Spending Allocation under Nominal Uncertainty: A Model of Effective Price Rigidity*

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Abstract

This paper reverses the typical firm-centered view on the source of co-movement of output and inflation, assuming instead that consumers face greater frictions than firms in responding to relative price changes. We model consumers as uncertain whether the fluctuations in flexible prices posted by local monopolists are aggregate or idiosyncratic in origin. An aggrgeate increase in local prices may then drive correlated forecasting mistakes, coordinating households' decisions to reallocate spending from local monopolistic to distant competitive markets at the cost of individual-specific shopping effort. We obtain three theoretical results and empirically test a tight prediction of our model. First, a distortion in competition arises, in that firms respond strategically to households' confusion over the source of posted price volatility, resulting in higher markups. Second, in the more realistic case of households being relatively less informed than firms, our new channel of frictional shopping features positive output-inflation co-movement, which is negative otherwise. Third, we show a new rationale for policies to reduce uncertainty, such as nominal stabilization or communication, counting on an attenuation of the competitive distortion. Finally, we use retailer scanner data to validate a distinctive prediction of the model, namely that the difference between posted and effective price inflation moves together with an inflationary shock.

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1 Introduction

At least since Phelps (1970) and Lucas (1972), most of the literature on the sources of co-movement between output and inflation has been firm-centric, modeling firms as relatively more constrained than households in responding to inflationary shocks. On the one side, firms cannot fully pass on a rise in wages to final prices because of some combination of physical and informational frictions.[1] At the same time, since households typically buy a representative consumption basket, they are trivially informed about the aggregate price level, hence, they correctly evaluate the opportunity cost of expanding demand.

Nevertheless, household survey data indicate that households make large and persistent errors in forecasting inflation (Mankiw et al., 2004; Reis, 2021), suggesting they may not be much better informed than firms about aggregate prices (Coibion and Gorodnichenko, 2015; Andrade et al., 2020). In particular, D'Acunto et al. (2019) show that consumers' inflation perception mostly relies on their shopping experience; that is, they form their view of the evolution of the aggregate price level by extrapolation from the prices posted at the limited set of stores they shop at.[2] This fact suggests that incomplete knowledge of the aggregate price level may well stem from shopping frictions. Households fail to take advantage of realized price differences for the same good at different stores (Kaplan and Menzio, 2015), which produces a considerable variation in household-level inflation (Kaplan and Schulhofer-Wohl, 2017) and, likely, households' inflation forecasts.

Motivated by this evidence, this paper models a new channel for inflation-output co-movement based on household uncertainty on the cost-opportunity of consumption, which does not require, although can accommodate, frictions in firms' pricing. At the core of our

[1] Typical physical frictions are menu costs (Golosov and Lucas (2007), Midrigan (2011), Alvarez et al. (2016)) whereas informational frictions stem from dispersed information (Woodford (2003), Angeletos and La'O (2009), Mackowiak and Wiederholt (2009)). Most of the empirical work at the micro-level has investigated firms' pricing in the attempt to differentiate between these two different versions of the same firm-centric view. See Alvarez et al. (2018) and Baley and Blanco (2019) for a review.

[2] For example, in the Bank of England Inflation Attitudes Survey in 2016, nearly half the respondents (41%) declared that change in stores' prices is the primary factor leading them to change inflation expectations, while media reports (21%) got less attention. The findings of the Chicago Booth Expectations and Attitudes Survey are similar (D'Acunto et al., 2019): again, one's personal shopping experience is reported as the most important factor in inflation expectations, before family and friends, TV/radio, newspapers, etc. Mosquera-Tarrío (2019) provides evidence that when there is high uncertainty about the aggregate level of prices, households' inflation expectations are closely bound to changes in the consumer's actual consumption basket.

model is the idea that *spending reallocation* towards low-markup sellers requires shopping effort, whose likely benefits households assess on the basis of the inflation perceived through market exposure at local sellers, i.e. by *learning from local prices.* We nest a tractable version of this mechanims in an otherwise frictionless monetary economy. In our model, firms' pricing is flexible, and each consumer is initially matched to a local shop. Consumers observe the price of a good at their local shop at no cost, as representative of local market exposure. Based solely on the information deriving from this local price, each consumer decides whether to switch – at a cost – to a platform where they can buy the same good at a better price; the shopping cost is specific to the individual, to capture the idea that once consumers do exert shopping effort, they will end up shopping at the most competitive suppliers. However, in seeking to gauge the likely benefit of effort, consumers are uncertain over the level of the competitive prices they could get outside their local market, because they do not know whether a local price change is due to aggregate or idiosyncratic causes.

This basic setting is used to discuss three theoretical implications of the mechanism posited. We then validate the mechanism empirically using retail scanner data, which provides corroborating evidence.

First, we show that there is a distortion in competition, in that firms respond strategically to households' confusion about the source of posted price volatility, resulting in higher markups. If consumers see a local price change as due to a general change in inflation, they will be less inclined to change stores; that is, because of uncertainty the elasticity of demand decreases, inducing a local monopolist to apply higher markups on average. This is a key novelty of our setting: in its optimal pricing, the firm takes account of the *signaling power* of its own posted price in shifting consumers' expectations. We derive the optimal pricing policy under discretion – the standard setting in macro models – and show that it boils down to an endogenous markup over the marginal cost. In particular, the markups of the local shops not only account for market power, i.e. average switching costs as normally accounted for in the literature, but also for signaling power. In spite of higher markups, we show that the unconditional average of firms' profits decreases with signaling power. This is because signaling generates a coordination failure between households and firms whose resolution would require commitment on the part of the firm.

Second, when consumers are less informed than firms, our frictional shopping model features positive comovement between output and inflation. An inflationary shock induces a correlated misperception, across locations, of an increase in the local relative price, resulting in two contrasting effects on aggregate demand. On the one hand, households with sufficiently low shopping cost decide to reallocate spending from the local to the competitive market, in which case their consumption increases because of the lower markup. There is a correlated reallocation from local monopolists towards low-markup firms, so that effective inflation can rise or fall less rapidly than posted price inflation. This reallocation causes a positive co-movement between inflation and the *extensive* margins in aggregate demand. On the other hand, households with relatively high shopping costs stay in their local markets and reduce consumption in favor of saving and leisure, implying a negative co-movement between inflation and the *intensive* margins of aggregate demand. This result is in stark contrast with the typical predictions of firm-centric models, in which the key assumption that consumers are better informed than firms guarantees positive co-movement in intensive margins. Crucially, while the extensive and intensive margins of demand have opposite implications for the co-movement of aggregate output and inflation, we show that for a large part of the parameter space, including the standard calibrations, the extensive margins dominate: a sufficient condition is that the elasticity of inter-temporal substitution is not larger than one. Intuitively, because of our first result, signaling power implies higher markups than would otherwise be the case, producing a large gain in consumption on the extensive margin.

Third, we provide a new rationale for policies to reduce uncertainty, in the form of nominal stabilization or communication, relying on an attenuation of the competitive distortion. Specifically, in our model uncertainty reduction improves welfare through two channels: first, it increases the efficiency of the choices of households and firms; and second, it lowers markups, thanks to the narrowing of the information gap between households and firms. Both channels have first-order impact on welfare[3], but we show that for reasonable calibrations (by order of magnitude), almost all the variation in welfare comes via the markup channel. We also discuss a hypothetical trade-off between communicating to consumers and to firms, showing

[3] By contrast, in typical New-Keynesian models the optimality of monetary policy rests on the second-order losses that inflation causes, because stickiness generates an inefficient dispersion in prices.

that the former is preferable.

Lastly, we derive an analytical expression for the gap between inflation in the effective price paid, obtained by weighting firms by their spending shares, and the counterfactual posted price inflation, where expenditure reallocation across retailers is muted. In our model inflation has first-order effects on the gap: effective inflation moves less than posted price inflation with the variation in aggregate inflation, owing to the greater (lesser) reallocation of expenditure from high to low price retailers when prices rise (fall). This is a distinctive feature of our mechanism, as it is well known that in standard new Keynesian models of price rigidity inflation has no first-order effect on the gap between effective and posted price inflation through the reallocation of expenditure.[4] We make minimal modifications to the empirical setting of Coibion et al. (2015), along the variants proposed by Gagnon et al. (2017), to test our prediction against the null hypothesis of no first-order relationship between average inflation and the inflation gap. We document that the difference between posted and effective price inflation is strongly co-variant with inflation: at our preferred specification, a 1-percentage-point change in posted price inflation is associated with a change of 0.66 points in effective price inflation.

Review of the Literature. Our setting shows how to overturn the basic island logic of Lucas while preserving inflation-output co-movement. We allow not only firms but also households, to be uncertain and introduce frictional spending reallocation, which generates strategic price-setting with informational consequences. When households are less informed than firms, our shopping frictions are essential to get expansionary inflation, which would be contractionary otherwise.

Various scholars have motivated informational frictions either by market segmentation (Lucas, 1972; Lorenzoni, 2009; Angeletos and La'O, 2009) or by some form of inattention (Sims, 2003; Mankiw and Reis, 2002; Woodford, 2003; Mackowiak and Wiederholt, 2009). In all these models, the degree of aggregate rigidity at the macro level is heightened to the degree of informational rigidity at the micro level. In our model, instead, aggregate price

[4] In a typical New-Keynesian model, effective inflation increases less than posted price inflation when inflation rises, but falls more than posted price inflation when inflation falls, so that average inflation has no first-order effect on the gap. See Chapter 2.1 in Galí (2015) for details on the log-linearization of effective price inflation.

rigidity is the result of the aggregate behavior of households seeking to protect their perceived purchasing power against price hikes. Thus, price rigidity in effective prices obtains in the aggregate even if individual firms' pricing is fully flexible. Imperfect learning by households about future wealth has been introduced in the literature on noisy news as an explanation for demand-driven business fluctuations (Lorenzoni, 2009; Jaimovich and Rebelo, 2009). More recently, the literature has discussed the impact of household uncertainty in the context of the so called "forward guidance puzzle" (Angeletos and La' O, 2020; Farhi and Werning, 2019; Gabaix, 2020; McKay et al., 2016). The idea is that households' uncertainty explains the reduced impact of monetary shocks on consumption by introducing cognitive discounting into the Euler equation. However, this factor only dampens the power of monetary policy; it does not alter the firm-centered mechanism at the base of its non-neutrality. Mackowiak and Wiederholt (2015) allow rational inattention for both households and firms. In their model, inattention has an impact on the intertemporal substitution, attenuating the reaction of the macroeconomy to demand shocks, whereas imperfect spending allocation has no first-order impact as in the prototypical New-Keynesian model.

Inspired by Phelps and Winter (1970), a number of scholars, notably Rotemberg and Woodford (1999) and Ravn et al. (2006), have pointed out the importance of modeling the buyer-seller relationship to assess the propagation of monetary shocks over the business cycle. The availability of detailed individual data on shopping behavior has now stimulated a growing body of literature modeling the impact of shopping dynamics on the macro-economy. See for instance Coibion et al. (2015), Kaplan and Menzio (2016), Nevo and Wong (2015), Michelacci et al. (2019) and Paciello et al. (2019). In particular, Coibion et al. (2015) and Kaplan and Menzio (2016) stress the connection between employment and inflation by modeling shopping effort as a substitute for the disutility of labor. With respect to these two works, we formalize a potentially complementary mechanism that takes inflation as shifting the perceived benefits of switching retailers instead of unemployment as a determinant of its opportunity cost.

To the best of our knowledge, this is the first general equilibrium model that posits the signaling role of monopolistic competitive prices. Amador and Weill (2010) and Gaballo (2018) have considered firms learning from prices in fully competitive markets, where the strategic use of signaling is impossible. In particular, Amador and Weill (2010) shows how greater precision of public information may crowd-out the aggregation of private information in prices, reducing welfare. Similarly, in our model, as firms learn more from public information, consumers learn less from local prices. However, this externality is never strong enough to generate a welfare loss as in Amador and Weill (2010). The increase in public information brought by policies of uncertainty reduction is always beneficial when consumers are less informed than firms. We show instead a new externality generated by learning from prices in monopolistic competitive markets, namely a reduction in demand elasticity. Chahrour and Gaballo (2019) and Angeletos and Lian (2019) also present models where households learn from prices, but these models are in real values, so that inflation has no role. L'Huillier (2020) presents a partial equilibrium model where a fraction of consumers learn from prices, inducing monopolists to set rigid prices; in that context, positive output-inflation co-movement obtains, as informed consumers benefit from suboptimal prices. Matejka (2015) gets a similar result in a rational inattention model.

There is a long tradition of modeling the interaction between sellers and buyers according to game theory. The work most resembling ours is Benabou and Gertner (1993); Fishman (1996) checks the impact of inflation volatility on consumers' search incentives when they learn from strategically set prices. As far as we know, ours is the first paper which in an otherwise standard general equilibrium model embeds some of the themes of this interesting literature. Unlike ours, however, these works typically characterize an endogenous rigidity of the pricing function, which fully internalizes households' search incentive. In this context, when search costs are low enough the nominal uncertainty induced by inflation volatility generally improves welfare, as markups optimally decline when the search incentive is stronger. In contrast, we maintain pricing under discretion, as is typical in macro models, yielding a full price pass-through of firms' perceived marginal costs. And we show that learning from prices generates a commitment problem on the firm's side, which leads to an ex-ante suboptimal increase in markups as the volatility of the aggregate component of inflation increases. In our model, then, inflation volatility is unambiguously bad when consumers are less informed than firms. This finding provides a new rationale for stabilization policies. The importance of commitment in models where consumers learn from strategic prices was noted by Janssen and Shelegia (2015) in the context of a double marginalization problem. In their rich game, the correlation in retail prices is induced by a single manufacturer trading under discretion with two retailers. In our general equilibrium model, the correlation is induced by the stochastic properties of inflation and the commitment problem directly concerns final sellers.

2 The model

We model households' frictional behavior by making two changes in an otherwise standard setup. First, there is spending allocation under nominal uncertainty, as households purchase a homogeneous consumption good in a segmented product market. In particular, having observed the posted price of a local monopolist, the household decides whether or not to exert shopping effort to buy in a competitive market, whose price cannot be observed otherwise. Second, centralized competitive labor and capital markets do not open until the product market closes. This means that when they make consumption decisions, households are uncertain about aggregate prices, including their own wages. As we will see, the uncertainty over real wage implies a conservative benchmark for assessing the impact of frictional spending reallocation.

These assumptions are convenient for modeling households' nominal uncertainty due to incomplete observation of product price dynamics, which lies at the core of our contribution. In our model, households set demand using only the information revealed by the prices at which they buy, and supply then adjusts to satisfy this level of demand. In practice, this assumption captures the major influence that local retail prices may have on households' expectations because of the frequency of individuals' exposure to shopping prices by comparison with wage revisions or financial contracts.

At the end of the period, uncertainty vanishes as households observe all prices. There are two main reasons for positing one-period uncertainty. First, the novelty of our mechanism relates to the microfoundations of the non-neutrality of money rather than to its gradual propagation.[5]. Second, by narrowing our framework down to the essentials, we get full closed-form tractability of our economy, allowing transparent characterization of its equilibrium and

[5] Persistent uncertainty could be simply introduced by assuming, for example, that changes in inflation are due to persistent and temporary components that are indistinguishable to households.

policy implications.

Before presenting the formal model, a final remark is worth. We have chosen to place our main friction on the demand side of final good markets. Nothing would prevent thinking that a similar mechanism could also be in play for firms on the demand side of an intermediate good market. In this case, final price rigidity would be the result of the cumulative effect of these frictions at different market layers. We do not pursue this line of modelling for simplicity, but also because we want to stress the relevance of households' frictional behavior suggested by the data. Our choice is also functional to the kind of empirical test we can perform: in the last section we will use scanner data on final goods market to validate a distinctive prediction of our model; as far as we know, data of similar quality are not available for intermediate good markets.

2.1 Households

The economy consists of a continuum of islands $j \in [0,1]$, each inhabited by a unitary mass of households indexed by $i \in [0,1]$ and one local firm indexed by j. Household i living in island j chooses consumption, $c_{ijt} \in \mathcal{R}_+$, labor supply, $\ell_{ijt} \in \mathcal{R}_+$, bond demand, $b_{ijt} \geq \underline{b}$, and shopping effort, $s_{ijt} \in \{0,1\}$, to maximize their preferences,

$$E\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{c_{ij\tau}^{1-\frac{1}{\gamma}} - 1}{1-\frac{1}{\gamma}} - \varphi\, \ell_{ij\tau} - \psi_{ij}\, s_{ij\tau}\right) \Big| \, \Omega_{ijt}^{u}\right], \tag{1}$$

with $\beta \in [0,1)$, $\gamma > 0$ and $\varphi > 0$ denoting, respectively, the time discount factor, the elasticity of intertemporal substitution and the disutility of working; $\psi_{ij} \in [0,\Psi]$ denotes the household-specific disutility cost from distant shopping distributed across households in an island according to a Pareto cumulative distribution

$$G(\psi) = 1 - \left(1 - \frac{\psi}{\Psi}\right)^{\xi}, \tag{2}$$

identical in all islands, with $\xi > 0$. The household has to satisfy the following budget constraint in each period,

$$c_{ijt}\,\mathcal{P}(s_{ijt}) + \frac{b_{ijt}}{R_t} = W_t\,\ell_{ijt} + b_{ijt-1} - T_t, \tag{3}$$

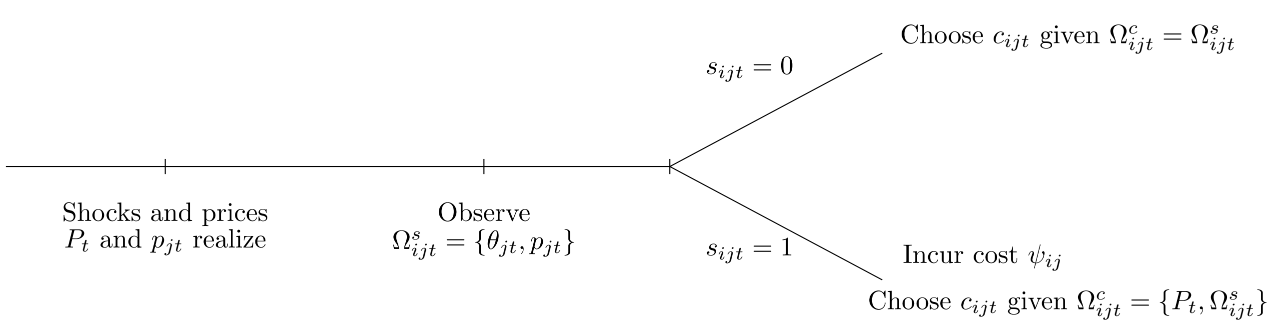
where

$$\mathcal{P}(s_{ijt}) = \begin{cases} P_t & \text{if } \; s_{ijt} = 1 \\ p_{jt} & \text{otherwise} \end{cases} \tag{4}$$

denotes the price for consumption, i.e. p_{jt} if the household shops locally, or P_t otherwise; R_t is the risk free nominal rate on bonds, W_t is the nominal wage and T_t is a lump-sum the government transfer.

Within a period, markets open and close at different stages. The product market opens first. In this market, households make their shopping and consumption decisions sequentially, as illustrated in Figure 1. The information set available to households evolves depending on their choices. In particular, Ω^u_{ijt} denotes the information available to household i in island j at time t, with $u \in \{s, c\}$ indexing households' information set at the stage of the shopping choice (s) and the consumption choice (c).

Figure 1: The household shopping and consumption problem



Initially, households decide *where* to buy. The household ij chooses its shopping effort s_{ijt} subject to $\Omega^s_{ijt} = \{p_{jt}, \theta_{jt}\} \cup \Omega_{t-1}$, or simply Ω^s_{jt}, where Ω_{t-1} is the history of realizations of the aggregate state, P_τ, for all τ up to period $t-1$, and θ_{jt} is the prior information about the current aggregate state common to all households in island j, summarized by

$$\theta_{jt} = \pi_t + \nu_{jt},$$

with $\pi_t \equiv \ln \Pi_t - E[\ln \Pi_t | \Omega_{t-1}]$, denoting the unexpected innovation in inflation, $\Pi_t \equiv P_t/P_{t-1}$, where $\nu_{jt} \sim N(0, \sigma_\nu^2)$ is an i.i.d. noise across islands. In particular, let $V_{jt}(\psi_{ijt}; s_{ijt})$ denote the present discounted value of utility in (1) of household i in island j evaluated at the beginning of period t, before shopping decisions are made, as a function of the individual shopping cost, ψ_{ijt}, and shopping behavior, s_{ijt}. Households shop at the local store if their search cost is such that

$$E\left[V_{jt}(\psi_{ijt}; 0) - V_{jt}(\psi_{ijt}; 1) \,|\, \Omega_{jt}^s\right] \geq 0. \tag{5}$$

After the shopping decision is taken, households choose *how much* to consume. Consumption c_{ijt} is chosen subject to $\Omega_{ijt}^c = \left\{\mathcal{P}(s_{ijt}), \Omega_{jt}^s\right\}$ because the household observes the price in the competitive market in case $s_{ijt} = 1$. The intratemporal first order condition for consumption is given by

$$c_{ijt}^{-\frac{1}{\gamma}} = E\left[\frac{\mathcal{P}(s_{ijt})}{W_t} \,\middle|\, \Omega_{ijt}^c\right], \tag{6}$$

so that consumption at t depends on the perceived cost in units of labor.

When the product market closes, the labor and financial markets open simultaneously: the household chooses b_{ijt} and ℓ_{ijt} given the (common) information set $\Omega_t = \{P_t\} \cup \Omega_{t-1}$, where we have posited that observation of market prices W_t and R_t provides perfect information about the aggregate state summarized by P_t. Because the disutility in labor is linear, the equilibrium nominal wage satisfies the following equation,

$$1 = \beta \, R_t \, W_t \, E_t\left[\frac{1}{W_{t+1}}\right], \tag{7}$$

so that the household (once at this stage, perfectly informed) is indifferent about the combination of labor and borrowing with which to finance its consumption expenditure in the product market to satisfy the budget constraint in (3). This further implies that the distribution of wealth across households is irrelevant to consumption decisions and that heterogeneity of wealth does not affect aggregate labor supply. Therefore, without loss of generality, we study an equilibrium where all households have the same wealth, $b_{ijt} = B_t$ for all i, j in each period, where B_t is the exogenous supply of bonds. In this equilibrium heterogeneity of households' consumption is accommodated with contemporaneous variation in the supply of labor. Furthermore, by comparing (6) and (7), we note that the current wage, which the household needs to forecast when deciding on consumption, is in fact a sufficient statistics of the intertemporal opportunity cost of current consumption. Finally, notice that in the limiting case where $\sigma_\nu \to 0$, then when the product market opens the household is perfectly informed about the aggregate state, so that the timing of household decisions within a period becomes immaterial.

2.2 Firms

There are two types of firms: distant and local. The representative distant firm transforms one unit of labor into one unit of consumption good which is sold at a price that equals the marginal cost of production,

$$P_t = W_t. \tag{8}$$

Equation (8) implies that price inflation in the competitive market, Π_t, coincides with wage inflation. Each local firm j transforms one unit of labor into z_{jt} units of the consumption good, with by z_{jt} denoting labor productivity that varies independently across firms and over time according to a log-normal distribution, i.e. $\ln z_{jt} \sim N(\bar{z}, \sigma_z^2)$. The information set of the firm at the time it sets its price comprises its idiosyncratic productivity, z_{jt}, all realizations of past nominal wages, $\{W_\tau\}_{\tau<t}$, and a signal about the change to wage inflation,

$$\vartheta_{jt} = \pi_t + u_{jt},$$

with $u_{jt} \sim N(0, \sigma_u^2)$ i.i.d. across islands and independent from the realization of ν_{jt} and z_{jt}. The local firm posts the price p_{jt} that maximizes expected profits,

$$\max_{p_{jt}} \; E\left[\varphi\, k_{jt}(p_{jt}) \,\middle|\, \Omega_{jt}^f\right], \tag{9}$$

with φ denoting the marginal value to households of one unit of consumption, $\Omega^f_{jt} \equiv \{\vartheta_{jt}, z_{jt}, \Omega_{t-1}\}$ being the information set of the firm, and

$$k_{jt}(p_{jt}) = \mathcal{D}(p_{jt}) \left(\frac{p_{jt}}{P_t} - z_{jt} \right), \tag{10}$$

denoting real profits; $\mathcal{D}(p_{jt}) \equiv \mathcal{N}(p_{jt})\, \mathcal{C}(p_{jt})$ is firm demand, which depends on the mass of households purchasing from the local retailer in island j, $\mathcal{N}(p_{jt})$, and the quantities demanded by each customer, $\mathcal{C}(p_{jt})$.

2.3 Monetary and fiscal policies

We assume that monetary policy targets an AR(1) process for inflation so that

$$\ln \Pi_t = \chi \ln \Pi_{t-1} + \pi_t, \tag{11}$$

where the innovation to inflation is normally distributed, $\pi_t \sim N(0, \sigma^2_\pi)$. Shocks to inflation can originate either from demand or from monetary policy. The Appendix offers a microfoundation for (11) in terms of a monetary policy target on real balances of bonds, B_t/P_t, where π_t originates from shocks to the propensity to save. However, the particular microfoundation behind (11) is actually non relevant to our analysis, as long as it preserves a normally distributed inflation. Given the information structure of our economy, all effects of the nominal shock on real outcomes will come simultaneously, so that the extent of correlation in $\ln \Pi_t$ captured by χ is irrelevant to output.[6] But the volatility of inflation shocks, σ^2_π, is a key parameters, in that it determines both the extent of common variation in marginal cost across islands and the variation in nominal wages over time. The path of the nominal interest rate R_t consistent with (11) is the one that solves the household's saving problem in equation (7),

$$\ln R_t = -\ln \beta + \chi \, \Delta \ln \Pi_t - .5\, \sigma^2_\pi,$$

[6] The parameter χ can be used to match the correlation between forecast of future inflation with estimates of current inflation, a robust feature of the data.

while $T_t = B_{t-1} - \frac{B_t}{R_t} - P_t K_t$ is the fiscal transfer that guarantees the equilibrium in the bond market for any given path of B_t, and given aggregate real profits K_t. In practice, because at the time the saving and the labor decisions are made the economy aggregates as a representative household economy, the particular path taken by B_t and T_t is irrelevant to equilibrium consumption.

3 Equilibrium

In this section we guess and verify the existence of an equilibrium where all variables are log-normally distributed. We then use this equilibrium to characterize firm profits and consumer demand.

The key element shaping the equilibrium allocation in our economy is the relative degree of uncertainty of households and firms about the nominal price level. On the one hand, when firms decide their price, they take the demand elasticity of households as given, know local productivity, but are uncertain on wages; this uncertainty affects the extent to which posted prices reflect the actual realization of wages. In particular, since both the change in inflation, π_t, and the firm's signal, ϑ_{jt}, are normally distributed, firms' uncertainty about inflation innovations is normally distributed with mean $E[\pi_t \,|\, \Omega_t^f] = \delta \vartheta_{jt}$ and variance $\mathcal{F} \equiv V(\pi_t \,|\, \Omega_t^f) = (1-\delta)\sigma_\pi^2$ with $\delta \equiv \sigma_\pi^2/(\sigma_\pi^2 + \sigma_u^2)$.

On the other hand, when households decide about shopping, they see the nominal local price – which for them is a given and signals firms' knowledge – but not the price level in the competitive sector; this uncertainty affects the elasticity of their local demand. Anticipating that, in equilibrium, the logarithm of the local prices $\ln p_{jt}$ will be normally distributed, like households' signals θ_{jt}, households' uncertainty about inflation innovations is normally distributed with mean

$$E[\pi_t \,|\, \Omega_t^s] = \rho\, \theta_{jt} + \omega\, (\ln p_{jt} - E[\ln p_{jt} | \Omega_{t-1}]), \tag{12}$$

and variance $\mathcal{S} \equiv V(\pi_t \,|\, \Omega_t^s) = (1 - \rho - \delta\, \omega)\, \sigma_\pi^2$, where $\rho \in (0,1)$ and $\omega \in (0,1)$ denote respectively the loadings on the exogenous and endogenous signals and are functions of the volatility of the fundamental shocks, σ_π and σ_z, as well as of the signal noises, σ_u and σ_ν:

$$\rho = \frac{\sigma_\nu^{-2}}{\sigma_f^{-2} + \sigma_\nu^{-2} + \sigma_\pi^{-2}} \qquad \text{and} \qquad \omega = \frac{1}{\delta} \frac{\sigma_f^{-2}}{\sigma_f^{-2} + \sigma_\nu^{-2} + \sigma_\pi^{-2}},$$

with $\sigma_f^2 = \sigma_u^2 + \sigma_z^2/\delta^2$. The coefficient $\omega \in (0,1)$ captures the manner in which firms' uncertainty spills over onto households' uncertainty. We will refer to this as *signaling power*, meaning the extent to which local prices affect households' expectations. Greater ω means that local prices are more informative about inflation innovations, so that price expectations correlate more strongly with them. In this sense, a local monopolist, through its pricing, has the power to displace households' expectations. ω is greater when the volatility of idiosyncratic shocks σ_z^2 is smaller relative to that of the aggregate state σ_π^2, and when the precision of firms' information about the aggregate state increases relative to that of households, i.e. σ_u^2/σ_ν^2 diminishes.

In equilibrium, the elasticities of firms' pricing and consumers' demand are mutually consistent given firms and households' uncertainty and equilibrium signaling power. In particular, local prices p_{jt} obtain as a constant markup over log-normally distributed marginal cost. Despite the elasticity of demand for local firms is endogenous (it depends on the precision of information available to consumers and firms) our functional forms ensure it is a constant, so that optimal fixed markups obtain as a consequence.

The next proposition characterizes this equilibrium, focusing on the three key equilibrium objects: households' shopping and consumption policies and the optimal local prices.

Proposition 1. *Let $C^* \equiv \varphi^{-\gamma}$ and $\xi \equiv \frac{\lambda}{\gamma-1}$ with $\lambda > 0$. If $\Psi = \frac{\varphi^{1-\gamma}}{\gamma-1}$ and mean log-productivity is such that $\bar{z} < Z$ for some finite Z, there exists a Gaussian equilibrium where:*

- *household i remains matched to island j if and only if $\psi_{ij} > \hat{\psi}_{jt}$, with $\hat{\psi}_{jt}$ solving*

$$1 - \frac{\hat{\psi}_{jt}}{\Psi} = e^{(1-\gamma)\left(\ln p_{jt} - E\left[\ln P_t \,|\, \Omega_{jt}^s\right] + \frac{1}{2}\mathcal{S}\right)}, \tag{13}$$

so that the mass of households purchasing in island j is given by

$$\mathcal{N}(p_{jt}) = e^{-\lambda\left(\ln p_{jt} - E\left[\ln P_t \,|\, \Omega_{jt}^s\right] + \frac{1}{2}\mathcal{S}\right)}; \tag{14}$$

- *the consumption of household i initially matched to a island j is*

$$c_{ijt} = \begin{cases} C^* \, e^{-\gamma\left(\ln p_{jt} - E\left[\ln W_t |\, \Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)} \equiv \mathcal{C}(p_{jt}) & \textit{if} \;\; \psi_{ij} > \hat{\psi}_{jt} \\ C^* & \textit{otherwise} \end{cases}; \tag{15}$$

- *the firm in island j faces constant elasticity of demand $(\gamma+\lambda)(1-\omega)$, so that the profit-maximizing price is*

$$p_{jt} = \frac{\mu}{z_{jt}} \, e^{E\left[\ln W_t |\, \Omega^p_{jt}\right] + \frac{1}{2}\mathcal{V}}, \tag{16}$$

where $\mathcal{V} = (2(\gamma+\lambda)\rho - 1)\,\mathcal{F}$ and μ is a constant markup given by

$$\mu = \begin{cases} \frac{(\gamma+\lambda)(1-\omega)}{(\gamma+\lambda)(1-\omega)-1} & \textit{if} \quad \omega < \frac{\gamma+\lambda}{\gamma+\lambda-1} \\ +\infty & \textit{otherwise} \end{cases}. \tag{17}$$

Proof. See Appendix B.1. □

With all equilibrium objects now defined, let us discuss how households' uncertainty affects their choices and the economy in a single island.

First, take the shopping decision. The threshold $\hat{\psi}_{jt}$ in (13) solves equation (5) with an equality so that only households with a high enough shopping cost will buy at the local seller. It is straightforward to notice that the log of $\hat{\psi}_{ijt}$ is equal to $1-\gamma$ times the log of the perceived relative local price. Therefore, the greater the perceived increase in relative price, the lesser the mass of household purchasing in island j, given by $\mathcal{N}(p_{jt}) = 1 - G(\hat{\psi}_{jt})$ in (14).[7] Next, the optimal consumption demand in (15) is obtained by using equations (4) and (6). Finally, we discuss the optimal firm pricing in equations (16)-(17). To this aim, let us derive here the expectation of the logarithm of the price level, conditional on the information sets $\Omega^{(\cdot)}_{jt}$, of shoppers, i.e. $E\left[\ln P_t |\, \Omega^s_{jt}\right]$, and firms, i.e. $E\left[\ln W_t |\, \Omega^f_{jt}\right]$, used respectively in equations (13)-(15) and (16). For given mean $E[\pi_t \,|\, \Omega^{(\cdot)}_{jt}]$ and variance $V(\pi_t \,|\, \Omega^{(\cdot)}_{jt})$ of posterior beliefs

[7]Note that, formally, this equilibrium requires that equilibrium prices in local markets be larger than the price in the competitive market with a probability arbitrarily close to 1, so that the shopping friction is binding for some consumers, i.e. $\hat{\psi}_{jt} > 0$, almost surely in all islands . This is achieved by assuming that average firm productivity in local islands is low enough, $\bar{z} < Z$. We notice that given the markup $\mu > 1$, and uncertainty in pricing, $\mathcal{V}$, and shopping, $\mathcal{S}$, Z is finite and need not be particularly small. In fact, in our calibration $\bar{z} = -\sigma^2/2$ so that $E(z) = 1$ would suffice.

about the current wage inflation innovation, we have:

$$E[\ln P_t \,|\, \Omega_{jt}^{(\cdot)}] = E[\ln P_t \,|\, \Omega_{t-1}] + E[\pi_t \,|\, \Omega_{jt}^{(\cdot)}], \tag{18}$$

with $V(\ln P_t \,|\, \Omega_t^{(\cdot)}) = V(\pi_t \,|\, \Omega_t^{(\cdot)})$ providing a bijective correspondence with the mean and variance of the distribution of posterior beliefs about the current competitive price level conditional on the information set $\Omega_t^{(\cdot)}$. Moreover, notice that because of (8), the expectations of P_t and W_t are identical.[8] Combining equations (12) and (18), we obtain that households' observation of an increase in (the logarithm of) local prices $\ln p_{jt}$ increases their perceived relative price, $\ln p_{jt} - E\left[\ln P_t | \, \Omega_{jt}^s\right]$, by a factor $1 - \omega$. This result, together with equations (13)-(15) implies that local firms face a constant demand elasticity equal to $(\lambda + \gamma)(1 - \omega)$. Hence, the profit-maximizing price in (16) is characterized by a constant markup over nominal marginal cost which by assumption is log-normally distributed, verifying our conjecture in equation (12).

The next two subsections discuss the economic forces behind the equilibrium, focusing on the impact of households' uncertainty about a perceived relative price change on the economy of a single island. The first considers how signaling power impacts on equilibrium markups and firms' profits. The second focuses on the variations of island-specific consumption in response to changes in local prices and inflation perceptions.

3.1 Signaling power and discretionary pricing

The signaling power of local prices affects the equilibrium pricing policy of firms. According to (17), the optimal markup is increasing in the elasticity of households' expectations to local prices, as measured by $\omega \in (0, 1)$, because demand elasticity decreases in signaling power. In fact, the possibility that local prices may increase because of economy-wide inflation rather than local conditions makes households more reluctant exert shopping effort as they expect a similar price increase to occur in the competitive market.

However, counter-intuitively, in equilibrium greater signaling power entails lower average

[8] The term $E[P_t \,|\, \Omega_{t-1}] = P_{t-1} + \chi \ln \Pi_{t-1}$ is readily interpretable as the expectation of the current price level in the competitive market conditional on past information.

profits. This is because by pricing under discretion the firm cannot prevent a failure of coordination with consumers. We illustrate this result formally in Proposition 2 and graphically in Figure 2.

Proposition 2. *Firms' real profits are decreasing in signaling power, that is*

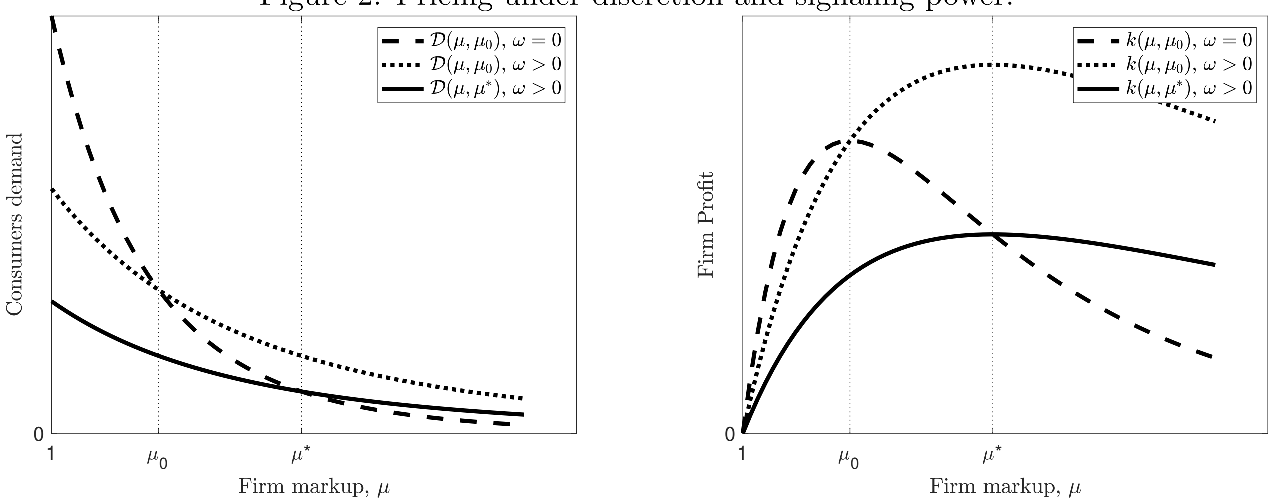
$$E\left[\frac{\partial k_{jt}}{\partial \omega}\right] < 0.$$

provided $\omega \in (0, 1]$.

Proof. See Appendix B.2.

Figure 2 illustrates the intuition behind the proposition. The two panels plot consumers' demand $\mathcal{D}(\mu, \mu^e)$ and firms' profit $k(\mu, \mu^e)$ as functions of a given markup μ (possibly off-equilibrium) and of the consumers' expected markup μ^e. The dotted line denotes the standard case of no signaling power, $\omega = 0$. Denote the optimal markup in this case as $\mu_0 = (\gamma + \lambda)/(\gamma + \lambda - 1)$.

Figure 2: Pricing under discretion and signaling power.



Suppose the firm sets the markup μ_0 when $\omega > 0$. Such a strategy cannot be an equilibrium, as a firm setting discretionary prices has an incentive to deviate in the presence of signaling power. This incentive is illustrated by the dashed line taking as given consumers' expectation of the deterministic component of prices $E[p_{jt} \,|\, \Omega_{t-1}]$ in (12), i.e. $\mu^e = \mu_0$. In this situation, any increase in the markup above μ_0 will be interpreted – in proportion to ω – as a change in aggregate inflation, lowering consumers' demand elasticity. This appears in the left-hand panel as a flattening of consumers' demand and in the right-hand panel as a higher profit curve for markups greater than μ. The latter shows the possibility of increasing profits by raising the markup to $\mu^* = (\gamma+\lambda)(1-\omega)/((\gamma+\lambda)(1-\omega)-1)$.

However, in a rational expectation equilibrium, consumers' expectation $E[p_{jt}\,|\,\Omega_{t-1}]$ must correctly evaluate the actual markup, so that $\mu^e = \mu^*$. This implies a downward shift in demand and in firms' profits from the dashed to the solid curve. The resulting equilibrium profits are lower than with no signaling power.

To sum up, signaling power may cause households to confuse an off-equilibrium move in markups with a positive inflationary shock, flattening their demand schedule. This induces firms to increase markups as they cannot commit not to exploit this power. In response, consumers' demand shifts downwards compressing profits.

3.2 Island consumption and local price variations

The decisions of households on *where* (shopping choice) and *how much* (the consumption choice) to buy are both functions of the perceived real local price $E\left[p_{jt}/P_t|\,\Omega^s_{jt}\right]$. The equilibrium local demand can be expressed as

$$\mathcal{D}(p_{jt}) = C^*\, e^{-(\lambda+\gamma)\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}, \tag{19}$$

according to (14) and (15). In terms of shopping choice, a higher perceived relative local price increases the gain from shifting to the competitive market, with an elasticity of the mass of customers to such a perceived price equal to λ. In terms of consumption choice, a higher local price is incentivates the saving of stayers according to an intertemporal elasticity of substitution equal to γ. Thus, in reaction to a rise in the perceived real local price, local consumption shrinks with elasticity $\lambda+\gamma$. At the same time, consumption in the competitive market increases by an amount equal to the mass of consumers induced to leave the local market multiplied by their new level of consumption. In particular, let $\mathcal{C}_{jt} \equiv \mathcal{N}(p_{jt})\mathcal{C}(p_{jt}) + (1-\mathcal{N}(p_{jt}))\,C^*$ denote the total consumption of type-j households. Clearly, the variation in the local prices reallocates demand from the local to the competitive market.

Hence, the net effect is ambiguous, depending on the strength of demand elasticities, as according to Proposition 3.

Proposition 3. *The elasticity of island-j households' total consumption, $\mathcal{C}_{jt}$, to a marginal increase in the perceived relative local price, $p^e_{jt} = E\left[p_{jt}/P_t | \, \Omega^s_{jt}\right]$ evaluated at $p^e_{jt} = \mu$, is strictly positive if and only if:*

$$\lambda \left[\frac{(\gamma+\lambda)(1-\omega)}{(\gamma+\lambda)(1-\omega)-1} \right]^{\gamma} > \lambda + \gamma. \tag{20}$$

A sufficient condition for (20) to be satisfied is that $\gamma \leq 1$.

Proof. See Appendix B.3.

Condition (20) requires that the elasticity of the increase in island-j demand in the competitive market – equal to the elasticity of switching λ multiplied by the consumption gain from the absence of markup – has to be greater than the elasticity of the decrease in local demand $\lambda + \gamma$.

To better appreciate the economics behind this condition is instructive to consider initially the log-utility case, $\gamma = 1$, and no signaling power, $\omega = 0$. In this case, the left-hand side of (20) matches the right-hand side, meaning there is zero net change in island-j consumption. This is intuitive. First, with log-preferences consumers' spending is independent of the price, so the left-hand side of (20) also measures the elasticity of the contraction in real revenues in the local market.[9] Second, given the linear technology, the left-hand side of (20) also measures the elasticity of the reduction in local firms' total cost. Given that the markup is chosen optimally by the firm to equate variations in revenues with variations in costs, neutrality necessarily obtains in this case.

The presence of some signaling power, $\omega > 0$, breaks this neutrality by allowing the markup to increase – and hence the relative consumption gain from shopping in the competitive market – without affecting the contribution to the change in demand of the extensive margins relative to the overall change, i.e. $\varepsilon_{\mathcal{N}}/(\varepsilon_{\mathcal{N}} + \varepsilon_{\mathcal{C}}) = \lambda/(\gamma + \lambda)$, with $\varepsilon_{\mathcal{N}}$ and $\varepsilon_{\mathcal{N}}$ denoting the price elasticity of the extensive and intensive margins of demand, respectively. More generally, by increasing markups the presence of signaling power increases the consumption gain of

[9] That is, expressed in competitive price units.

switchers, magnifying the expansionary effect of an increase in real local prices. As a result, expansionary effects obtain beyond the case $\gamma \leq 1$ as the signaling power gets stronger.

4 Aggregate consumption and inflation

This section examines the role of imperfect learning in coordinating business fluctuations across islands due to aggregate nominal disturbances. More generally, we show that, within our setting, the magnitude and direction of the effects of inflationary shocks depend on the relative uncertainty of households and firms.

The mechanism is simple. Firms' markup is invariant, in equilibrium, to aggregate shocks. However, the effective markup paid by consumers does vary with the aggregate shock, as it gives rise to correlated switching choices. When households are less informed than firms, as wage inflation hits the competitive and local markets, households' inflation expectations across islands rise less than local prices, leading to an aggregate reallocation of spending from local to competitive markets.

Formally, the effective consumer price index is defined as

$$P_t^{eff}\, C_t = \left(1 - E_t\left[\mathcal{N}(p_{jt})\right]\right) P_t\, C^* + E_t\left[\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})p_{jt}\right]. \tag{21}$$

The relevant weights in the computation of the consumption price index depend both on demand per customer, captured by C^* and $\mathcal{C}(p_{jt})$ for the competitive and monopolistic sellers respectively, and on the customer share of each type of seller, captured by, $\mathcal{N}(p_{jt})$. Denote by $\bar{\mathcal{N}} = E\left[\mathcal{N}(p_{jt})\right]$, $\bar{\mathcal{C}} = E\left[\mathcal{C}(p_{jt})/C^*\right]$, and $\bar{\mathcal{P}} = E\left[p_{jt}/P_t\right]$ respectively the average mass of customers purchasing from local stores, the average consumption at local relative competitive sellers, and the average price paid in local islands relative to competitive prices.[10]

[10] Their analytical expressions are given by

$$\bar{\mathcal{N}} = \mu^{-\lambda}\, e^{\lambda\, \mathcal{Z} + .5\, \lambda^2\, \mathcal{Q}}, \qquad \bar{\mathcal{C}} = \mu^{-\gamma}\, e^{\gamma\, \mathcal{Z} + .5\, \gamma^2\, \mathcal{Q} + \lambda\, \gamma\, \mathcal{Q}}, \qquad \bar{\mathcal{P}} = \mu\, e^{-\bar{z} + .5\, \mathcal{Q}_p},$$

where $\mathcal{Z} = (1-\omega)\, \bar{z} + .5[(1-\omega)^2\, \sigma_z^2 + \mathcal{V} + \mathcal{S}]$, $\mathcal{Q} = [(1-\omega)^2\, \delta/\omega + \rho]\, \mathcal{S}$, and $\mathcal{Q}_p = (1 - 2\,(\lambda+\gamma)(1-\omega))\,(\sigma_z^2 + \delta\, \mathcal{V})$. Higher markups reduce both the number of customers and their consumption in local islands with elasticities λ and γ respectively. $\mathcal{Z}$, $\mathcal{Q}$ and $\mathcal{Q}_p$ capture, respectively, the role of the variation of idiosyncratic productivity and estimation errors by consumers and firms. Given that we work under the assumption that the average

We are now ready to establish the main results of the paper concerning the pass-through of nominal shocks to effective prices and consumption.

Proposition 4. *Let $\zeta \equiv \omega\,\delta + \rho$. The market share of local islands is given by*

$$N_t = \frac{\bar{\mathcal{N}}\,\bar{\mathcal{C}}\,e^{-(\lambda+\gamma)\,(\delta-\zeta)\,\pi_t}}{1 - \bar{\mathcal{N}}\,e^{-\lambda\,(\delta-\zeta)\,\pi_t} + \bar{\mathcal{N}}\,\bar{\mathcal{C}}\,e^{-(\lambda+\gamma)\,(\delta-\zeta)\,\pi_t}}. \tag{22}$$

The effective price paid by consumers, P_t^{eff}, is given by

$$\frac{P_t^{eff} - W_t}{W_t} = N_t\,\left(\bar{\mathcal{P}}\,e^{-(1-\delta)\,\pi_t} - 1\right). \tag{23}$$

Aggregate consumption, C_t, is given by

$$\frac{C_t - C^*}{C^*} = \bar{\mathcal{N}}\,e^{-\lambda\,(\delta-\zeta)\,\pi_t}\,\left(\bar{\mathcal{C}}\,e^{-\gamma\,(\delta-\zeta)\,\pi_t} - 1\right). \tag{24}$$

The proposition illustrates the three novel predictions of our mechanism as regards the effect of inflationary shocks on aggregate consumption. First, they affect aggregate consumption if and only if there is an information gap between consumers and firms, i.e. $\zeta \neq \delta$. Second, the sign of the comovement of inflation with consumption depends on the sign of $\delta - \zeta$ as well as on the relative strength of the extensive and intensive margins of demand, i.e. λ versus γ. Third, nominal shocks have real effects on aggregate consumption *even if* posted prices are fully flexible and characterized by full pass-through of wage shocks, i.e. $\delta = 1$.

We can now elaborate on these three results. Recall that the coefficients ζ and δ denote, respectively, the sensitivity of consumers' and firms' perceptions of the nominal price level, i.e. $E[\ln P_t|\,\Omega_{jt}^s]$ and $E[\ln P_t|\,\Omega_{jt}^f]$, to a change in the (log of the) nominal price level, $\ln P_t$. Crucially, the difference $\delta - \zeta$ measures the elasticity of the relative local price perceived by the consumer, $p_{jt}^e = E\left[p_{jt}/P_t|\,\Omega_{jt}^s\right]$, to a change in the local price p_{jt} originating from a change in aggregate nominal marginal cost. When $\delta = \zeta$, perceptions of relative prices do not respond to aggregate nominal shocks. Even if the pass-through of the nominal shock to firm's prices is small, i.e. δ is low, the effect on consumption is zero in this case. In other

productivity of local sellers is at most equal to that in the competitive sector ($\bar{z} \leq -.5\,\sigma_z^2$), their effect on consumption is typically second-order with respect to the effect of the markup.

words, so long as consumers and firms have the same sensitivity of inflation perceptions to inflationary shocks, nominal shocks are neutral (i.e. they do not move consumption) even if the pass-through to consumer prices is limited. Notice that the case $\delta = \zeta$ is possible when the precision of the private signal available to firms is greater than that of the signal available to consumers because consumers are able to learn about inflation from local prices on top of their private signal.

When $\delta > \zeta$ ($\delta < \zeta$) firms are relatively better (less) informed than consumers resulting in an increase (decrease) of the perceived relative price p^e_{jt}, when local price increases come in response to inflationary shocks. A change in the perceived relative price affects aggregate consumption through two channels: on the one hand, an increase in p^e_{jt} increases savings of non-switching consumers; and, on the other, it also increases the mass of consumers who switch. Because of the lower markup in the competitive sector, consumer switching increases total consumption.[11] While these two forces affect consumption in opposite directions, Proposition 3 tells us that the second dominates when the elasticity of intertemporal substitution γ is not too large for a given extensive margin elasticity λ and signaling power ω. Let us consider a first order Taylor expansion of (24) when $\sigma^2_\pi \to 0$ and $\mathcal{Z} = 0$,

$$\ln C_t - \ln \bar{C} \approx (\lambda\, \mu^\gamma - \gamma - \lambda)\, (\delta - \zeta)\, \kappa_c\, \pi_t, \tag{25}$$

with $\kappa_c = 1/(1 + \mu^\gamma\, (\mu^\lambda - 1)) > 0$. The first term on the right-hand-side of the equation is the object of interest of Proposition 3, capturing the net effect on demand of a shock to the real price perceived by consumers in an island, p^e_{jt}. The second term, $\delta - \zeta$, captures the effect of the shock to π_t on p^e_{jt}. If firms are better informed than consumers, $\delta > \zeta$, then an increase in aggregate inflation, π_t, is perceived by consumers as an increase in local prices relative to the aggregate. If instead consumers are better informed that firms, $\delta < \zeta$, we have the opposite. Combining the results of Propositions 3 and 4, it follows that inflationary shocks expand aggregate consumption for a standard parametrization of elasticity of intertemporal substitution, i.e. $\gamma \leq 1$, when firms are better informed than consumers, i.e. $\delta > \zeta$. We emphasize that, holding ζ fixed, the real effects of nominal shocks on consumption

[11] Notice that this follows from equation 24 because $\bar{\mathcal{C}} < 1$.

are maximized when firms' posted prices are fully responsive to nominal shocks. This result is in stark contrast with most of the existing literature.

It is helpful to study the special case where consumers cannot leave their island, obtained by setting $N_t = \bar{\mathcal{N}} = 1$ and $\lambda = 0$ in equations (23)-(24). This benchmark is particularly interesting because demand responds only to inter-temporal substitution incentives, which is the typical channel in the New-Keynesian narrative of demand driven business cycles. In this benchmark, the pass-through of wage inflation to the consumer price index is incomplete and is fully determined by the pass-through of wage shocks to firms' prices, itself governed by the parameter δ. The smaller δ, the greater the inertia in consumer prices. In order for inflationary shocks to expand aggregate demand, this benchmark needs consumers to be better informed than firms, i.e. $\zeta > \delta$. The motivation is straightforward. Wage inflation needs to be associated with a fall in the relative price p^e_{jt} perceived by consumers, so that households take advantage of the reduction in markups at local sellers thanks to the incomplete price pass-through and increase their demand. In the canonical island model of Lucas (1972), widely adopted in the macro literature, consumers are fully informed about inflation, i.e. $\zeta = 1$, so that the effects of nominal shocks on aggregate demand are maximized, for given pass-through to consumer prices. Table 4 summarizes the qualitative results of this discussion under the

Table 1: Comovement consumption-inflation

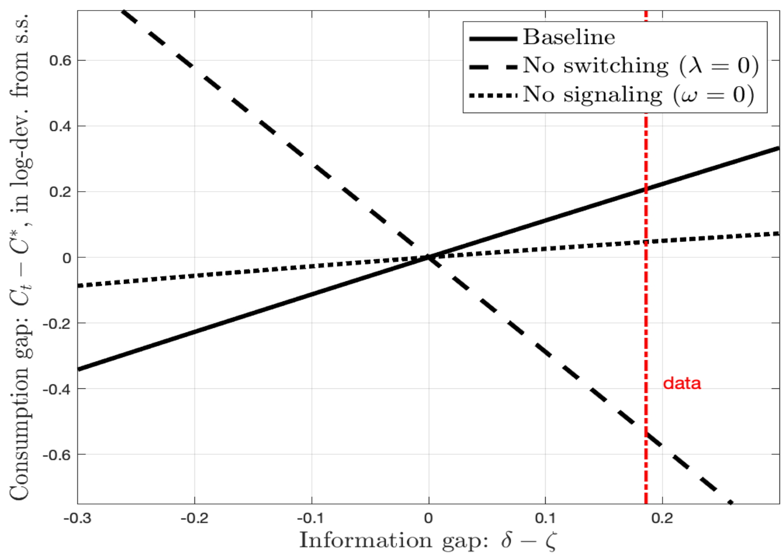
	With ext. margin: $\lambda > 0$	W/o ext. margin: $\lambda = 0$
Consumers better informed: $\zeta > \delta$	Negative	Positive
Firms better informed: $\delta > \zeta$	Positive	Negative

Note: Here we assume γ small enough so that the extensive margin dominates inter-temporal incentives. When $\omega > 0$, a sufficient condition is $\gamma \leq 1$.

assumption that γ is small enough so that the extensive margin dominates the intertemporal substitution incentives. Essentially, we propose a model in which the positive comovement of inflation and consumption stems from firms being better informed than consumers and from strategic demand reallocation between high- and low-markup sellers (the bottom-left box), whereas to date the literature has tended to posit frameworks in which demand responds only to inter-temporal incentives and consumers are better informed than firms (top-right box).

Figure 3 illustrates the working of equation (25) at our preferred calibration, plotting the response of the consumption gap $C_t - C^*$ to a 1% wage inflation shock, π_t, as a function of the information gap, $\delta - \zeta$. We set $\delta = 0.6$, consistent with estimates by Cavallo (2018).[12] The parameter governing the signaling power is set to $\omega = 0.19$, in order to target the sensitivity of expected inflation to experienced log-prices reported by D'Acunto et al. (2019), as shown in Table 2. The elasticity of demand along the intensive and extensive margin is set respectively

Figure 3: The response of consumption to a 1% shock to wage inflation



Note: The calibration is such that $\gamma = .5, \lambda = 4$, $\delta = 0.6$ and $\omega = 0.19$; ρ is left to vary on the horizontal axis.

to $\gamma = 1/2$ and $\lambda = 4$, implying a markup of $\mu = 1.38$.[13] The coefficient determining consumer information precision, ρ, is allowed to vary so as to have different values of the information gap $\delta - \zeta$ on the horizontal axis. The vertical red line indicates the value of ρ consistent with our estimates of the comovement of effective inflation and wage inflation reported in Section 6. In our model, the magnitude of the response of consumption to inflation increases with the information advantage of firms over consumers, $\delta - \zeta$ on the horizontal axis. At our preferred

[12] Cavallo (2018) estimates the short-run and long-run pass-trough of the nominal exchange rate to retail prices in the US for the period 2013-2017 using online data from a large number of multi-channel retailers.

[13] The intertemporal elasticity of substitution is set to $1/\gamma = 1/2$, standard in the macroeconomic literature. The elasticity of demand along the extensive margin is set to $\lambda = 4$, within the range of estimates found in the literature (see Kumar and Leone (1988) and Paciello et al. (2019)).

calibration of the information gap, a 1% shock to wage inflation, π_t, increases the consumption gap $C_t - C^*$ by 0.2% by comparison with a steady state with zero inflation. If we remove the signaling power, $\omega = 0$, the response of consumption is still increasing in $\delta - \zeta$ but with a much smaller slope, because the implied markup falls to $\mu = 1.28$. In the counterfactual where cross-sellers reallocation is not allowed, i.e $\lambda = 0$, the response of consumption is decreasing in $\delta - \zeta$, and at our preferred calibration a 1% inflation shock would imply a fall in consumption of 0.5%.

Finally, we discuss the implications of our model for the pass-through of nominal shocks to consumer prices. To this end, let the counterfactual posted price be defined as the average price paid by consumers if the market share of local sellers is held constant at its value in absence of aggregate shocks, i.e. $N_t = \bar{N}$,

$$\frac{P_t^{pos} - W_t}{W_t} = \bar{N}\,\left(\bar{\mathcal{P}}\,e^{-(1-\delta)\,\pi_t} - 1\right). \tag{26}$$

The pass-through of wage inflation to posted prices is increasing in the precision of information available to firms, as measured by δ. The pass-through to effective consumer prices in our model also factors in the comovement between inflation and the extent of reallocation from high- to low-markup sellers, as captured by N_t in (23). In general, N_t co-moves positively with inflation if $\zeta > \delta$, thus resulting in stronger co-movement of the effective price with wage inflation, whereas it co-moves negatively with inflation if $\delta > \zeta$, in which case it implies weaker co-movement and hence more rigid effective prices. Interestingly, even if firm pass wage inflation fully through posted prices, $\delta = 1$, the effective consumer price can still display rigidity in responding to nominal shocks if consumers are not perfectly informed, $\zeta < 1$. Notice further that $\delta = 1$ maximizes the effects of nominal shocks on consumption for given information available to consumers in our model. Therefore, firms' posted prices can be fully flexible despite a high degree of positive co-movement between inflation and consumption. Indeed, in our environment greater rigidity in firms' pricing, i.e. lower δ, results in *less* co-movement of consumption and inflation, particularly when signaling power is stronger, i.e. ω is larger.

5 Inflation Uncertainty and Welfare

Now we turn to the implications of our model for welfare, first characterizing its first-order determinants and then studying how a reduction in inflation uncertainty may affect it. Lastly, we derive the effects of a reduction in uncertainty for households only and not for firms, and vice-versa.

5.1 Welfare and Profits

Given that we posit that shocks last for only one period, without any loss of generality we can take the unconditional expectation of the present-flow utility of a household as our measure of welfare. Dropping the time subscripts, welfare is thus defined as

$$\mathcal{W} \equiv E\left[u(P) + \mathcal{N}(p_{jt})(u(p_{jt}) - u(P)) - \int_0^{\hat{\psi}(p_{jt})} \psi g(\psi) d\psi\right]$$

where $u(\mathcal{P}(s_{ijt})) = (\mathcal{C}(\mathcal{P}(s_{ijt}))^{1-\gamma^{-1}} - 1)/(1-\gamma^{-1}) - \varphi\, \ell_{ijt}$ denotes utility net of switching costs with $\Delta_{jt} \equiv E[u(p_{jt}) - u(P_t)|\Omega^s_{jt}]$ being the utility gain expected by households at the time of their shopping choice. Let $\tilde{\mathcal{W}} \equiv \Psi - \gamma/(\gamma-1)$ be the value of the welfare in a *perfect competition benchmark* with no shopping disutility, i.e. one in which all households buy effortlessly from the competitive market so that profits are zero. We can now state a new proposition.

Proposition 5. *Welfare measured in deviations from the perfect competition benchmark is given by*

$$\mathcal{W} - \tilde{\mathcal{W}} \;\; = \;\; \varphi\,(\mu^* - 1)\left(E\left[\mathcal{D}(p_{jt})\frac{p_{jt}}{P_t}\right] - \tilde{C}\right) + \varphi\, E\left[\mathcal{D}(p_{jt})\left(\frac{p_{jt}}{P_t} - e^{-z_{jt}}\right)\right] < 0$$

where $\mu^* = (\gamma+\lambda)/(\gamma+\lambda-1)$ is the markup in the absence of signaling power. Consider two allocations characterized by two different levels of welfare $\mathcal{W}_1$ and $\mathcal{W}_2$ respectively; the first-order approximation of their difference around $\mathcal{W}_1$ is given by

$$\mathcal{W}_1 - \mathcal{W}_2 \approx (\lambda+\gamma)\,(\mu^e_1 - 1)\,(\mu^e_2 - \mu^e_1)\,, \tag{27}$$

where

$$\mu_n^e \equiv \mu_n + \frac{1}{2}\mathcal{S}_n + \frac{1}{2}\mathcal{V}_n$$

with $n = \{1, 2\}$*, is the deterministic log-component of local prices at* W_n*.*

Proof. See Appendix B.5. □

The proposition defines welfare as the sum of two components. The first is the difference between the expected real local demand and the perfect competitive level $\tilde{C}$ at $\tilde{\mathcal{W}}$. This difference is negative and impacts on welfare proportionally to the net markup with no signaling power $\mu^* - 1$. The second term is average real profits, which are positive, although not sufficiently to reverse the overall sign of $\mathcal{W} - \tilde{\mathcal{W}}$.

Both components go to zero as the elasticity of demand goes to infinity, i.e. $\lim_{\gamma+\lambda\to\infty} \mu^* = 1$ at which we also have $\lim_{\gamma+\lambda\to\infty} K_t = 0$. In this case, market power is absent because either agents can shop in the competitive market at no cost (case $\lambda \to \infty$) or else they prefer saving everything then consuming to any price above the competitive price (case $\gamma \to \infty$). As a result, infinite demand elasticity entails our welfare benchmark $\tilde{\mathcal{W}}$.

The second part of the proposition means that first-order changes in welfare are driven by changes to the deterministic log-component of local prices, which in our economy constitute a direct measure of the efficiency wedge between social cost and social benefit of consumption in our economy. We can distinguish two channels through which nominal uncertainty affects welfare: the classical *allocational* channel and a novel *markup* channel. The latter, which is peculiar to our setting, is captured by variations in markup μ due to changes in signaling power ω; as we have seen, this is the product of households confusion between strategic and fundamental motives in local firms' pricing. The former, instead, is captured by variations in the terms $\mathcal{S}$ and $\mathcal{V}$, directly mapping into variances in the forecasting errors that households and local firms' make in setting their choices.

5.2 Policies of Uncertainty Reduction

A natural way for policy to affect welfare in this economy is by reducing uncertainty about inflation innovations. This may be done either through inflation stabilization, i.e. by setting nominal interest rates in reaction to changes in inflation, or thought public communication, i.e. by releasing a public news about inflation innovations. The two policies are equivalent since the anticipated component of inflation variation is neutral in our economy.

To understand how nominal stabilization affects welfare we need to study its effect through both the allocational and the markup channels. First of all, let notice that the relationship between local markup and stabilization, which is driven by changes in signaling power, is not straightforward. This can be appreciated by simply looking at the expression for ω that follows

$$\omega = \underbrace{\frac{1}{\delta}}_{\text{scale}} \times \underbrace{\frac{\sigma_f^{-2}}{\sigma_f^{-2} + \sigma_\nu^{-2} + \sigma_\pi^{-2}}}_{\text{relative precision}}, \tag{28}$$

where $\sigma_f^{-2} = 1/(\sigma_u^2 + \sigma_z^2/\delta^2)$ measures the the precision of the local prices. Signaling power depends on two factors: a scale and a relative precision factor. The latter captures the intrinsic informativeness of the price signal about inflation innovations; in practice it measures how much households would weight firms' signals if they were observing them *directly*. In fact, households only observe local prices, which reflect firms' signals up to the firms' own weight δ; thus, the optimal weight that households apply embodies a scale factor discounting δ.

When there is a reduction in σ_π^2, i.e. the ex-ante uncertainty about unanticipated inflation, the relative precision factor goes down as more precise public information reduces households' need for other sources of information. However, the same goes for firms, which, accordingly, respond less to their own signal; this induces a higher scale factor, which increases the signaling power.

The following proposition establishes the analytical condition governing the effect of nominal stabilization on welfare.

Proposition 6. *As the volatility of inflation innovation σ_π^2 decreases:*

i) the local markup μ decreases, provided that

$$\sigma_z^2 \; > \; \bar{\sigma}_z^2(\sigma_\pi^2) \; \in \; \left[0, \, \frac{\sigma_u^4}{\sigma_u^2 + \sigma_\nu^2}\right) \tag{29}$$

where $\bar{\sigma}_z^2(\sigma_\pi^2)$ is a threshold dependeing on σ_π^2 with $\bar{\sigma}_z' \geq 0$, which is satisfied whenever

$\mathcal{S} > \mathcal{F}$ (or equivalently $\delta > \zeta$);

ii) households' and firms' uncertainty, $\mathcal{S}$ and $\mathcal{F}$ respectively, always decreases.

Proof. See Appendix B.6. □

This proposition bears on the effect of changes in σ_π^2, spotlighting the crucial role of the dispersion in local productivity σ_z^2. With no such dispersion, households would learn firms' signal exactly, so that $\sigma_f^2 = \sigma_u^2$. In this case, we necessarily have that $\delta < \zeta$ as households receive one more signal than firms. This further implies that as σ_π^2 decreases, δ decreases faster than the relative precision of the price signal, measured by $\omega\delta$, meaning that ω increases. Dispersion in local productivity σ_z^2 blurs households' inference about firms' signals. Only when dispersion is sufficiently high is it possible for firms to be more informed than households. In such a case, as σ_π^2 decreases, σ_f^2 increases, so that the relative precision of prices, $\omega\delta$, diminishes more rapidly than δ, which implies that ω too declines.

Parts ii) and iii) of the proposition are useful to distinguish the type of externality defined here from other models of learning from prices studied in the literature. In particular, Amador and Weill (2010) have shown that communication can generate perverse welfare effects by crowding out the aggregation of private information in prices, which attenuates instead of aggravating agents' uncertainty.[14] In our model, instead, households' ex post uncertainty, and more generally the allocational loss in firms' and households' choices, always diminishes as communication increases; however, signaling power can deteriorate. And if signaling power is a dominant component of overall welfare losses, then welfare could actually decrease in response to nominal stabilization. The proposition itself makes it clear that this is not the case when firms are better informed than households, which is the relevant case as discussed in Section 4.

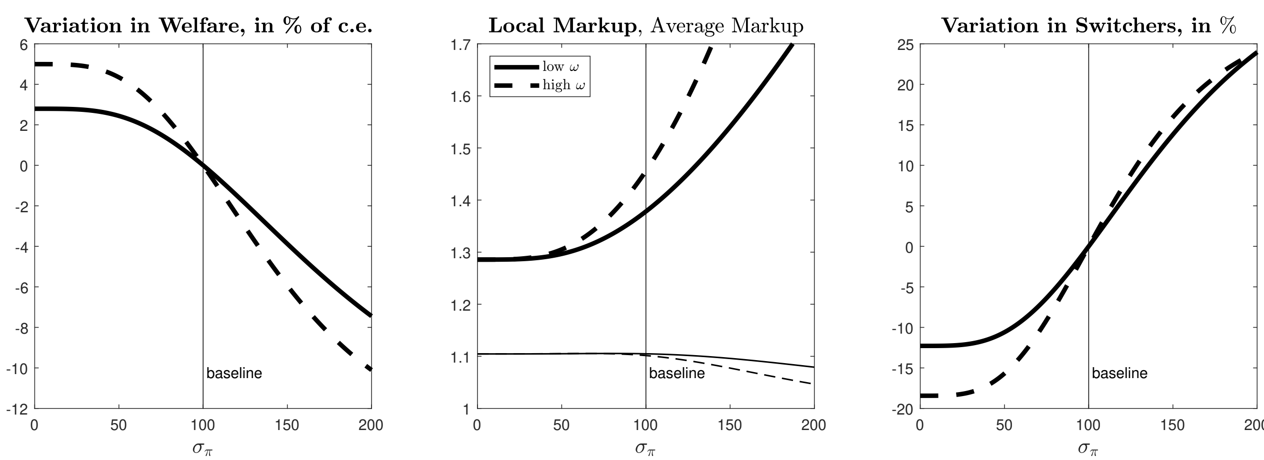
Figure 4 illustrates how stabilization affects welfare, markups and the fraction of switchers. The x-axis shows the value of σ_π normalized to 100 at a baseline value. The baseline configuration is as in Figure 3. We then distinguish two different calibrations of σ_ν, σ_z to match different values of ω: the solid line denotes lower values than the dashed line. At baseline values, the solid line has $\omega = 0.19$, the dashed line $\omega = 0.29$. These two values of are taken

[14]For some evidence of this mechanism in the context of forward guidance see Ehrmann et al. (2019).

directly from the estimates of D'Acunto et al. (2019). Finally, without loss of generality we normalize $\varphi = 1$.

The first important implication of the figure is that the first-order effects of inflation stabilization act almost exclusively through the markup channel. In fact, the *allocational* welfare loss $\mathcal{S} + \mathcal{V}$ is of the order of the variance of inflation innovation, hence negligible in our calibration. This is illustrated in the first panel of Figure 4. The variation in welfare is expressed in equivalent consumption growth units; that is, the y-axis measures the equivalent variation in consumption for both switchers and non-switchers relative to the respective baseline consumption that – with everything else fixed at baseline levels – would equate any level of welfare to the baseline level. The procedure to calculate this value is described in Appendix B.7. Complete stabilization of inflation volatility would be equivalent to roughly 3% of baseline consumption with "low" ω value and 5% with the "high" ω value. By contrast, a doubling of inflation volatility would decrease welfare by almost 8% of consumption equivalent in the "low" setting and by more than 10% in the "high" setting.

Figure 4: Policies of uncertainty reduction: variation in Welfare, in % of c.e.



Note: The figure shows how welfare varies with σ_π^2. The vertical dotted line corresponds to the baseline value of $\sigma_\pi = 0.35\%$. Welfare is calculated in equivalent consumption growth with respect to baseline consumption. We fixed $\gamma = .5, \lambda = 4$ and $\varphi = 1$. Calibration is such that at baseline, $\delta = 0.6$, $\rho = 0.1$ and $\omega = 0.29$ for the dashed line and $\omega = 0.19$ for the solid line.

Welfare is decreasing in inflation volatility because, in our calibration with $\delta > \rho$, this increases signaling power. This can be seen in the second panel of Figure 4. Complete stabilization produces symmetry in firms' and households' information, so that no signaling power is present without unanticipated aggregate nominal fluctuations; with no signaling power, the markup is 1.29. As inflation volatility increases, so does the dependence of households' expectations on firms' local prices, making local markups *inefficiently* high. A higher local markup induces households to shop in the competitive market and exert shopping effort that, absent signaling power¸ they would not have exerted. The third panel of Figure 4 shows that the inefficient increase in the mass of switchers is in the range of $10-20\%$. Thus, as inflation volatility increases, a rapid contraction of the customer base drives real profits down even as local markups are being raised. For the same reason, the average markup paid falls below 1.1 (its value in the absence of signaling power), as inflation volatility increases.

5.3 Targeted Communication

In the foregoing we have discussed the welfare impact of variations in ex-ante uncertainty for both households and firms at the same time. Now we look at the welfare impact of a variation in ex-post uncertainty of either firms or households, independently. We can think of the latter as the effect of communication targeted to households rather but not firms, as opposed to common communication. The following proposition captures the key features of such targeted communication.

Proposition 7. *Local markups μ are:*

i) increasing in the relative precision of households' exogenous signals, σ_ν^{-2};

ii) decreasing in the relative precision of firms' exogenous signals, σ_u^{-2};

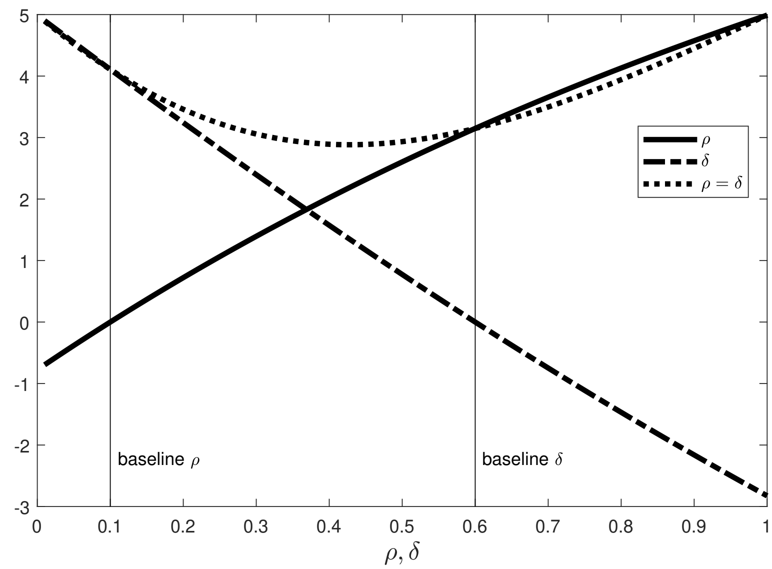
iii) at the no-signal-power level with either uninformative firms' signals or perfectly informative households' signals, i.e.

$$\lim_{\sigma_u^{-2}\to 0} \mu = \lim_{\sigma_\nu^{-2}\to\infty} \mu = \mu^*.$$

Proof. See Appendix B.8. □

To illustrate this, figure 5 we reproduces the "high" baseline of Figure 4 letting ρ and δ vary, these being measures of the relative precision of households' and firms' signal respectively, vary. More precisely, on the x-axis we measure the possible range of ρ and δ from zero to one. The y-axis, as before, measures the variation of welfare expressed in equivalent consumption growth units. The solid line considers variations of ρ, keeping σ_π, σ_z and δ fixed; by construction, it takes value 0 at the baseline value of $\rho = 0.1$. The dotted-dashed line considers variations of δ, keeping σ_π, σ_z and ρ fixed; by construction, it takes value 0 at the baseline value of $\delta = 0.6$. Finally, the dotted line considers variations of $\delta = \rho$, keeping σ_π, σ_z fixed.

Figure 5: Targeted communication: variation in Welfare, in % of c.e.



Note: The figure shows how welfare varies when ρ and δ vary, first independently (denoted by solid and dotted-dashed lines respectively) and then jointly at the same value (denoted by the dotted line). The vertical solid lines correspond to the baseline value of $\sigma_\pi^2 = 0.35\%^2$. Welfare is calculated in equivalent consumption growth with respect to baseline consumption. We fixed $\gamma = .5, \lambda = 4$ and $\varphi = 1$. Calibration is such that at baseline, $\delta = 0.6$, $\rho = 0.1$ and $\omega = 0.29$.

Higher ρ means greater relative precision of the signal available to households, hence less sensitivity of their expectations about local prices. This lowers inefficiently-high markups and so increases welfare. The opposite happens when, instead, δ is higher. The lack of commitment pushes firms to inefficiently increase markups, decreasing welfare. Interestingly, the same level of welfare is obtained by letting firms be totally uninformed or households perfectly informed. This happens because in both cases the signaling power is minimal.

It is instructive, finally, for purposes of comparison, to examine the case where $\rho = \delta$. At both extremes, welfare is the greatest when signaling power is least. The extreme in which both firms and households are perfectly informed corresponds to no ex-ante uncertainty: communication, or stabilization, is perfect. However, the other extreme, in which both firms and households are totally uninformed, does not correspond to the case of arbitrarily high ex-ante uncertainty on inflation innovations.[15]. In both situations agents' posteriors deteriorate: in case $\sigma_\pi^{-1} \to 0$, this is due to greater variance of inflation innovations; in case $(\rho, \delta) \to (0, 0)$, this is due to greater noise variance in signal. However, whereas in the latter case private signals loose weight as prior knowledge becomes relatively more precise, in the former, the opposite occurs. Only when firms have no private information, so that households do not have nothing to learn from them, does the inefficiency in markups, which is at the hearth of our mechanism, disappear.

6 Effective and Posted Price Inflation: Evidence

In this section we derive and validate a key testable prediction of our theory: namely, that wage inflation, π_t, has a first order effect on the gap between effective and posted price inflation, i.e. between $\pi_t^{eff} = \ln P_t^{eff} - \ln P_{t-1}^{eff}$ and $\pi_t^{pos} = \ln P_t^{pos} - \ln P_{t-1}^{pos}$, respectively. To see this we can use equations (23) and (26) to obtain a closed form expression for $\pi_t^{eff} - \pi_t^{pos}$ as a function of π_t^{pos} in a neighbour of $\pi_t = 0$:

$$\pi_t^{eff} - \pi_t^{pos} \approx \underbrace{-(\delta - \zeta)\,\kappa_\pi}_{\alpha}\, \pi_t^{pos}, \tag{30}$$

with

$$\kappa_\pi = \frac{\bar{\mathrm{N}}}{1 - \mu\,\bar{\mathrm{N}}\,(1-\delta)}\,\frac{\lambda + \gamma\,(1-\bar{\mathrm{N}})}{1 - \bar{\mathrm{N}}(1-\bar{\mathrm{N}})} > 0$$

[15]In this case, signaling power converges to

$$\lim_{\sigma_\pi^{-2} \to 0} \omega = \frac{\sigma_\nu^2}{\sigma_u^2 + \sigma_\nu^2 + \sigma_z^2}.$$

Intuitively, as long as uncertainty about inflation is increasing, there is less to learn from firms so one could expect the signaling power to vanish with sufficiently high volatility of inflation innovations. But, as long as firms observe noisy realizations of inflation, there is always something to learn from them, which is why signaling power converges to a finite value as inflation volatility increases.

if and only if $\lambda > 0$.[16] When firms are relatively better informed than consumers about nominal inflation, i.e. $\delta > \zeta$, there will be a negative comovement of the inflation gap, $\pi_t^{eff} - \pi_t^{pos}$, with posted price inflation, π_t^{pos}.

Our model predicts that inflation has first order effects on this gap: effective inflation increases (decreases) less than posted price inflation when inflation rises (falls) due to the greater (lesser) reallocation of expenditure from high- to low-price retailers. An aggregate inflation shock is confused with an idiosyncratic price shock, implying more or less reallocation from high to low markup firms, depending on the sign of the perceived change in local prices relatively to the aggregate. It is the combination of this confusion with the possibility of reallocating demand in accordance with changes in perceived relative prices that makes inflation having a first order effect on the gap. This in fact is a distinctive feature of our mechanism: in a typical New-Keynesian model, effective inflation increases less rapidly than posted price inflation when inflation rises, but decreases more rapidly than posted price inflation when inflation falls, so that average inflation has no first order effects on the gap.[17]

Next we test the null hypothesis that, when inflation increases, the effective price inflation experienced by consumers increases less than posted price inflation, i.e. $\alpha < 0$ in equation (30). We adopt the methodology and data proposed by Coibion et al. (2015) and studied further by Gagnon et al. (2017).[18] Whereas those authors have focused on the effect of unemployment on store switching, the focus here is on the effect of inflation. The "posted price", $p_{mscj,t}$ is the price for an item in week t, with m, s, c and j denoting respectively the market, the store, the product category and the UPC, and is obtained as $p_{mscj,t} = TR_{mscj,t}/TQ_{mscj,t}$, where $TR_{mscj,t}$ and $TQ_{mscj,t}$ are, respectively, the total revenues and quantities sold in the week. Each combination of product category and market is a "stratum". Posted price inflation

[16] Notice that $\bar{\mathbb{N}} = 1$ when $\lambda = 0$. The expression for κ_π is derived in the Appendix.

[17] See Section 2.1 of Chapter in Galí (2015) for details on the log-linearization of effective price inflation.

[18] Data on transaction prices comes Information Resources Inc. ("IRI") and includes weekly price and quantity information from 2001 to 2011 on items, each item defined as the interaction of an Universal Product Code (UPC) pertaining to one of 31 product categories and a seller pertaining to about 2,000 supermarkets and drugstores in 50 markets in the U.S. The product categories include housekeeping, personal care, food at home, cigarettes and photographic supplies.

aggregated at the stratum level is constructed as

$$\pi^{pos}_{mc,t} = \sum_{(s,j)\in I_{mc}} w_{mscj,t} \ln\left(\frac{p_{mscj,t}}{p_{mscj,t-1}}\right),$$

where I_{mc} is the set of items, i.e. different combination of UPCs j and stores s, in stratum $\{m, c\}$, and $w_{mscj,t}$ is its weight. The "effective price" of item j in market m and product category c is obtained by aggregating all expenditure across stores in a market m, whose set is denoted by S_m, $p^{eff}_{mscj,t} = \sum_{s\in S_m} TR_{mscj,t} / \sum_{s\in S_m} TQ_{mscj,t}$. Effective price inflation aggregated at the stratum level is then given by

$$\pi^{eff}_{mc,t} = \sum_{j\in J_{mc}} w_{mcj,t} \ln\left(\frac{p^{eff}_{mcj,t}}{p^{eff}_{mcj,t-1}}\right),$$

where J_{mc} is the set of UPCs j sold in the stratum $\{m, c\}$ and $w_{mcj,t}$ is its weight. All weights are computed at a yearly frequency so that the reallocation of consumer spending from high-price to low-price stores is captured by $\pi^{eff}_{mc,t}$ but not $\pi^{pos}_{mc,t}$. The weights $w_{mscj,t}$ are computed using the revenue shares of the item in a stratum in a year. Similarly, the weights $w_{mcj,t}$ are computed using the revenue shares of each UPC in a stratum in a year. The data is aggregated at the monthly frequency and used to estimate the following relationship:

$$\pi^{eff}_{mc,t} - \pi^{pos}_{mc,t} = \alpha\, \pi^{pos}_{mc,t} + \iota\, \sigma_{mc,t} + \varrho\, u_{m,t} + f_t + h_{mc} + error_{mc,t}, \tag{31}$$

where $\pi_{mc,t}$ and $\sigma_{mc,t}$ are, respectively, the average inflation and inflation dispersion in each stratum, and $u_{m,t}$ is the seasonally-adjusted unemployment in market m and month t; f_t and h_{mc} are time and stratum fixed effects, respectively. The top panel of Table 2 reports estimates of the baseline specification of equation (31). The bottom panel considers the case in which the measure of average posted price inflation and its dispersion on the right hand side is replaced by $\pi_{mc,t} = \pi^{unw}$ computed by aggregating the inflation rates of each item in the stratum with equal weights, i.e. $\pi^{unw}_{mc,t} = \frac{1}{N_{mc}} \sum_{(s,j)\in I_{mc}} \ln\left(\frac{p_{mscj,t}}{p_{mscj,t-1}}\right)$, with N_{mc} denoting the number of items in the stratum. As all items have the same weight, variation in $\pi^{unw}_{mc,t}$ is less likely than $\pi_{mc,t}$ to be driven by idiosyncratic shocks. Finally, the first two columns of Table 2 assign

Table 2: Gap between effective and posted price inflation

	Baseline			
Inflation mean, $\pi^{pos}_{mc,t}$	-0.40^{***}	-0.40^{***}	-0.33^{***}	-0.33^{***}
	(0.018)	(0.018)	(0.019)	(0.020)
Inflation dispersion, $\sigma^{pos}_{mc,t}$	0.003^{*}	0.003^{*}	0.004^{*}	0.004^{*}
	(0.0018)	(0.0018)	(0.0016)	(0.0016)
Unemployment, $u_{m,t}$		-0.15^{***}		0.009
		(0.04)		(0.05)
	Unweighted inflation			
Inflation mean, $\pi^{unw}_{mc,t}$	-0.40^{***}	-0.40^{***}	-0.36^{***}	-0.36^{***}
	(0.023)	(0.024)	(0.025)	(0.025)
Inflation dispersion, $\sigma^{unw}_{mc,t}$	0.003	0.003	0.006^{***}	0.006^{***}
	(0.002)	(0.002)	(0.002)	(0.002)
Unemployment, $u_{m,t}$		-0.14^{***}		0.016
		(0.04)		(0.04)
Stratum F.E.	Yes	Yes	Yes	Yes
Month F.E.	Yes	Yes	Yes	Yes
Weighted regressions	No	No	Yes	Yes

equal weight to the different strata, while the last two columns implement weighted panel regressions. The estimates of α reported in Table 2 are negative and statistically different from zero at the 1% significance level across all specifications; they are fairly insensitive to the measure of inflation used, and they change very little when unemployment is included. Our baseline specification with panel regression and posted price inflation implies $\alpha = -0.33$, which is the value we use to calibrate the model in the previous sections.[19] Not surprisingly, the effect of unemployment on the inflation gap is consistent with the findings of Gagnon et al. (2017) and is significant only when observations from the different markets are weighted equally.[20] The evidence on the relationship between the dispersion of inflation rates and the inflation gap is also inconsistent, with the estimated coefficient being mildly positive but not

[19]The vertical line in 3 corresponds to a value of $\rho = 0.1$ implying $\alpha = -0.33$ given all the other parameter values.

[20]They also argue that the results for unemployment depend on sample selection. In the Appendix we report results for different sample specifications, as suggested by these authors, and show that the estimate of α is consistently negative across the different specifications.

always statistically different from zero.

7 Conclusions

What is the source of output-inflation co-movement? To answer this question, a long tradition in macroeconomics puts firms' pricing frictions at the center of business cycle models. At first glance, this may appear inevitable in the formalization of the Keynesian logic. One needs firms as price setters in order for demand, not supply, to determine the actual quantity produced in equilibrium. Higher inflation translates into higher perceived real income, because inflation increases less than nominal income due to sticky posted prices.

This paper, instead, sets out a novel theory in which the evolution of effective consumer prices, not posted prices, is the engine of the New-Keynesian logic. This approach overturns the standard island setting of Lucas by assuming that uncertainty about the aggregate versus the idiosyncratic origin of fluctuations in posted prices affects demand but not supply decisions.

As consumers learn from posted prices, firms gain signaling power in pricing, which they use to maximize profits. A rise in posted prices may be interpreted as a signal of general inflation, discouraging consumers from searching for lower prices. In this context, higher inflation expands demand when it increases the share of consumers who re-locate from high- to low-markup firms, but it reduces demand when consumers do not relocate because higher inflation is associated with lower real income.

We have shown that in this framework what matters for the sign and the magnitude of the real effects of nominal shocks is the difference in the precision of the information available to firms and consumers, not simply the degree of precision of the information. Inflationary shocks are expansionary for output only if firms are better informed than consumers about the aggregate state; otherwise they are recessionary.

Moreover, firms exploit their signaling power by increasing their markups over marginal cost. Stabilization of the nominal price level has first-order impact, as it lowers markups by reducing firms' signaling power. Increasing the precision of the information available to consumers increases welfare, whereas better provision of information to firms may reduce it, because this has the opposite effect on firms' signaling power.

Finally we have produced corroborating evidence of a negative reallocation bias due to different???? inflationary shocks between effective and posted price inflation.

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A Mutual fund and monetary policy

There is a representative mutual fund operating in a competitive financial market. The mutual fund owns all firms in the economy, borrows in one period nominal bonds, B_t, from households, and lends in one period nominal bonds, Q_t, to the government. The flow budget constraint of the mutual fund is such that

$$\frac{B_t - Q_t}{R_t} + P_t K_t = B_{t-1} - Q_{t-1}, \tag{32}$$

where $K_t = \int_0^1 k_{jt}\, dj$ is the aggregate dividend, and $B_t = \int_0^1 \int_0^1 b_{ijt}\, di\, dj$ in equilibrium. There are exogenous shocks to the supply of assets to households which evolves according to

$$\Delta \ln B_t = \chi\, \Delta \ln B_{t-1} + b_t, \tag{33}$$

with $b_t \sim N(0, \sigma_b^2)$ i.i.d over time. The government finances the net repayment of debts to the mutual fund with a lump-sum tax on households,

$$T_t = Q_{t-1} - \frac{Q_t}{R_t}. \tag{34}$$

Monetary policy controls the equilibrium nominal interest rate R_t in the bond market through Q_t to achieve a given target for real supply of assets to the private sector,

$$\ln \frac{B_t}{P_t} = \phi\, \ln B_t. \tag{35}$$

This target effectively determines the path of the nominal price level in the economy, $\ln P_t = (1 - \phi)\, \ln B_t$. The case $\phi = 1$ corresponds to the case of full price level stabilization. Combining equations (33)-(35) we obtain the implied policy target for inflation,

$$\ln \Pi_t = \chi\, \ln \Pi_{t-1} + (1 - \phi)\, b_t. \tag{36}$$

This specification of monetary policy ensures that innovations to inflation are Gaussian distributed, $\pi_t = \ln \Pi_t - \chi \ln \Pi_{t-1} \sim N(0, \sigma_\pi^2)$, with $\sigma_\pi^2 = (1 - \phi)^2 \sigma_b^2$. This assumption, together with the functional form of distribution of shopping preferences $G(\psi)$, ensures the existence of Gaussian equilibrium which we can characterize analytically. Moreover, the parameter $\rho > 0$ allows to capture the relationship between inflation perceptions and expected inflation observed in the data.

We notice that the effect of an increase in B_t is isomorphic to an increase of the discount factor β, a popular reduce form way of modeling demand shocks. In this sense, we will refer to an increase in B_t as to a negative demand shock. In particular, changes in R_t transmits to demand in the product market by affecting the equilibrium nominal wage through equations (38)-(39), and the equilibrium price level P_t through (8).

First order conditions of the household problem. Let λ_{ijt} denote the Lagrangian multiplier on the household's budget constraint. The first order conditions to the household problem read:

$$c_{ijt}^{-\frac{1}{\gamma}} = E\left[\lambda_{ijt}\,\mathcal{P}(s_{ijt})\,|\,\Omega^c_{ijt}\right], \tag{37}$$

$$\lambda_{ijt} = E\left[\lambda_{ijt+1}\beta\,R_t\,|\,\Omega_t\right], \tag{38}$$

$$\varphi = \lambda_{ijt}\,W_t, \tag{39}$$

$$s_{ijt} = 1 \quad \text{iff} \quad \psi_{ij} \leq \hat{\psi}_{jt}, \tag{40}$$

together with the budget constraint in (3), where $\hat{\psi}_{jt}$ is the preference threshold at which the household is indifferent between staying or leaving the island.

Equilibrium definition. Given the initial aggregate state $\Omega_{t-1} = \{Q_{t-1}, B_{t-1}, \Delta \ln B_{t-1}\}$, and realizations of aggregate and idiosyncratic state, respectively b_t and $\{z_{jt}\}_{j\in[0,1]}$, and prior information $\{\theta_{jt}, \vartheta_{jt}\}_{j\in[0,1]}$, an equilibrium of this economy at time t is a collection of prices $\{P_t, W_t, R_t, \{p_{jt}\}_{j\in[0,1]}\}$, household choices $\{s_{ijt}, c_{ijt}, \ell_{ijt}\}_{i,j\in[0,1]\times[0,1]}$ and quantities $\{Q_t, T_t, D_t\}$ such that:

- P_t is given by equation (11) given P_{t-1}, Π_{t-1} and the realization of b_t;
- R_t solves equations (38)-(39) and T_t guarantees the equilibrium in the bond market in (32), consistently with the monetary policy target in (35)

$$\ln R_t = -\ln \beta + \rho\,\Delta \ln \Pi_t - .5\,\sigma_\pi^2, \tag{41}$$

$$T_t = B_{t-1} - \frac{B_t}{R_t} - P_t K_t; \tag{42}$$

- $W_t = P_t$ is given by (8), Q_t is given by (34) and $K_t = \int_0^1 k_{jt}\,dj$;
- in each island j, p_{jt} and k_{jt} solve the firm problem in equation (9); $b_{ijt} = B_t$ while l_{ijt} guarantees that the budget constraint in equation (3) holds $\forall i$; c_{ijt} and s_{ijt} solve, respectively, equation (6) and equations (40)-(5).

B Proofs

B.1 Proof of Proposition 1

Conjecture that the optimal pricing policy by firms is a constant markup over expected marginal cost, as in (16). Then p_{jt} is log-normal distributed because the expected nominal marginal cost is log-normal distributed. Given that P_t and W_t are log-normal distributed, it immediately follows from equations (6) and (5) that c_{ijt} and $\hat{\psi}_{jt}$ are also log-normal distributed. It is immediate to show from (6) that optimal consumption is given by

$$c_{ijt} = \begin{cases} \varphi^{-\gamma}\,e^{-\gamma\,(\ln p_{jt} - E[\ln P_t|\Omega^s_{ijt}] + \frac{1}{2}\mathcal{S})} & \text{if } \; s_{ijt} = 1 \\ \varphi^{-\gamma} & \text{if } \; s_{ijt} = 0 \end{cases}.$$

We next show that the optimal threshold to shop is given by equation (13). We want to compute the following object:

$$\Delta_{jt} = E\left[\frac{c_{ijt}^{1-\frac{1}{\gamma}}-1}{1-\frac{1}{\gamma}} - \varphi\,\ell_{ijt}\middle|\Omega^s_{ijt}, s_{ijt}=1\right] - E\left[\frac{c_{ijt}^{1-\frac{1}{\gamma}}-1}{1-\frac{1}{\gamma}} - \varphi\,\ell_{ijt}\middle|\Omega^s_{ijt}, s_{ijt}=0\right]$$

where labor ensures that the budget holds, i.e.

$$\ell_{ijt} = \frac{1}{W_t}\left(p_{jt}\,c_{ijt} - D_t\right).$$

Conjecture that the price function is given by equation (16). So we get:

$$\begin{aligned}
\Delta_{jt} &= E\left[\frac{\left(\varphi^{-\gamma}\,e^{-\gamma\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})}\right)^{1-\frac{1}{\gamma}} - \varphi^{1-\gamma}}{1-\frac{1}{\gamma}} - \frac{\varphi}{W_t}\left(p_{jt}\,\varphi^{-\gamma}\,e^{-\gamma\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})} - \varphi^{-\gamma}\,P_t\right)|\Omega^s_{ijt}\right] \\
&= \varphi^{1-\gamma}\,E\left[\frac{\left(e^{-\gamma\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})}\right)^{1-\frac{1}{\gamma}} - 1}{1-\frac{1}{\gamma}} - \left(e^{\ln p_{jt}-\ln P_t-\gamma\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})} - 1\right)|\Omega^s_{ijt}\right] \\
&= \varphi^{1-\gamma}\left[\frac{e^{(1-\gamma)\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})} - 1}{1-\frac{1}{\gamma}} - \left(e^{(1-\gamma)\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})} - 1\right)\right] \\
&= \varphi^{1-\gamma}\,\frac{e^{-(\gamma-1)\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})} - 1}{\gamma-1}.
\end{aligned}$$

Hence the threshold for shopping is such that $\Delta_{jt} + \hat{\psi}_{jt} = 0$, implying:

$$\hat{\psi}_{jt} = \frac{\varphi^{1-\gamma}}{\gamma-1} - \frac{\varphi^{1-\gamma}}{\gamma-1}\,e^{-(\gamma-1)\,(\ln p_{jt}-E[\ln P_t|\Omega^s_{ijt}]+\frac{1}{2}\mathcal{S})}.$$

By setting $\Psi = \frac{\varphi^{1-\gamma}}{\gamma-1}$ we obtain the optimal shopping policy of (13). Finally we verify our conjecture about the firm pricing policy. The mass of customers of firm in island j is given by $\mathcal{N}(p_{jt}) = 1 - G(\hat{\psi}_{jt})$ with G given in (2). The assumption $\frac{\xi}{\gamma-1} = \lambda < \infty$ guarantees that $G(\psi)$ is well defined at $\gamma = 1$: $\lim_{\gamma\to 1} G(\psi) = 1 - e^{-\lambda\,\psi}$. The assumption $\bar{z} < Z$ guarantees that $\hat{\psi}_{jt} > 0$ in all islands with probability arbitrarily close to 1. In particular, let

$$F = Pr\left(\ln\mu + \frac{1}{2}\mathcal{V} + \frac{1}{2}\mathcal{S} > (1-\omega)\,\ln z_{jt} - (1-\omega)\,\delta\,\vartheta_{jt} + \rho\,\theta_{jt}\right).$$

We have that F is increasing in $\bar{z}$, and $\lim_{\bar{z}\to Z} F = 1$ for some $Z > -\infty$, given $\ln\mu + \frac{1}{2}\mathcal{V} + \frac{1}{2}\mathcal{S} > 0$. Using (13) and $\lambda \equiv (\gamma-1)\,\xi$ and $F \approx 1$, we obtain that the elasticity of $\mathcal{N}(p_{jt})$ with respect to p_{jt} is equal in approximately all islands to $-(1-\omega)\,\lambda$, where ω denotes the elasticity of $E\left[\ln P_t|\,p_{jt},\,\theta_{jt}\right]$ with respect to p_{jt}, with the consumers correctly conjecturing that the pricing policy is characterized by the markup μ in (approximately) all islands. The demand of each customer $\mathcal{C}(p_{jt})$ is obtained evaluating equation (15) at $s_{ijt} = 1$, which readily implies an elasticity of demand -per-customer to p_{jt} equal to $-(1-\omega)\,\gamma$. Therefore, the overall elasticity of firm demand to a change in its own price is constant and given by $-(\lambda+\gamma)(1-\omega)$.

To find the optimal price, let us write the first order condition of real profits with respect to p_{ijt} taking as given past price p_{ijt-1}

$$\frac{\partial E\left[k_{jt}\middle|\Omega_{jt}^{f}\right]}{\partial p_{jt}}=E\left[-(\gamma+\lambda)(1-\omega)\frac{\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})}{p_{jt}}\left(\frac{p_{jt}}{W_t}-\frac{1}{z_{jt}}\right)+\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})\frac{1}{W_t}\middle|\Omega_{jt}^{f}\right]=0, \quad (43)$$

in which, notice, we used

$$\frac{\partial E\left[\ln p_{jt}\middle|\,\Omega_{jt-1}\right]}{\partial p_{jt}}=0.$$

Therefore, the optimal price is

$$p_{jt} \quad = \quad \mu\frac{E\left[\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})\middle|\Omega_{jt}^{f}\right]}{E\left[\mathcal{N}(p_{jt})\mathcal{C}(p_{jt}))\frac{1}{W_t}\middle|\Omega_{jt}^{f}\right]}\frac{1}{z_{jt}},$$

where

$$\mu=\begin{cases}\frac{(\gamma+\lambda)(1-\omega)}{(\gamma+\lambda)(1-\omega)-1} & \text{with} \quad \omega<\frac{\gamma+\lambda-1}{\gamma+\lambda},\\ +\infty & \text{with} \quad \omega\geq\frac{\lambda+\gamma-1}{\gamma+\lambda},\end{cases}$$

because of the assumption that z_{jt} is known to the firm. In a log-normal equilibrium the optimal price can be expressed as

$$\log p_{jt}=\ln\mu+E[\ln W_t|\Omega_{jt}^{f}]-\ln z_{jt}+\frac{1}{2}\mathcal{V}, \quad (44)$$

where

$$\mathcal{V}=V\left(\ln\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})\,\middle|\,\Omega_{jt}^{f}\right)-V\left(\ln\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})-\ln W_t\,\middle|\,\Omega_{jt}^{f}\right),$$

depends on the conditional correlation of wages and local demand. Let us now compute $\mathcal{V}$. First let us express:

$$\begin{aligned}V\left(\ln\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})\,\middle|\,\Omega_{jt}^{f}\right) &= (\gamma+\lambda)^2V\left(-\rho\,\vartheta_{jt}\,\middle|\,\Omega_{jt}^{f}\right)=(\gamma+\lambda)^2\rho^2(1-\delta)\sigma_\pi^2\\ V\left(\ln\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})-\ln W_t\,\middle|\,\Omega_{jt}^{f}\right) &= (\gamma+\lambda)^2V\left(-\rho\,\vartheta_{jt}\,\middle|\,\Omega_{jt}^{f}\right)+V\left(\ln P_t\,\middle|\,\Omega_{jt}^{f}\right)\\ &\quad +2(\gamma+\lambda)Cov\left(-\rho\,\vartheta_{jt};\ln P_t\,\middle|\,\Omega_{jt}^{f}\right)\\ &= \left((\gamma+\lambda)^2\rho^2+1-2(\gamma+\lambda)\rho\right)(1-\delta)\sigma_\pi^2.\end{aligned}$$

Therefore, we have

$$\mathcal{V}=-\left(1-2(\gamma+\lambda)\rho\right)(1-\delta)\sigma_\pi^2.$$

B.2 Proof of proposition 2

We rewrite average ex-post real profits (10) as

$$E\left[k_{jt}\right] \quad = \quad E\left[\mathcal{D}(p_{jt})\left(\frac{p_{ijt}}{W_t}-\frac{1}{z_{jt}}\right)\right]$$

By deriving the expression above with respect to $\mu = \mu_t$ at <u>any</u> t we get

$$\frac{\partial E\left[k_{jt}\right]}{\partial \mu} \quad = \quad E\left[-(\gamma+\lambda)\,\frac{1}{p_{jt}}\,\mathcal{D}(p_{jt})\left(\frac{p_{ijt}}{W_t}-\frac{1}{z_{jt}}\right)\,+\,\mathcal{D}(p_{jt})\,\frac{1}{W_t}\right].$$

Now we note that,

$$\frac{\partial E\left[k_{jt}\right]}{\partial \mu} \quad = \quad E\left[\underbrace{E\left[-(\gamma+\lambda)\,\frac{1}{p_{jt}}\,\mathcal{D}(p_{jt})\left(\frac{p_{ijt}}{W_t}-\frac{1}{z_{jt}}\right)\,+\,\mathcal{D}(p_{jt})\,\frac{1}{W_t}\,\middle|\,\Omega^f_{jt}\right]}_{\equiv f\left(\Omega_t,\Omega^f_{jt}\right)}\right],$$

Imosing the first order condition for firm profits, we obtain that in then case $\omega \in (0,1]$ then $f\left(\Omega_t,\Omega^f_{jt}\right) < 0$ for any possible $\left(\Omega_t,\Omega^f_{jt}\right)$, from which the result in the proposition.

B.3 Proof of proposition 3

Let us denote $p^e_{jt} \equiv e^{\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right]+\frac{1}{2}\mathcal{S}}$ then we have

$$\begin{aligned}
\frac{\partial \mathcal{C}_{jt}}{\partial \ln p^e_{jt}}\Big|_{p^e_{jt}=\mu} \quad &= \quad \mathcal{N}'(p^e_{jt})\left(\mathcal{C}(p^e_{jt})-\varphi^{-1}\right)+\mathcal{N}(p^e_{jt})\mathcal{C}'(p^e_{jt})\\
&= \quad -\lambda\mathcal{N}(p^e_{jt})\left(\mathcal{C}(p^e_{jt})-\varphi^{-1}\right)-\gamma\mathcal{N}(p^e_{jt})\mathcal{C}(p^e_{jt})\\
&= \quad -\left(\lambda\left(1-\left(\frac{(\gamma+\lambda)(1-\omega)}{(\gamma+\lambda)(1-\omega)-1}\right)^{\gamma}\right)+\gamma\right)\mathcal{N}(p^e_{jt})\mathcal{C}(p^e_{jt})\\
&= \quad -\left(\lambda+\gamma-\lambda\left(\frac{(\gamma+\lambda)(1-\omega)}{(\gamma+\lambda)(1-\omega)-1}\right)^{\gamma}\right)\mathcal{N}(p^e_{jt})\mathcal{C}(p^e_{jt}),
\end{aligned}$$

from which the result in the proposition.

B.4 Proof of Proposition 4

Aggregate consumption is given by:

$$\begin{aligned}
C_t &= \varphi^{-\gamma}(1-E_t\left[\mathcal{N}(p_{jt})\right])+E_t\left[\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})\right],\\
&= \varphi^{-\gamma}-\varphi^{-\gamma}\,E_t\left[e^{-\lambda\left(\ln p_{jt}-E\left[\ln P_t|\,\Omega^s_{jt}\right]+\frac{1}{2}\mathcal{S}\right)}\right]+\varphi^{-\gamma}\,E_t\left[e^{-(\lambda+\gamma)\left(\ln p_{jt}-E\left[\ln P_t|\,\Omega^s_{jt}\right]+\frac{1}{2}\mathcal{S}\right)}\right]
\end{aligned}$$

where

$$\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] = \ln\mu - z_{jt}(1-\omega)+\frac{1}{2}\mathcal{V}+(\delta-\zeta)\,\pi_t+\delta\,(1-\omega)\,u_{jt}-\rho\,\nu_{jt}$$

Let $\mathcal{Q} = (1-\omega)^2\sigma_z^2 + \delta^2\,(1-\omega)^2\,\sigma_u^2 + \rho^2\sigma_\nu^2$ and $\tilde{\mu} = \ln\mu - \bar{z}\,(1-\omega) + \frac{1}{2}\mathcal{V} + \frac{1}{2}\mathcal{S}$. Equilibrium output is given by

$$\begin{aligned}\frac{C_t}{C^*} &= 1 - e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{\lambda\,(\zeta-\delta)\,\pi_t} + e^{-(\lambda+\gamma)\,(\tilde{\mu}-\frac{\lambda+\gamma}{2}\mathcal{Q})+(\lambda+\gamma)\,(\zeta-\delta)\,\pi_t}\\ &= 1 - e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - e^{-\gamma\,(\tilde{\mu}-\frac{\gamma}{2}\mathcal{Q})+\gamma\,\lambda\mathcal{Q}+\gamma\,(\zeta-\delta)\,\pi_t}\right).\end{aligned}$$

The effective price is given by

$$P_t^{eff} = P_t\,C^*(1 - E_t\,[\mathcal{N}(p_{jt})])\frac{1}{C_t} + E_t\,[\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})p_{jt}]\,\frac{1}{C_t}.$$

We have

$$\begin{aligned}P_t^{eff}\frac{C_t}{C^*} &= P_t - P_t\,E_t\left[e^{-\lambda\left(\ln p_{jt}-E\left[\ln P_t|\,\Omega_{jt}^s\right]+\frac{1}{2}\mathcal{S}\right)}\right] + \,E_t\left[e^{-(\lambda+\gamma)\left(\ln p_{jt}-E\left[\ln P_t|\,\Omega_{jt}^s\right]+\frac{1}{2}\mathcal{S}\right)+\ln p_{jt}}\right]\\ &= P_t - P_t\,e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{\lambda\,(\zeta-\delta)\,\pi_t} + \,P_t E_t\left[e^{-(\lambda+\gamma)\left(\ln p_{jt}-E\left[\ln P_t|\,\Omega_{jt}^s\right]+\frac{1}{2}\mathcal{S}\right)+\ln p_{jt}-\ln P_t}\right]\\ &= P_t - P_t\,e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{\lambda\,(\zeta-\delta)\,\pi_t} + \,P_t e^{-(\lambda+\gamma)\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{(\lambda+\gamma)\,(\zeta-\delta)\,\pi_t}\,e^{-(1-\delta)\,\pi_t}\bar{\mathcal{P}}\end{aligned}$$

where we have used that

$$\begin{aligned}&-(\lambda+\gamma)\,\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega_{jt}^s\right] + \frac{1}{2}\mathcal{S}\right) + \ln p_{jt} - \ln P_t =\\ &= -(\lambda+\gamma)\,\left(\ln\mu + \frac{1}{2}\mathcal{V} - (1-\omega)z_{jt} + \delta(1-\omega)\,u_{jt} - \rho\,\nu_{jt}\right) + \ln\mu - z_{jt} + \delta\,u_{jt} + ((\lambda+\gamma)\,(\zeta-\delta) + \delta - 1)\pi_t\end{aligned}$$

and taking expectations

$$E_t\left[e^{-(\lambda+\gamma)\left(\ln p_{jt}-E\left[\ln P_t|\,\Omega_{jt}^s\right]+\frac{1}{2}\mathcal{S}\right)+\ln p_{jt}-\ln P_t}\right] = e^{-(\lambda+\gamma)\,(\tilde{\mu}-\frac{\lambda+\gamma}{2}\mathcal{Q})+\lambda\gamma\,\mathcal{Q}}\,e^{(\lambda+\gamma)\,(\zeta-\delta)\,\pi_t}\,e^{-(1-\delta)\,\pi_t}\bar{\mathcal{P}}$$

where

$$\bar{\mathcal{P}} \equiv e^{\ln\mu-\bar{z}-((\lambda+\gamma)(1-\omega)-.5)\,(\sigma_z^2+\delta^2\,\sigma_u^2)} = e^{\ln\mu-\bar{z}-((\lambda+\gamma)(1-\omega)-.5)\,(\sigma_z^2+\delta\,\mathcal{V})}$$

Then

$$\begin{aligned}\frac{P_t^{eff}}{P_t} &= \frac{1 - e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})+\lambda\gamma\,\mathcal{Q}}\,e^{\lambda\,(\zeta-\delta)\,\pi_t} + e^{-(\lambda+\gamma)\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{(\lambda+\gamma)\,(\zeta-\delta)\,\pi_t}\,e^{-(1-\delta)\,\pi_t}\bar{\mathcal{P}}}{1 - e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - e^{-\gamma\,(\tilde{\mu}-\frac{\gamma}{2}\mathcal{Q})+\gamma\,\lambda\mathcal{Q}+\gamma\,(\zeta-\delta)\,\pi_t}\right)}\\ &= \frac{1 - e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - e^{-\gamma\,(\tilde{\mu}-\frac{\gamma}{2}\mathcal{Q})+\gamma\,\lambda\mathcal{Q}+\gamma\,(\zeta-\delta)\,\pi_t}\,e^{-(1-\delta)\,\pi_t}\bar{\mathcal{P}}\right)}{1 - e^{-\lambda\,(\tilde{\mu}-\frac{\lambda}{2}\mathcal{Q})}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - e^{-\gamma\,(\tilde{\mu}-\frac{\gamma}{2}\mathcal{Q})+\gamma\,\lambda\mathcal{Q}+\gamma\,(\zeta-\delta)\,\pi_t}\right)}\\ &= \frac{1 - \bar{\mathcal{N}}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - \bar{\mathcal{C}}\,e^{\gamma\,(\zeta-\delta)\,\pi_t}\,\bar{\mathcal{P}}\,e^{-(1-\delta)\,\pi_t}\right)}{1 - \bar{\mathcal{N}}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - \bar{\mathcal{C}}\,e^{\gamma\,(\zeta-\delta)\,\pi_t}\right)}\\ &= 1 + \frac{\bar{\mathcal{N}}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\bar{\mathcal{C}}\,e^{\gamma\,(\zeta-\delta)\,\pi_t}\,\left(\bar{\mathcal{P}}\,e^{-(1-\delta)\,\pi_t} - 1\right)}{1 - \bar{\mathcal{N}}\,e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - \bar{\mathcal{C}}\,e^{\gamma\,(\zeta-\delta)\,\pi_t}\right)}\\ &= 1 + \frac{\bar{\mathcal{N}}\,\bar{\mathcal{C}}\,e^{(\lambda+\gamma)\,(\zeta-\delta)\,\pi_t}}{1 - \bar{\mathcal{N}}\,e^{\lambda\,(\zeta-\delta)\,\pi_t} + \bar{\mathcal{N}}\,\bar{\mathcal{C}}\,e^{(\lambda+\gamma)\,(\zeta-\delta)\,\pi_t}}\,\left(\bar{\mathcal{P}}\,e^{-(1-\delta)\,\pi_t} - 1\right) = 1 + N_t\,\left(\bar{\mathcal{P}}\,e^{-(1-\delta)\,\pi_t} - 1\right)\end{aligned}$$

B.5 Proof of proposition 5

Welfare is given by

$$\mathcal{W} \equiv E\left[u(P) + \mathcal{N}(p_{jt})(u(p_{jt}) - u(P)) - \int_0^{\hat{\psi}(p_{jt})} \psi g(\psi) d\psi\right].$$

Given that $\mathcal{N}(p_{jt})$ is measurable with respect to Ω^s_{jt}, we can write $E[\mathcal{N}(p_{jt})(u(p_{jt}) - u(P))] = E[\mathcal{N}(p_{jt})E[u(p_{jt}) - u(P)|\Omega^s_{jt}]]$ where

$$\Delta_{jt} \equiv E[u(p_{jt}) - u(P)|\Omega^s_{jt}] = \Psi\left(e^{-(\gamma-1)\,(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})} - 1\right). \tag{45}$$

where $\Psi = \varphi^{1-\gamma}/(\gamma-1)$, so that

$$\mathcal{N}(p_{jt})\Delta_{jt} = \Psi\left(e^{-(\gamma+\lambda-1)\,(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})} - e^{-\lambda\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}\right). \tag{46}$$

We note that

$$E[\mathcal{N}(p_{jt})] = E\left[e^{-\gamma(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})}\right]$$

so that

$$E[\mathcal{N}(p_{jt})\Delta_{jt}] + \Psi E[\mathcal{N}(p_{jt})] = \Psi e^{-(\gamma+\lambda-1)\,(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})}. \tag{47}$$

Let us calculate now the cumulative shopping effort cost

$$\begin{aligned}
-\int_0^{\hat{\psi}(p_{jt})} \psi g(\psi) d\psi &= -\int_0^{\hat{\psi}(p_{jt})} \xi \frac{\psi}{\Psi}\left(1 - \frac{\psi}{\Psi}\right)^{\xi-1} d\psi \\
&= -\left[-\frac{\Psi + \psi\xi}{\xi+1}\left(\frac{\Psi - \psi}{\Psi}\right)^{\xi}\right]_0^{\hat{\psi}} \\
&= \frac{\Psi + \hat{\psi}\xi}{\xi+1}\left(1 - \frac{\hat{\psi}}{\Psi}\right)^{\xi} - \frac{\Psi}{\xi+1} \\
&= -\frac{\Psi}{\xi+1}\left(1 - \mathcal{N}(p_{jt})\right) - \frac{\xi}{\xi+1}\Delta_{jt}\mathcal{N}(p_{jt})
\end{aligned}$$

exploiting the fact $\hat{\psi}_{jt} = -\Delta_{jt}$. We know that

$$u(P_t) = \Psi - \frac{\gamma}{\gamma-1} + \varphi K_t$$

where note $E[K_t] = E[k_{jt}]$. Once denoting $\tilde{\mathcal{W}} = \Psi - \gamma/(\gamma-1)$ we can write

$$\begin{aligned}
\mathcal{W} - \tilde{\mathcal{W}} - \varphi E[K_t] &= E\left[\Delta_{jt}\mathcal{N}(p_{jt}) - \frac{\Psi}{\xi+1}\left(1-\mathcal{N}(p_{jt})\right) - \frac{\xi}{\xi+1}\Delta_{jt}\mathcal{N}(p_{jt})\right] \\
&= E\left[\frac{1}{\xi+1}\Delta_{jt}\mathcal{N}(p_{jt}) + \frac{\Psi}{\xi+1}\mathcal{N}(p_{jt}) - \frac{\Psi}{\xi+1}\right] \\
&= \frac{\Psi(\gamma-1)}{\lambda+\gamma-1}E\left[e^{-(\gamma+\lambda-1)\left(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S}\right)}\right] - \frac{\Psi(\gamma-1)}{\lambda+\gamma-1} \\
&= \Psi(\gamma-1)(\mu^*-1)\left(E\left[e^{-(\gamma+\lambda-1)\left(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S}\right)}\right] - 1\right) \\
&= \varphi(\mu^*-1)\left(E\left[\mathcal{D}(p_{jt})\frac{p_{jt}}{P_t}\right] - C^*\right)
\end{aligned}$$

and finally

$$\begin{aligned}
\mathcal{W} - \tilde{\mathcal{W}} &= \varphi(\mu^*-1)\left(E\left[\mathcal{D}(p_{jt})\frac{p_{jt}}{P_t}\right] - C^*\right) + \varphi E[K_t] \\
&= \varphi(\mu^*-1)\left(E\left[\mathcal{D}(p_{jt})\frac{p_{jt}}{P_t}\right] - C^*\right) + \varphi E\left[\mathcal{D}(p_{jt})\left(\frac{p_{jt}}{P_t} - e^{-z_{jt}}\right)\right].
\end{aligned}$$

We can approximate a first-order welfare loss at $\ln\bar{\mu}^e = \ln\bar{\mu} + \bar{\mathcal{S}}/2 + \bar{\mathcal{V}}/2$ and $z = 0$ by using results of the first-order approximation of profits:

$$\begin{aligned}
\mathcal{W} - \bar{\mathcal{W}} &\approx -\left((\lambda+\gamma-1)\mu^*\bar{\mu}^e - (\lambda+\gamma)\right)(\mu^e - \bar{\mu}^e) \\
&\approx -(\lambda+\gamma)\left(\bar{\mu}^e - 1\right)(\mu^e - \bar{\mu}^e).
\end{aligned}$$

To prove that the sign is negative let us first look at the derivative w.r.t. μ (this holds by considering μ^e with $\mathcal{S} > \mathcal{V}$)

$$\begin{aligned}
\frac{\partial\mathcal{W}}{\partial\mu} &= \frac{\partial\Psi\,(\gamma-1)\,\mu^{-(\lambda+\gamma)}\,\left(\mu^*\,\mu - 1\right)}{\partial\mu} \\
&= \Psi\,(\gamma-1)\,\mu^{-(\lambda+\gamma)}\left(-(\gamma+\lambda)\frac{1}{\mu}(\mu^*\,\mu-1) + \mu^*\right) \\
&= \Psi\,(\gamma-1)\,\mu^{-(\lambda+\gamma)}\left(-(\gamma+\lambda)\left(\frac{\gamma+\lambda}{\gamma+\lambda-1} - \frac{(\gamma+\lambda)(1-\omega)-1}{(\gamma+\lambda)(1-\omega)}\right) + \frac{\gamma+\lambda}{\gamma+\lambda-1}\right) \\
&= \Psi\,(\gamma-1)\,\frac{1}{\omega-1} \leq 0
\end{aligned}$$

For sufficiently small σ_π, we have that

$$\mathcal{W} - \tilde{\mathcal{W}} = \Psi(\gamma-1)\left(\mu^{-(\gamma+\lambda)}\left(\mu^*\,\mu - 1\right) - (\mu^*-1)\right) < 0,$$

which can be established by noting that, because of the sign of $\frac{\partial\mathcal{W}}{\partial\mu}$, we have

$$\max_{\mu}\{\mathcal{W} - \tilde{\mathcal{W}}\} = \Psi(\gamma-1)\left(\mu^{*-(\gamma+\lambda)}\left(\mu^*+1\right) - 1\right)(\mu^*-1) < 0$$

where

$$\mu^{*-(\gamma+\lambda)} = \left(\frac{\gamma+\lambda}{\gamma+\lambda-1}\right)^{-(\gamma+\lambda)} < \frac{1}{\mu^*+1} = \frac{\gamma+\lambda-1}{2(\gamma+\lambda)-1}$$

holds for any $\gamma+\lambda > 1$.

B.6 Proof of proposition 6

Households forecast error variance $\mathcal{S}$. Does an increase in the precision of public communication can increase, instead of decreasing, overall uncertainty of households $\mathcal{S}$? This could be the case when the direct increase in precision via σ_π^{-1} is overturned by the indirect loss of precision via σ_f^{-1} similarly to Amador and Weill (2010). Mathematically, an increase in the precision of public information decreases households' forecast error variance whenever

$$\frac{\partial \mathcal{S}}{\partial \sigma_\pi^{-2}} = -\left(\frac{\partial \sigma_\pi^{-2} + \partial \sigma_f^{-2}}{\partial \sigma_\pi^{-2}}\right)\mathcal{S}^2 = -\left(1 - 2\sigma_u^2\sigma_z^2 \frac{\sigma_\pi^{-2}\sigma_u^2+1}{\left(\sigma_\pi^{-4}\sigma_u^4\sigma_z^2 + 2\sigma_\pi^{-2}\sigma_u^2\sigma_z^2 + \sigma_u^2+\sigma_z^2\right)^2}\right)\mathcal{S}^2 < 0,$$

which is always the case, since

$$\left(\sigma_\pi^{-4}\sigma_u^4\sigma_z^2 + 2\sigma_\pi^{-2}\sigma_u^2\sigma_z^2 + \sigma_u^2+\sigma_z^2\right)^2 - 2\sigma_u^2\sigma_z^2\left(\sigma_\pi^{-2}\sigma_u^2+1\right) > 0,$$

is the same as

$$\frac{\sigma_\pi^8\sigma_u^4 + \sigma_\pi^8\sigma_z^4 + 2\sigma_\pi^6\sigma_u^4\sigma_z^2 + 4\sigma_\pi^6\sigma_u^2\sigma_z^4 + 2\sigma_\pi^4\sigma_u^6\sigma_z^2 + 6\sigma_\pi^4\sigma_u^4\sigma_z^4 + 4\sigma_\pi^2\sigma_u^6\sigma_z^4 + \sigma_u^8\sigma_z^4}{\sigma_\pi^8} > 0.$$

So despite an externality similar to the one emphasized by Amador and Weill (2010) is present in our setting, it is not sufficiently strong to let households' forecast error variance being increasing in the precision of public information.

Signaling power and nominal stabilization. Let us state first explicit expressions for ω and $\delta\,\omega$ that we will be useful as we go on:

$$\begin{aligned}
\delta\omega &= \frac{\sigma_\pi^6\sigma_\nu^2}{\sigma_\pi^6\sigma_u^2 + \sigma_\pi^6\sigma_\nu^2 + \sigma_\pi^6\sigma_z^2 + \sigma_\pi^4\sigma_u^2\sigma_\nu^2 + \sigma_\pi^2\sigma_u^4\sigma_z^2 + 2\sigma_\pi^4\sigma_u^2\sigma_z^2 + \sigma_\pi^4\sigma_\nu^2\sigma_z^2 + \sigma_u^4\sigma_\nu^2\sigma_z^2 + 2\sigma_\pi^2\sigma_u^2\sigma_\nu^2\sigma_z^2} \\
\omega &= \frac{\sigma_\pi^4\sigma_\nu^2\left(\sigma_\pi^2+\sigma_u^2\right)}{\sigma_\pi^6\sigma_u^2 + \sigma_\pi^6\sigma_\nu^2 + \sigma_\pi^6\sigma_z^2 + \sigma_\pi^4\sigma_u^2\sigma_\nu^2 + \sigma_\pi^2\sigma_u^4\sigma_z^2 + 2\sigma_\pi^4\sigma_u^2\sigma_z^2 + \sigma_\pi^4\sigma_\nu^2\sigma_z^2 + \sigma_u^4\sigma_\nu^2\sigma_z^2 + 2\sigma_\pi^2\sigma_u^2\sigma_\nu^2\sigma_z^2}
\end{aligned}$$

Now, we compute the derivative,

$$\frac{\partial\omega}{\partial\sigma_\pi^2} = \frac{\partial(1/\delta)}{\partial\sigma_\pi^2}\delta\omega + \frac{1}{\delta}\left(\frac{\partial(\delta\omega)}{\partial\sigma_f^2}\frac{\partial\sigma_f^2}{\partial\sigma_\pi^2} + \frac{\partial(\delta\omega)}{\partial\sigma_\pi^2}\right),$$

by looking at its components separately:

$$
\begin{aligned}
\frac{\partial(1/\delta)}{\partial\sigma_\pi^2} &= \frac{\partial\left(\frac{1}{\left(\frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}}\right)}\right)}{\partial\sigma_\pi^2} = -\frac{\sigma_u^2}{\sigma_\pi^4} < 0 \\
\frac{\partial(\delta\omega)}{\partial\sigma_f^2} &= \frac{\partial\left(\frac{\sigma_f^{-2}}{\sigma_f^{-2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}\right)}{\partial\sigma_f^2} = -\left(\sigma_\pi^{-2}+\sigma_\nu^{-2}\right)\left(\frac{\sigma_f^{-2}}{\sigma_f^{-2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}\right)^2 = -\left(\sigma_\pi^{-2}+\sigma_\nu^{-2}\right)\delta^2\omega^2 < 0 \\
\frac{\partial\sigma_f^2}{\partial\sigma_\pi^2} &= -\frac{2}{\sigma_\pi^6}\sigma_u^2\sigma_z^2\left(\sigma_\pi^2+\sigma_u^2\right) < 0 \\
\frac{\partial\left(\delta\omega\right)}{\partial\sigma_\pi^2} &= \frac{\partial\left(\frac{\sigma_f^{-2}}{\sigma_f^{-2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}\right)}{\partial\sigma_\pi^2} = \frac{\sigma_\pi^{-4}}{\sigma_f^{-2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}\frac{\sigma_f^{-2}}{\sigma_f^{-2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}} = \frac{\sigma_\pi^{-4}}{\sigma_f^{-2}}\delta^2\omega^2 > 0
\end{aligned}
$$

Let us first compute the derivative

$$
\begin{aligned}
\frac{\partial(\delta\omega)}{\partial\sigma_f^2}\frac{\partial\sigma_f^2}{\partial\sigma_\pi^2}+\frac{\partial(\delta\omega)}{\partial\sigma_\pi^2} &= \left(\frac{2\left(\sigma_\pi^{-2}+\sigma_\nu^{-2}\right)\sigma_u^2\sigma_z^2\left(\sigma_\pi^2+\sigma_u^2\right)}{\sigma_\pi^6}+\frac{\sigma_\pi^{-4}}{\sigma_f^{-2}}\right)\delta^2\omega^2 \\
&= \left(\frac{2\left(\sigma_\pi^{-2}+\sigma_\nu^{-2}\right)\sigma_u^2\sigma_z^2\left(\sigma_\pi^2+\sigma_u^2\right)}{\sigma_\pi^6}+\sigma_\pi^{-4}\left(\sigma_u^2+\frac{\sigma_z^2}{\left(\frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}}\right)^2}\right)\right)\delta^2\omega^2 \\
&= \underbrace{\frac{\left(\sigma_\pi^4\sigma_u^2\sigma_\nu^2+2\sigma_\pi^2\sigma_u^4\sigma_z^2+2\sigma_\pi^4\sigma_u^2\sigma_z^2+\sigma_\pi^4\sigma_\nu^2\sigma_z^2+4\sigma_\pi^2\sigma_u^2\sigma_\nu^2\sigma_z^2+3\sigma_u^4\sigma_\nu^2\sigma_z^2\right)\sigma_\pi^{-8}}{\sigma_\nu^2}}_{=A}\delta^2\omega^2
\end{aligned}
$$

so that

$$
\begin{aligned}
\frac{\partial\omega}{\partial\sigma_\pi^2}\frac{\sigma_\pi^2}{\omega} &= \left(\frac{\partial(1/\delta)}{\partial\sigma_\pi^2}\omega\delta+\frac{1}{\delta}\left(\frac{\partial(\delta\omega)}{\partial\sigma_f^2}\frac{\partial\sigma_f^2}{\partial\sigma_\pi^2}+\frac{\partial(\delta\omega)}{\partial\sigma_\pi^2}\right)\right)\frac{\sigma_\pi^2}{\omega} \\
&= \left(-\frac{\sigma_u^2}{\sigma_\pi^{-4}}\omega\delta+\frac{1}{\delta}A\delta^2\omega^2\right)\frac{\sigma_\pi^2}{\omega} \\
&= -\sigma_u^2\sigma_\pi^{-2}\delta+\sigma_\pi^2A\delta\omega.
\end{aligned}
$$

Let us re-order terms

$$
\frac{\partial\omega}{\partial\sigma_\pi^2}\frac{\sigma_\pi^2}{\omega} = -\sigma_u^2\sigma_\pi^{-2}\delta+\sigma_u^2\sigma_\pi^{-2}\delta\omega+\left(2\sigma_\pi^2\sigma_\nu^{-2}\sigma_u^4+2\sigma_\pi^4\sigma_\nu^{-2}\sigma_u^2+\sigma_\pi^4+4\sigma_\pi^2\sigma_u^2+3\sigma_u^4\right)\sigma_z^2\sigma_\pi^{-6}\delta\omega
$$

then we have

$$
\lim_{\sigma_z\to 0}\frac{\partial\omega}{\partial\sigma_\pi^2}\frac{\sigma_\pi^2}{\omega} < 0
$$

which takes value 0 when

$$-\sigma_u^2\sigma_\pi^{-2}+\frac{\left(\sigma_u^2\sigma_\pi^{-2}+\left(2\sigma_\pi^2\sigma_\nu^{-2}\sigma_u^4+2\sigma_\pi^4\sigma_\nu^{-2}\sigma_u^2+\sigma_\pi^4+3\sigma_u^4+4\sigma_\pi^2\sigma_u^2\right)\sigma_z^2\sigma_\pi^{-6}\right)\sigma_\pi^4\sigma_\nu^2\left(\sigma_\pi^2+\sigma_u^2\right)}{\sigma_\pi^6\sigma_u^2+\sigma_\pi^6\sigma_\nu^2+\sigma_\pi^6\sigma_z^2+\sigma_\pi^4\sigma_u^2\sigma_\nu^2+\sigma_\pi^2\sigma_u^4\sigma_z^2+2\sigma_\pi^4\sigma_u^2\sigma_z^2+\sigma_\pi^4\sigma_\nu^2\sigma_z^2+\sigma_u^4\sigma_\nu^2\sigma_z^2+2\sigma_\pi^2\sigma_u^2\sigma_\nu^2\sigma_z^2}=0$$

that is at

$$\sigma_z^*=\frac{\sigma_\pi^3\sigma_u^2}{\left(\sigma_\pi^2+\sigma_u^2\right)\sqrt{\sigma_\pi^2\sigma_u^2+\sigma_\pi^2\sigma_\nu^2+2\sigma_u^2\sigma_\nu^2}},$$

where we note $\partial\sigma_z^*/\partial\sigma_\pi^2>0$.

For

$$\sigma_z>\sigma_z^*$$

then the signaling power is increasing in σ_π^2. Let us look at the two limits

$$\begin{aligned}\lim_{\sigma_\pi\to 0}\sigma_z^* &= 0,\\ \lim_{\sigma_\pi\to\infty}\sigma_z^* &= \frac{\sigma_u^2}{\sqrt{\sigma_u^2+\sigma_\nu^2}}.\end{aligned}$$

Let us look at when firms are more informed than households i.e. when

$$\begin{aligned}\left(\sigma_u^2+\frac{\sigma_z^2}{\left(\frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}}\right)^2}\right)^{-1}+\sigma_\nu^{-2}+\sigma_\pi^{-2} &< \sigma_u^{-2}+\sigma_\pi^{-2}\\ \left(\sigma_u^2+\frac{\sigma_z^2}{\left(\frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}}\right)^2}\right)^{-1} &< \sigma_u^{-2}-\sigma_\nu^{-2}\\ \sigma_u^2+\frac{\sigma_z^2}{\left(\frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}}\right)^2} &> \frac{1}{\sigma_u^{-2}-\sigma_\nu^{-2}}\\ \sigma_z^2 &> \left(\frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}}\right)^2\left(\frac{1}{\sigma_u^{-2}-\sigma_\nu^{-2}}-\sigma_u^2\right)\\ \sigma_z^2 &> \frac{-\sigma_\pi^4\sigma_u^4}{\left(\sigma_\pi^2+\sigma_u^2\right)^2\left(\sigma_u^2-\sigma_\nu^2\right)}\end{aligned}$$

which requires $\sigma_u<\sigma_\nu$.

Therefore, the condition

$$\frac{-\sigma_\pi^4\sigma_u^4}{\left(\sigma_\pi^2+\sigma_u^2\right)^2\left(\sigma_u^2-\sigma_\nu^2\right)}>\frac{\sigma_\pi^6\sigma_u^4}{\left(\sigma_\pi^2+\sigma_u^2\right)^2\left(\sigma_\pi^2\sigma_u^2+\sigma_\pi^2\sigma_\nu^2+2\sigma_u^2\sigma_\nu^2\right)},$$

which is the same as

$$2\frac{\sigma_\pi^4\sigma_u^6}{\left(\sigma_\pi^2+\sigma_u^2\right)^2}\frac{\sigma_\pi^2+\sigma_\nu^2}{\left(\sigma_\nu^2-\sigma_u^2\right)\left(\sigma_\pi^2\sigma_u^2+\sigma_\pi^2\sigma_\nu^2+2\sigma_u^2\sigma_\nu^2\right)}>0,$$

establishes that whenever firms are more informed than households (which requires $\sigma_\nu>\sigma_u$) then more precision in public information always decreases ω, i.e. the signaling power. This concludes our proof.

B.7 Consumption-equivalent welfare measure

Suppose a welfare measures $\bar{\mathcal{W}} = \mathcal{W}_{baseline} + k$ where $\mathcal{W}_{baseline}$ is a convenient reference. Let us denote

$$\mathcal{K}(C^*, C) \equiv E\left[(1 - N(p_{jt}))\frac{(C^*)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + N(p_{jt})\frac{C(p_{jt})^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}\right]$$

where notice $\mathcal{K}(x\,C^*, x\,C) = x^{1-\frac{1}{\gamma}}\mathcal{K}(C^*, C)$. We want to find the g_c such that

$$\mathcal{K}(g_c\,C^*, g_c\,C_{baseline}) - \mathcal{K}(C^*, C_{baseline}) = k$$

where $C_{baseline}$ is local consumption corresponding to $\mathcal{W}_{baseline}$. We get

$$g_c = \left(\frac{k + \mathcal{K}(C^*, C_{baseline})}{\mathcal{K}(C^*, C_{baseline})}\right)^{\frac{\gamma}{1-\gamma}}.$$

B.8 Proof of proposition 7

The proof of the proposition directly follows from inspection of the derivatives of signaling power, ω, with respect to σ_u^2 and σ_ν^2

$$\begin{aligned}
\frac{\partial\omega}{\partial\sigma_u^2} &= \frac{\left(-\sigma_\pi^4\sigma_\nu^2\right)\left(\sigma_u^4\sigma_\pi^2\sigma_z^2 + \sigma_u^4\sigma_\nu^2\sigma_z^2 + 2\sigma_u^2\sigma_\pi^4\sigma_z^2 + 2\sigma_u^2\sigma_\pi^2\sigma_\nu^2\sigma_z^2 + \sigma_\pi^8 + \sigma_\pi^6\sigma_z^2 + \sigma_\pi^4\sigma_\nu^2\sigma_z^2\right)}{\left(\sigma_u^4\sigma_\pi^2\sigma_z^2 + \sigma_u^4\sigma_\nu^2\sigma_z^2 + \sigma_u^2\sigma_\pi^6 + \sigma_u^2\sigma_\pi^4\sigma_\nu^2 + 2\sigma_u^2\sigma_\pi^4\sigma_z^2 + 2\sigma_u^2\sigma_\pi^2\sigma_\nu^2\sigma_z^2 + \sigma_\pi^6\sigma_\nu^2 + \sigma_\pi^6\sigma_z^2 + \sigma_\pi^4\sigma_\nu^2\sigma_z^2\right)^2} < 0 \\
\frac{\partial\omega}{\partial\sigma_\nu^2} &= \frac{\left(\sigma_\pi^6\left(\sigma_\pi^2 + \sigma_u^2\right)\right)\left(\sigma_\pi^4\sigma_u^2 + \sigma_\pi^4\sigma_z^2 + 2\sigma_\pi^2\sigma_u^2\sigma_z^2 + \sigma_u^4\sigma_z^2\right)}{\left(\sigma_\pi^6\sigma_u^2 + \sigma_\pi^6\sigma_z^2 + \sigma_\nu^2\sigma_\pi^6 + 2\sigma_\pi^4\sigma_u^2\sigma_z^2 + \sigma_\nu^2\sigma_\pi^4\sigma_u^2 + \sigma_\nu^2\sigma_\pi^4\sigma_z^2 + \sigma_\pi^2\sigma_u^4\sigma_z^2 + 2\sigma_\nu^2\sigma_\pi^2\sigma_u^2\sigma_z^2 + \sigma_\nu^2\sigma_u^4\sigma_z^2\right)^2} > 0
\end{aligned}$$

To show the limit results note that

$$\lim_{\sigma_u^2\to\infty}\omega = \lim_{\sigma_\nu^2\to 0}\omega = 0.$$

C Empirical evidence

We first derive the expressions for effective and posted price inflation. From

$$\ln(P_t^{eff}/P_t) \approx N_t\left(e^{-(1-\delta)\,\pi_t}\mu - 1\right)$$

we obtain

$$\pi_t^{eff} - \pi_t \approx \bar{N}\left(d\ln N_t - \mu\,(1-\delta)\,\pi_t\right)$$

and using the expression for N_t it can be rewritten as

$$\pi_t^{eff} - \pi_t \approx \bar{N}\,(\lambda+\gamma)(\zeta-\delta)\pi_t + \bar{N}\,(\zeta-\delta)\frac{\mu^{-\lambda}\lambda - \mu^{-\lambda-\gamma}(\lambda+\gamma)}{1-\mu^{-\lambda}+\mu^{-\lambda-\gamma}}\,\pi_t - \bar{N}\,(1-\delta)\,\mu\,\pi_t$$

where we have used

$$d\ln N_t = (\lambda+\gamma)(\zeta-\delta)\pi_t + \frac{\bar{\mathcal{N}}}{1-\bar{\mathcal{N}}+\bar{\mathcal{N}}\bar{\mathcal{C}}}\lambda(\zeta-\delta)\pi_t - \frac{\bar{\mathcal{N}}\bar{\mathcal{C}}}{1-\bar{\mathcal{N}}+\bar{\mathcal{N}}\bar{\mathcal{C}}}(\lambda+\gamma)(\zeta-\delta)\pi_t$$

implying

$$\pi_t^{eff} - \pi_t = -(1-\delta)\,\bar{N}\,\mu\,\pi_t + (\zeta-\delta)\,\frac{\lambda+\gamma\,(1-\mu^{-\lambda})}{1-\mu^{-\lambda}(1-\mu^{-\gamma})}\,\bar{N}\,\pi_t.$$

We define the posted price inflation as inflation keeping $N_t = \bar{N}$:

$$\pi_t^{pos} - \pi_t \equiv -\bar{N}\mu\,(1-\delta)\,\pi_t$$

Then

$$\pi_t^{eff} - \pi_t^{pos} = \frac{\lambda+\gamma\,(1-\mu^{-\lambda})}{1-\mu^{-\lambda}(1-\mu^{-\gamma})}\,\bar{N}\,(\zeta-\delta)\,\pi_t.$$

We use the same data and methodologies of Coibion et al. (2015) and Gagnon et al. (2017), denoted as CGH and GLSS respectively, to construct measures of effective and posted price inflation. Data on transaction prices comes Information Resources Inc. ("IRI") and includes weekly price and quantity information from 2001 to 2011 on items, each item defined as the interaction of an Universal Product Code (UPC) pertaining to 31 product categories and a store pertaining to about 2,000 supermarkets and drugstores across 50 U.S. markets.[1] We use the methodology detailed in the Online Appendix and the codes made available in the Data Set of GLSS to construct the measures of effective and posted price inflation. In particular, each combination of product category and market is a "stratum", corresponding to the level at which we, as well as CGH and GLSS, evaluate the variation in posted and price inflation. In each stratum there is a collection of weekly "price trajectories", each corresponding to an item, and containing information on the number of units sold, total revenues as well as a promotion flag and other characteristics.[2]

The weights $w_{mcj,t}$ used to aggregate effective price changes across UPCs are computed either using the revenue shares of each UPC in a stratum in a year, "Market-specific" weights, or the revenue share in the category across all markets in a year, "Common" weights. Similarly, the weights $w_{mscj,t}$ used to aggregate posted price changes across items are computed either using the revenue shares of each item in a stratum in a year, or the revenue share of the item in the category across all markets in a year.[3] Crucially, weights are computed at a yearly frequency so that the reallocation of consumer spending from high-price to low-price stores is captured by $\pi_{mc,t}^{eff}$ but not $\pi_{mc,t}^{pos}$. Finally, CGH and GLSS aggregate the micro data to a

[1]The product categories include housekeeping, personal care, food at home, cigarettes and photographic supplies.

[2]All observations pertaining to private labels are dropped. We refer to the GLSS for further details.

[3]CGH and GLSS also consider a third weighting scheme, "Uniform", where all UPCs or items receive the same weight in the stratum irrespectively of their sales. but focus their analysis on the Market-specific and Common weights because they control for the relative importance of items and UPCs in the computation of the π^{pos} and π^{eff} series providing more reliable estimates of store switching. See the discussion in GLSS for more details.

Table 3: Response of reallocation bias in inflation to local posted inflation

	Market (1)	Common (2)	Market (3)	Common (4)	Market (5)	Common (6)	Market (7)	Common (8)
				CGH				
Posted prices	-0.07^{***}	-0.11^{***}	-0.07^{***}	-0.11^{***}	-0.05	-0.10^{***}	-0.05	-0.11^{***}
	(0.023)	(0.021)	(0.023)	(0.021)	(0.038)	(0.034)	(0.038)	(0.034)
Unempl.			-0.13^{***}	-0.14^{***}			-0.09^{***}	-0.11^{***}
			(0.034)	(0.030)			(0.034)	(0.035)
				GLSS: without truncation				
Posted prices	-0.40^{***}	-0.46^{***}	-0.41^{***}	-0.46^{***}	-0.33^{***}	-0.38^{***}	-0.33^{***}	-0.38^{***}
	(0.018)	(0.019)	(0.018)	(0.019)	(0.019)	(0.018)	(0.020)	(0.019)
Unempl.			-0.16^{***}	-0.18^{***}			0.002	-0.04
			(0.04)	(0.04)			(0.004)	(0.05)
				GLSS: all data filters				
Posted prices	-0.06^{***}	-0.06^{***}	-0.09^{***}	-0.09^{***}	-0.04^{***}	-0.04^{***}	-0.07^{***}	-0.07^{***}
	(0.008)	(0.007)	(0.008)	(0.008)	(0.019)	(0.008)	(0.020)	(0.008)
Unempl.			0.02	0.02			0.04	0.05^{*}
			(0.03)	(0.03)			(0.03)	(0.03)
Stratum F.E.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month F.E.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Panel reg.	No	No	No	No	Yes	Yes	Yes	Yes

monthly frequency.

In Table 2 we only report results for the Market weights and the baseline sample selection proposed by GLSS. Table 3 reports estimates of equation (31) for different weighting schemes (even versus odd columns) and different sample selections.[4] The top panel refers to the CGH' sample which truncates all monthly item price movements that exceed, in absolute terms, 100 percent on an annualized basis. The middle panel refers to GLSS' sample which uses a less invasive method to control for outsize price adjustments, and is the same sample used in Table 2. The bottom panel implements a series of extra filtering of the data.[5] We refer to CGH and GLSS for a discussion about the advantages and disadvantages of the different approaches.

[4] In particular, columns 1-8 in Table 3 correspond to the sample specifications in columns 5-8 of GLSS. We have twice the number of columns because we estimate equation (31) with and without unemployment.

[5] A robust imputation procedure for missing observations, comparable definitions of monthly posted and effective price inflation, a proper stitching of the subsamples, and a simple control for clearance sales.

D Appendix - not for publication

D.1 Mapping from ω, δ and ρ to σ_z^2, σ_u^2 and σ_ν^2

$$\omega = \frac{1}{\delta}\frac{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}}{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}. \tag{48}$$

The remaining parameters determine the loadings of expectations on the exogenous signals,

$$\delta = \frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}} \quad \text{and} \quad \rho = \frac{\sigma_\nu^{-2}}{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}.$$

It is immediate to obtain expressions for the forecast errors of consumers and firms,

$$\mathcal{F} = \frac{1}{\sigma_u^{-2}+\sigma_\pi^{-2}} \quad \text{and} \quad \mathcal{S} = \frac{1}{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}.$$

Let $\sigma_f^2 = \sigma_u^2 + \sigma_z^2/\delta^2$. We can express σ_f^2, σ_u and σ_ν^2 as a function of δ, ω and ρ:

$$\begin{aligned}
\sigma_u^2 &= (1/\delta - 1)\sigma_\pi^2 \\
\delta\omega &= \frac{\sigma_f^{-2}}{\sigma_f^{-2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}} \\
\rho &= \frac{\sigma_\nu^{-2}}{\sigma_f^{-2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}
\end{aligned}$$

implies

$$\begin{aligned}
\sigma_f^2 &= \frac{1-\rho-\omega\,\delta}{\omega\,\delta}\sigma_\pi^2 \\
\sigma_\nu^2 &= \frac{\omega\,\delta}{\rho}\sigma_f^2 \\
\sigma_z^2 &= \frac{\delta}{\omega}(1-\omega-\rho)
\end{aligned}$$

D.2 Linear approximations in (25)-(30)

$$\ln(P_t^{eff}/P_t) \approx N_t\,\left(e^{-(1-\delta)\,\pi_t}\mu - 1\right)$$

we obtain

$$\pi_t^{eff} - \pi_t \approx \bar{N}\left(d\ln N_t - \mu\left(1-\delta\right)\pi_t\right)$$

and using the expression for N_t can be rewritten as

$$\pi_t^{eff} - \pi_t \approx \bar{N}\,(\lambda+\gamma)(\zeta-\delta)\pi_t + \bar{N}\,(\zeta-\delta)\frac{\mu^{-\lambda}\lambda - \mu^{-\lambda-\gamma}(\lambda+\gamma)}{1-\mu^{-\lambda}+\mu^{-\lambda-\gamma}}\,\pi_t - \bar{N}\,(1-\delta)\,\mu\,\pi_t$$

where we have used

$$d\ln N_t = (\lambda+\gamma)(\zeta-\delta)\pi_t + \frac{\bar{\mathcal{N}}}{1-\bar{\mathcal{N}}+\bar{\mathcal{N}}\bar{\mathcal{C}}}\lambda(\zeta-\delta)\pi_t - \frac{\bar{\mathcal{N}}\bar{\mathcal{C}}}{1-\bar{\mathcal{N}}+\bar{\mathcal{N}}\bar{\mathcal{C}}}(\lambda+\gamma)(\zeta-\delta)\pi_t$$

implying

$$\pi_t^{eff} - \pi_t = -(1-\delta)\,\bar{N}\,\mu\,\pi_t + (\zeta-\delta)\,\frac{\lambda+\gamma\left(1-\mu^{-\lambda}\right)}{1-\mu^{-\lambda}(1-\mu^{-\gamma})}\,\bar{N}\,\pi_t.$$

We define the posted price inflation as inflation keeping $N_t = \bar{N}$:

$$\pi_t^{pos} - \pi_t \equiv -\bar{N}\mu\left(1-\delta\right)\pi_t$$

Then

$$\pi_t^{eff} - \pi_t^{pos} = \frac{\lambda+\gamma\left(1-\mu^{-\lambda}\right)}{1-\mu^{-\lambda}(1-\mu^{-\gamma})}\,\bar{N}\left(\zeta-\delta\right)\pi_t.$$

Finally, using

$$d\ln\frac{C^*-C_t}{C^*} = -\frac{\bar{C}}{C^*-\bar{C}}\,d\ln C_t$$

and

$$d\ln\frac{C^*-C_t}{C^*} = \lambda\left(\zeta-\delta\right) - \frac{\bar{\mathcal{C}}}{1-\bar{\mathcal{C}}}\gamma(\zeta-\delta)$$

we obtain

$$d\ln C_t = \frac{C^*-\bar{C}}{\bar{C}}\left[\lambda - \frac{\bar{\mathcal{C}}}{1-\bar{\mathcal{C}}}\gamma\right](\delta-\zeta) = \frac{1-\bar{\mathcal{C}}}{1-\bar{\mathcal{N}}+\bar{\mathcal{N}}\bar{\mathcal{C}}}\left[\lambda - \frac{\bar{\mathcal{C}}}{1-\bar{\mathcal{C}}}\gamma\right](\delta-\zeta) = \frac{\delta-\zeta}{1-\bar{\mathcal{N}}+\bar{\mathcal{N}}\bar{\mathcal{C}}}\left[\lambda(1-\bar{\mathcal{C}})-\bar{\mathcal{C}}\gamma\right]$$

D.3 First-order approximation of profits

Given $\ln p^e_{jt} = \ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \mathcal{S}/2$ we rewrite average ex-post real profits (10) as

$$
\begin{aligned}
E\left[k_{jt}\right] &= E\left[\mathcal{D}(p^e_{jt})\left(\frac{p_{jt}}{W_t} - \frac{1}{z_{jt}}\right)\right] \\
&= E\left[E\left[\mathcal{D}(p^e_{jt})\frac{p_{jt}}{W_t}|\Omega^s_{jt}\right]\right] - E\left[\mathcal{D}(p_{jt})\frac{1}{z_{jt}}\right] \\
&= E\left[\mathcal{D}(p^e_{jt})p^e_{jt}\right] - E\left[\mathcal{D}(p^e_{jt})\frac{1}{z_{jt}}\right] \\
&= E\left[e^{-(\lambda+\gamma-1)\ln p^e_{jt}}\right] - E\left[e^{-(\lambda+\gamma)\ln p^e_{jt} - \ln z_{jt}}\right]
\end{aligned}
$$

Let us now compute a first order approximation at $\ln \bar{\mu}^e = \ln \bar{\mu} + \bar{\mathcal{S}}/2 + \bar{\mathcal{V}}/2$ and $z = 0$

$$
\begin{aligned}
E\left[\mathcal{D}(p^e_{jt})p^e_{jt}\right] &\approx E\left[\mathcal{D}(\bar{\mu}^e)\bar{\mu}^e - (\lambda+\gamma-1)\mathcal{D}(\bar{\mu}^e)\bar{\mu}^e\frac{(p^e_{jt} - \bar{\mu}^e)}{\bar{\mu}^e}\right] \\
&\approx \mathcal{D}(\bar{\mu}^e)\bar{\mu}^e - (\lambda+\gamma-1)\mathcal{D}(\bar{\mu}^e)\bar{\mu}^e E\left[\ln\frac{p^e_{jt}}{\bar{\mu}^e}\right] \\
&\approx \mathcal{D}(\bar{\mu}^e)\bar{\mu}^e - (\lambda+\gamma-1)\mathcal{D}(\bar{\mu}^e)\bar{\mu}^e\left(\ln\mu + \frac{1}{2}\mathcal{V} + \frac{1}{2}\mathcal{S} - \ln\bar{\mu}^e\right)
\end{aligned}
$$

in analogy, for the other term,

$$
E\left[\mathcal{D}(p^e_{jt})\frac{1}{z_{jt}}\right] \approx \mathcal{D}(\bar{\mu}^e) - (\lambda+\gamma)\mathcal{D}(\bar{\mu}^e)\left(\ln\mu + \frac{1}{2}\mathcal{V} + \frac{1}{2}\mathcal{S} - \ln\bar{\mu}^e\right).
$$

Given that $-(\gamma+\lambda-1)\bar{\mu}^e + (\gamma+\lambda) < 0$ for any $\bar{\mu}^e > \mu^* = (\gamma+\lambda)/(\gamma+\lambda-1)$ we conclude that for given λ and γ a first-order loss to profits is proportional to

$$
\ln\mu^e - \ln\bar{\mu}^e
$$

where

$$
\ln\mu^e = \ln\mu + \frac{1}{2}\left(\mathcal{S} - \mathcal{F} + 2(\gamma+\lambda)\,\rho\,\mathcal{F}\right).
$$

Note that $\bar{\mu}^e > \mu^*$ always whenever firms are more informed than households, i.e. $\bar{\mathcal{S}} > \bar{\mathcal{V}}$.

D.4 Algebra for Proof of Proposition 4

$$
\begin{aligned}
C_t &= \varphi^{-\gamma}(1 - E_t\left[\mathcal{N}(p_{jt})\right]) + E_t\left[\mathcal{N}(p_{jt})\mathcal{C}(p_{jt})\right], \\
&= \varphi^{-\gamma} - \varphi^{-\gamma}\, E_t\left[e^{-\lambda\left(\ln p_{jt} - E\left[\ln P_t |\, \Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}\right] + \varphi^{-\gamma}\, E_t\left[e^{-(\lambda+\gamma)\left(\ln p_{jt} - E\left[\ln P_t |\, \Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}\right]
\end{aligned}
$$

where

$$
\ln p_{jt} - E\left[\ln P_t |\, \Omega^s_{jt}\right] = \ln\mu - z_{jt}(1-\omega) + \frac{1}{2}\mathcal{V} + (\delta - \zeta)\,\pi_t + \delta\,(1-\omega)\,u_{jt} - \rho\,\nu_{jt}
$$

Let $\mathcal{Q} = (1-\omega)^2\sigma_z^2 + \delta^2\,(1-\omega)^2\,\sigma_u^2 + \rho^2\sigma_\nu^2$ and $\tilde{\mu} = \ln\mu + \frac{1}{2}\mathcal{V} + \frac{1}{2}\mathcal{S}$. Equilibrium output is given by

$$
\begin{aligned}
\frac{C_t}{C^*} &= 1 - e^{-\lambda\,(\tilde{\mu} - \frac{\lambda}{2}\mathcal{Q})}\, e^{\lambda\,(\zeta-\delta)\,\pi_t} + e^{-(\lambda+\gamma)\,(\tilde{\mu} - \frac{\lambda+\gamma}{2}\mathcal{Q}) + (\lambda+\gamma)\,(\zeta-\delta)\,\pi_t} \\
&= 1 - e^{-\lambda\,(\tilde{\mu} - \frac{\lambda}{2}\mathcal{Q})}\, e^{\lambda\,(\zeta-\delta)\,\pi_t}\,\left(1 - e^{-\gamma\,(\tilde{\mu} - \frac{\gamma}{2}\mathcal{Q}) + \gamma\,\lambda\mathcal{Q} + \gamma\,(\zeta-\delta)\,\pi_t}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{Q} &= (1-\omega)^2\sigma_z^2 + \delta^2\,(1-\omega)^2\,\sigma_u^2 + \rho^2\sigma_\nu^2 \\
&= (1-\omega)^2\,\delta^2(\sigma_z^2/\delta^2 + \sigma_u^2) + \rho^2\sigma_\nu^2 \\
&= [(1-\omega)^2\,\delta^2 + \rho\omega\delta]\frac{1-\rho-\omega\,\delta}{\omega\,\delta}\sigma_\pi^2 \\
&= [(1-\omega)^2\,\delta/\omega + \rho](1-\zeta)\sigma_\pi^2 = [(1-\omega)^2\,\delta/\omega + \rho]\mathcal{S}
\end{aligned}
$$