \pi_{t+3} + \beta E_t \upsilon_{t+2} + E_t \upsilon_{t+1} .$$

In the same manner, let us place equation (3.5'') inside equation (3.5):

$$\pi_t = \lambda x_t + \beta E_t [\lambda x_{t+1} + \beta E_t (\lambda x_{t+2} + \beta E_t \pi_{t+3} + \upsilon_{t+2}) + E_t \upsilon_{t+1}] + \upsilon_t$$

and after combining like terms and arranging we obtain equation (3.6') as follows:

$$(3.6') \pi_t = \lambda (x_t + \beta E_t x_{t+1} + \beta^2 E_t \lambda x_{t+2}) + \upsilon_t + \beta E_t \upsilon_{t+1} + \beta^2 E_t \upsilon_{t+2} + \beta^3 E_t \pi_{t+3} .$$

We therefore find that the inflation rate depends on the future series of the output gap expected today, $E_t x_{t+i}$, and the future series of the disturbances expected today, $E_t \upsilon_{t+i}$. We should note that in the equation, the expectations for inflation only exist for the end period, that is - only at infinity. This factor approaches zero at infinity because $\beta < 1$ and also because we assume that the disturbances do not diverge and there will therefore be convergence at infinity to the inflation target. If we continue this iteration to infinity, we will then obtain equation (3.6):

$$(3.6)\pi_t = \sum_{i=0}^{\infty} (\lambda\beta^i E_t x_{t+i} + E_t \upsilon_{t+i}).$$

****

APPENDIX 4:

In this appendix we will solve equation (3.12) as a multi-period differential equation. Let us begin with equation (3.12):

$$(3.12)\pi_t = -\frac{\lambda^2}{\alpha a}(e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)} - 1) + \beta E_t \pi_{t+1} + \upsilon_t$$

and after moving terms, we obtain:

$$1 - \frac{\alpha a}{\lambda^2}(\pi_t - \beta E_t \pi_{t+1} - \upsilon_t) = e^{a(\pi_t + \frac{a}{2}\sigma_\pi^2)}.$$

Using logs gives:

$$\frac{1}{a}\ln[1 - \frac{\alpha a}{\lambda^2}(\pi_t - \beta E_t \pi_{t+1} - \upsilon_t)] = \pi_t + {}^{a}\!/_{2}\,\sigma_\pi^2$$

and after using approximation (3.16) and moving terms, we obtain the following differential equation:

$$(3.12')\pi_t(1 + \frac{\alpha}{\lambda^2}) - \frac{\alpha}{\lambda^2}\beta E_t \pi_{t+1} - \frac{\alpha}{\lambda^2}\upsilon_t)] = -{}^{a}\!/_{2}\,\sigma_\pi^2.$$

We will now solve the differential equation: $y_t = y_c + y_p$

A. The fixed (particular) part – y_p

Dealing with the particular part implies what the inflation rate will be in the long run, with convergence. in this situation:

$$\lim_{j\to\infty} E_t \pi_{t+j} = \pi_t$$

$$\lim_{j\to\infty} \upsilon_{t+j} = 0.$$

Placing it in equation (3.12') and moving terms gives:

$$(A3.12)\pi_t = -\frac{\lambda^2}{\alpha(1-\beta) + \lambda^2}\,{}^{a}\!/_{2}\,\sigma_\pi^2 = y_p.$$

B. The dynamic (complementary) part – y_c

We will start with equation (3.12'), where the fixed expression equals zero:

$$(A3.13)\pi_t(1 + \frac{\alpha}{\lambda^2}) - \frac{\alpha}{\lambda^2}\beta E_t \pi_{t+1} - \frac{\alpha}{\lambda^2}\upsilon_t)] = 0$$

and after moving terms, we obtain:

$$(A3.13')\pi_t = (\frac{\alpha}{\lambda^2 + \alpha})(\beta E_t \pi_{t+1} + \upsilon_t)\,.$$

We will now perform forward iteration:

$$(A3.13'')E_t \pi_{t+1} = (\frac{\alpha}{\lambda^2 + \alpha})(\beta E_t \pi_{t+2} + E_t \upsilon_{t+1})$$

$$(A3.13''')E_t \pi_{t+2} = (\frac{\alpha}{\lambda^2 + \alpha})(\beta E_t \pi_{t+3} + E_t \upsilon_{t+2})\,.$$

We will place (A3.13''') inside (A3.13'') and obtain:

$$(A3.14)E_t \pi_{t+1} = (\frac{\alpha}{\lambda^2 + \alpha})^2 \beta^2 E_t \pi_{t+3} + (\frac{\alpha}{\lambda^2 + \alpha})^2 \beta E_t \upsilon_{t+2} + (\frac{\alpha}{\lambda^2 + \alpha}) E_t \upsilon_{t+1}\,.$$

We will now place (A3.14) inside (A3.13') and obtain:

$$(A3.14')\pi_t = (\frac{\alpha}{\lambda^2 + \alpha})^3 \beta^3 E_t \pi_{t+3} + (\frac{\alpha}{\lambda^2 + \alpha})^3 \beta^2 E_t \upsilon_{t+2} + (\frac{\alpha}{\lambda^2 + \alpha})^2 \beta E_t \upsilon_{t+1} + (\frac{\alpha}{\lambda^2 + \alpha}) \upsilon_t$$

That is, if we continue to infinity, the following behavior is obtained:

$$(A3.15)\lim_{j \to \infty} \pi_t = (\frac{\alpha}{\lambda^2 + \alpha})^{j+1} \beta^{j+1} E_t \pi_{t+j+1} + \sum_{j=0}^{\infty} (\frac{\alpha}{\lambda^2 + \alpha})^{j+1} \beta^j E_t \upsilon_{t+j}$$

Since $\frac{\alpha}{\alpha + \lambda^2} < 1$, the first expression on the right-hand side of the equation approaches zero, and only the second expression remains.

We will now join both parts of the differential equation, (A3.12) and (A3.15), and obtain equation (3.19) that describes the dynamics of the inflationary process and its point of convergence:

$$(3.19)\pi_t = \sum_{j=0}^{\infty} (\frac{\alpha}{\alpha + \lambda^2})^{j+1} \beta^j E_t \upsilon_{t+j} - \frac{\lambda^2}{\alpha(1-\beta) + \lambda^2} {}^{a}\!/_{2}\, \sigma_\pi^2\,.$$

This equation states that the inflationary process depends solely on disturbances. In addition, its end point depends on the variance of inflation, and this describes a fixed gap between the quadratic function and the Linex function throughout the entire convergence path, but the variance and asymmetry do not affect the dynamic of the convergence[39]. This means that the first expression in the right-hand side of the equation does not depend on the asymmetry factor *a*.

****

[39] Here I assume that the variance is fixed. Nevertheless, if during the convergence process the variance is not fixed, the variance will also be part of the dynamic process, and in this case the gap between the quadratic function and the Linex function will not be fixed. The solution in this interesting case is not presented in this paper.

APPENDIX 5:

In this appendix we will develop equation (3.21) until we obtain equation (3.22). Let us begin with equation (3.20):

$$(3.20)\pi_t = -\frac{\lambda^2}{\alpha a}(e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)} - 1) + \beta\rho\pi_t + \upsilon_t$$

We will move terms in order to isolate the exponent:

$$\Rightarrow \pi_t(1-\beta\rho) - \upsilon_t = -\frac{\lambda^2}{\alpha a}(e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)} - 1)$$

$$\Rightarrow 1 - \frac{\alpha a}{\lambda^2}[\pi_t(1-\beta\rho) - \upsilon_t] = e^{a(\pi_t+\frac{a}{2}\sigma_\pi^2)}.$$

Using logs gives:

$$\Rightarrow \frac{1}{a}\ln\{1 - \frac{\alpha a}{\lambda^2}[\pi_t(1-\beta\rho) - \upsilon_t] = \pi_t + a/2\,\sigma_\pi^2.$$

We will now use approximation $(3.16)\ln(1-\frac{\alpha a}{\lambda}\pi_t) \cong -\frac{\alpha a}{\lambda}\pi_t$ and after arranging the equation we obtain:

$$\Rightarrow -\frac{\alpha}{\lambda^2}[\pi_t(1-\beta\rho) - \upsilon_t] - \pi_t = a/2\,\sigma_\pi^2.$$

We will now isolate inflation against disturbances. In Appendix 4 we also showed that the variance is a fixed factor that does not affect the dynamics, so that in the dynamic we can ignore this factor:

$$\Rightarrow \pi_t(\frac{\alpha(1-\beta\rho)+\lambda^2}{\lambda^2}) = \frac{\alpha}{\lambda^2}\upsilon_t,$$

after moving terms we obtain the dynamic part of the equation:

$$\Rightarrow \pi_t = \frac{\alpha}{\alpha(1-\beta\rho)+\lambda^2}\upsilon_t = \alpha q\upsilon_t$$

and after adding the fixed part (shown in apendix 4) we obtain equation (3.21):

$$(3.21)\pi_t = \frac{\alpha}{\alpha(1-\beta\rho)+\lambda^2}\upsilon_t - \frac{\lambda^2}{\alpha(1-\beta)+\lambda^2}a/2\,\sigma_\pi^2 = \alpha q\upsilon_t - \lambda^2 A\,a/2\,\sigma_\pi^2.$$

****

APPENDIX 6:

In this appendix we will express equation (3.11') of the expected output gap in terms of the exogenous variables and parameters instead of expected inflation. The optimum conditions (3.11) also give us the expected output gap:

$$(3.11')E_t x_{t+1} = -\frac{\lambda}{\alpha a}(e^{a(E_t\pi_{t+1}-\pi^* + \frac{a}{2}\sigma_\pi^2)} - 1)$$

This formula expresses the expected output gap as a function of expected inflation. However, as we did for the output gap, here too the expected output gap can be expressed in terms of the parameters and exogenous variables alone, without the expected inflation rate appearing in the equation, as we see below.

We shall begin with equation (3.11'). After moving terms we obtain:

$$\Rightarrow -\frac{\alpha a}{\lambda}E_t x_{t+1} = (e^{a(E_t\pi_{t+1}-\pi^* + \frac{a}{2}\sigma_\pi^2)} - 1)$$

$$\Rightarrow (1-\frac{\alpha a}{\lambda})E_t x_{t+1} = e^{a(E_t\pi_{t+1}-\pi^* + \frac{a}{2}\sigma_\pi^2)}.$$

Using logs gives:

$$\Rightarrow \frac{1}{a}\ln[(1-\frac{\alpha a}{\lambda})E_t x_{t+1}] = E_t\pi_{t+1} + \frac{a}{2}\sigma_\pi^2.$$

We will use approximation (3.16):

$$(3.16)\ln(1-\frac{\alpha a}{\lambda}x_t) \cong -\frac{\alpha a}{\lambda}x_t$$

place it and obtain:

$$\Rightarrow -\frac{\alpha}{\lambda}E_t x_{t+1} = E_t\pi_{t+1} + \frac{a}{2}\sigma_\pi^2.$$

We can now set the expected inflation equation (3.18):

$$(3.18)E_t\pi_{t+1} = \frac{\alpha}{\alpha(1-\beta)+\lambda^2}\rho\upsilon_t - \frac{\lambda^2}{\alpha(1-\beta)+\lambda^2}{}^{a}\!/_{2}\,\sigma_\pi^2$$

and obtain:

$$\Rightarrow -\frac{\alpha}{\lambda}E_t x_{t+1} = \alpha q\rho\upsilon_t - \lambda^2 A\,{}^{a}\!/_{2}\,\sigma_\pi^2 + {}^{a}\!/_{2}\,\sigma_\pi^2$$

and after moving terms we obtain the expected output as a function of the parameters and exogenous variables:

$$\Rightarrow (3.26')E_t x_{t+1} = -\lambda q\rho\upsilon_t - \frac{\lambda a}{2\alpha}\sigma_\pi^2(1-\lambda^2 A).$$

Bibliography

Aiyagari, Rao S. and R. Anton Braun (1998). “Some Models to Guide the Fed”, *Carnegie-Rochester Conf. Ser. Public Policy 48*, pp. 1-42.

Ardeshir, J. Dalal and Moawia Alghalith (2002). "The Choice between Multiplicative and Additive Output Uncertainty", *CRIEFF Discussion Paper* No. 0209.

Al-Nowaihi, Ali, and Livio Stracca (2002). "Non-Standard Central Bank Loss Functions, Skewed Risks, and Certainty Equivalence", *ECB, Working Paper Series (March).*

Barro, R.J. and Gordon D.B. (1983). “A Positive Theory of Monetary Policy in a Natural Rate Model”, *The Journal of Political Economy 91*, 589-610.

Blinder, A. (1997). "What central bankers can learn from academia - and vice versa", *Journal of Economic Perspective 11.2*, pp 3-19.

Blinder, Alan S., Charles Goodhart, Philipp Hildebrand, David Lipton, and Jeremy B. Rudd (1988) "Asking About Prices: A New Approach to Understanding Price Stickiness", New York: Russell Sage Foundation.

Blinder, A. (1998). *Central Banking in Theory and Practice*, The Lionel Robbins Lectures.

Brainard, William C. (1967). “Uncertainty and the Effectiveness of Policy”, *Americen. Economic Review, 57*, pp. 411-425.

Carlstrom, Charles T. and Timothy S. Fuerst (1995). “Interest Rate Rules vs. Money Supply Rules: A Welfare Comparison in a Cash-in-Advance Model”, *Journal of Monetary Economics 36:2*, pp. 247-268.

Clarida, Richard and Mark Gertler (1997). "How the Bundesbank Conducts Monetary Policy", Romer, D. (eds.), *Reducing Inflation*. University of Chicago Press, Chicago.

Clarida, R., J. Gali and M.Gertler, (1999). "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature XXXVII*, pp. 1661-1707 (December).

Chadha J.S and P. Schellekens (1999). "Monetary Policy Loss Function: Two Cheers for the Quadratic", *Bank of England Working Paper No. 101* (also available as Utility Function for Central Bankers – The Not So Drastic Quadratic", *L.S.E. Financial Markets Groups Discussion Paper No. 308*.

Christofferson, P.F and Diebold F. (1994). “Optimal Prediction Under Asymmetric Loss”, *National Bureau of Economic Research Working Paper 167* (October).

Cukierman, Alex and V. Anton Muscatelli (2003). *Do Central Banks have Precautionary Demands for Expansions and for Price Stability? - A New Keynesian Approach* (April).

Dolado J.R. Maria-Dolores and M. Naviera (2000). "Asymmetries in Monetary Policy Rules: Evidence for four Central Banks", *CEPR, Discussion Paper No. 2441* (April) http://www.cepr.org

Fischer, Stanley (1994). "Modern Central Banking in Forrest Capie", in: Charles Goodhart, Stanley Fischer and Norbert Schnadt, *The Nature of Central Banking,* Cambridge University Press, Cambridge, U.K.

Friedman, Milton (1968). "The Role of Monetary Policy", *American Economic Review* (March).

Fuhrer, Jeff and George Moore (1995). "Inflation Persistence", *The Quarterly Journal of Economics* (February).

Goodhart, Charls A.E. (2001). “Monetary Transmission Lags and the Formulation of the Policy Decision on Interest Rates”, *Federal Reserve Bank of St. Louis Review*, July/August 2001, 165-181.

King, Robert G. and Alexander L. Wolman (1996). “Inflation Targeting in a St. Louis Model of the 21st Century”, *NBER Working Paper 5507*.

Ireland, Peter N. (1996a). “The Role of Countercyclical Monetary Policy”, *Journal of Political Economy 104:4*, pp. 704-723.

Kydland, F.E. and E.C.Prescot (1977). “Rules Rather than Discretion: the Inconsistency of Optimal Plans", *The Journal of Political Economy 85*, 473-492.

Laxton, Douglas and David Rose (1995). “Asymmetry in the U.S. Output-Inflation Nexus – Issues and Evidence”, *I.M.F Working Papers* (WP 95/76).

McCallum, Bennett (1993). "Discretion Versus Policy Rules in Practice: two Critical Points - A Comment", *Carnegie-Rochester Conference Series on Public Policy 39*, PP. 215-220

McCallum, Bennett and Edward Nelson (1999). "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis", *Journal of Money, Credit, and Banking,* Vol. 31. No. 3, Part 1 (August).

Minford, Patric and David Peel (1983). "Rational Expectations and the New Macroeconomics".

Mishkin, F. (1997). "The Causes and Propagation of Financial Instability: Lesson for Policymaker", in: *Maintaining Financial Stability in a Global Economy*, Kansas City.

Mishkin, F. and Adam S. Posen (1997). "Inflation Targeting: Lessons From Four Countries”, *Economic Policy Review, Federal Reserve Bank of New York.*

Nobay, A., Robert and David A. Peel (1998). "Optimal Monetary Policy in a Model of Asymmetric Central Bank Preference", *L.S.E. Financial Markets Group Discussion Paper No. 306.*

Persson, Torsten and Guido Tabellini (1997). “Political Economics and Macroeconomic Policy”, *mimeo, Institute for International Economics*, Stockholm.

Phelps, Admund (1972). *Inflation Policy and Unemployment Theory*, New York: Norton; London: Macmillan.

Phelps, Admund (1978). "Commodity-Supply Shock and Full-Employment Monetary Policy", *Journal of Money, Credit and Banking*, vol. 10 (May).

Rotemberg, Julio and Michael Woodford (1997). “An Optimization-Based Econometric Frame Work for the Evaluation of Monetary Policy”, *NBER Macroeconomics Annual*. Ben Bernanke and J. Rotemberg (eds.).

Ruge-Murcia, Francisco j. (2000). "Uncovering Financial Markets' Beliefs About Inflation Targets", *Journal of Applied Econometrics 15*, pp. 483-512 (May).

Ruge-Murcia, Francisco j. (2001). "Inflation Targeting Under Asymmetric Preferences", *IMF Working Paper* (WP/01/161) (October).

Ruge-Murcia, Francisco j. (2001). "The inflation bias when the central bank targets the natural rate unemployment", *European Economic Review* (April). (Also available at) www.elsevier.com/locate/econbase

Sargent, J. Thomas (1987). *Macroeconomic Theory – Economic Theory, Econometrics, and Mathematical Economics*, Second Edition.

Sargent, Thomas J. and Neil Wallace (1975). "Rational Expectation, the Optimal Monetary Instrument and the Optimal Money Supply Rule", *Journal of Political Economy 83*, pp. 241-254.

Sbordone, Argia (1998). "Prices and Unit Labor Costs: A New Test of Sticky Prices", *Journal of Monetary Economics*, Elsevier, vol. 49(2),pp. 265-292 (March).

Sbordone, Argia (2002). "An Optimizing Model of U.S Wage and Price Dynamics", *Proceeding*, *Federal Reserve Bank of San Francisco* (March).

Simon Hall, Chris Salmon, Tony Yates and Nicoletta Battini (1999). *Uncertainty and Simple Monetary Policy Rules – An Illustration for the United Kingdom*, Bank of England, ISSN 1368-5562.

Svensson, Lars (1997). "Optimal Inflation Targets, Conservative Central Banks, and Linear Inflation Contracts", *American Economic Review 87*, 98-114.

Svensson, Lars and M. Woodford (2000). "Indicator Variable for Optimal Policy", *NBER Working Paper No. 7953.*

Svensson, Lars (2003). *Optimal Policy with Low-Probability Extreme Events*, *Princeton University, CEPR and NBER.* 2nd version (June).

Svensson, Lars (2003a). "Monetary Policy and Real Stabilization", *National Bureau of Economic Research, Working Paper 9486* (February).

Svensson, Lars (2003b). "The Inflation Forecast and the Loss Function", *CEPR and NBER* (Version 1.2: January). www.princeton.edu/~svensson

Taylor, John B. (1993). *Conference Series on Public Policy*, 39, PP. 195-214

Taylor, John B. (1993) *Macroeconomic Policy in a World Economy: From Econometric Design to Practical Operation*, New York: W.W. Norton.

Taylor, John B. (1980) "Aggregate Dynamics and Staggered Contracts", *Journal of Political Economy*, *University of Chicago Press*, vol. 88(1), pp. 838-855 (December).

Taylor, John B. (1979) "Estimation and Control of Macroeconomic Model with Rational Expectations", *Econometrica 47*, pp. 1267-1286 (September).

Theil, Henry (1958). *Economic Forecast and Policy*, Amsterdam, North Holland.

Tinbergen, Jan (1952). "*On the Theory of Economic Policy*", 2nd ed. Amsterdam: North Holland.

Varian, H. (1974). "A Bayesian Approach to Real Estate Assessment", in: "*Studies in Bayesian Economics in Honor of L. J. Savage*, edited by S. E. Feinberg and A Zellner, North Holland: Amsterdam, pp.195-208.

Walsh, Carl (1995). "Recent Central-Bank Reforms and the Role of Price Stability as the Sole Objective of Monetary Policy", Bernanke & Rotemberg (eds), *NBER Macroeconomics Annual*.

Woodford, Michael (2003). *Interest and Prices – Foundation of a Theory of Monetary Policy*, Princeton University Press.

Zellner, Arnold (1986). "Bayesian Estimation and Prediction Using Asymmetric Loss Functions", *Journal of the American Statistical Association 81*. No. 394, 446-451 (Jun).