Bank of Israel **Research Department**

The Term Premium in a Small Open Economy: A Micro-Founded Approach

Alex Ilek and Irit Rozenshtrom*

Discussion Paper No. 2017.06
July 2017

Bank of Israel http://www.boi.org.il
[*] Alex Ilek, Bank of Israel Research Department – Email: alexei.ilek@boi.org.il, Tel.972-2-6552636. Irit Rozenshtrom, Bank of Israel Research Department – Email: irit.rozenshtrom@boi.org.il, Tel.972-2-6552569.

חטיבת המחקר, בנק ישראל ת״ד 780 ירושלים 91007
Research Department, Bank of Israel, POB 780, 91007 Jerusalem, Israel

פרמיית הסיכון על אג״ח במשק קטן ופתוח: ניתוח במסגרת מודל המבוסס על יסודות מיקרו-כלכליים

אלכס אילק ועירית רוזנשטרום

תקציר

מחקר זה מתמקד בפרמיית הסיכון על אג״ח ממשלתיות במשק קטן ופתוח, והוא בוחן אותה במסגרת מודל DSGE עם העדפות מסוג Epstein-Zin. אנו פותרים את המודל תוך שימוש בקירוב מסדר שלישי על מנת לאפשר לפרמיית הסיכון להשתנות על פני זמן, ובכך מרחיבים מחקרים על משקים סגורים למשק קטן ופתוח. אנו מוצאים כי כאשר המודל מאפשר לטכנולוגיה לזלוג מהעולם למשק הקטן (technological spillovers), הוא מסוגל לייצר בו-זמנית פרמיות סיכון חיוביות משמעותיות על אג״ח נומינליות בעולם ובמשק הקטן, שונות סבירה של המשתנים המקרו-כלכליים העיקריים, ומתאמים גבוהים בין המשתנים בכלכלות המקומית והעולמית, בהתאם לנתונים. המודל משמש אותנו כדי לבחון כיצד פתיחות המשק למסחר במוצרים ובנכסים פיננסיים (אג״ח) משפיעה על פרמיית הסיכון, ואנו מזהים שתי השפעות מנוגדות: מחד גיסא הפתיחות מקטינה את הפרמיה מפני שמשק פתוח מסוגל להתמודד עם זעזועים מקומיים טוב ממשק סגור. מאידך גיסא, בהינתן זליגה טכנולוגית, זעזועים טכנולוגיים בעולם יוצרים במשק פתוח פרמיית סיכון גבוהה יותר מאשר במשק סגור. במודל שלנו שתי השפעות אלו מקזזות זו את זו מבחינה כמותית, ולכן פרמיית הסיכון על אג״ח נומינליות ארוכות טווח במשק פתוח דומה לפרמיה במשק סגור בעל מאפיינים דומים.

The Term Premium in a Small Open Economy: A Micro-Founded Approach

Alex Ilek and Irit Rozenshtrom[*]
Research Department, Bank of Israel

July 2017

Abstract

We study the term premium on nominal and real government bonds in a small open economy within a micro-founded DSGE model with Epstein–Zin preferences. We solve the model using a third-order approximation to allow for time-varying risk premia. We thus extend previous work on closed economies to the case of a small open economy. We find that technological spillovers from the global economy to the small open economy are essential for the ability of the model to produce concurrently a substantial positive nominal term premium, realistic variability of the main macroeconomic variables, and high correlations between the global and domestic economies as evident in the data. We use the model to study the effect of the openness of the economy on bond risk premia. We identify two opposing effects of the openness of the economy on the nominal term premium. The better ability of the open economy to accommodate domestic shocks works to decrease the term premium in the open economy. By contrast, in the presence of technological spillovers from the global economy to the small open economy, the foreign technological shock generates a higher term premium in the open economy compared to a closed one. Quantitatively, in our model these effects roughly offset each other so that the term premium in the open economy is similar to the premium in an otherwise similar economy that is closed to trade in goods and financial assets.

1 Introduction

We study the term premium on long-term nominal and real government bonds in a small open economy within a structural micro-founded DSGE model. Previous works that tried to explain the term premium on government bonds using DSGE

---

[*] We thank Eliezer Borenstein for helpful comments and suggestions and Daniel Nathan for providing us with the estimates of the term premia derived from an affine term structure model for Israel. We also thank participants at the Bank of Israel research department seminar for their comments.

models focused on closed economies. A prominent work in this literature is Rudebusch and Swanson (2012), who showed that a (standard) DSGE model (of a closed economy) with Epstein–Zin preferences is able to generate a large (and variable) nominal term premium—in line with estimates from affine term-structure models—alongside realistic macroeconomic moments. In their model, the positive nominal term premium is explained by a persistent technological shock.

We extend the analysis to a small open economy. It is not clear a priori whether the success of a DSGE model of a closed economy in generating realistic term premia and macroeconomic moments carries over to the open-economy setup, for several reasons. First, in the open-economy setup we try to match not only macroeconomic as well as bond-pricing moments for the domestic and foreign economies separately, but also international correlations between the domestic economy and the world in line with the empirical evidence. Second, the openness of the economy may affect the required risk premia on financial assets since it may change the way domestic households may respond to unexpected shocks hitting the economy. Third, the term premium in the small open economy is affected by both global and domestic factors, and the implications of these for the term premium are not clear.

We construct a small-open-economy (henceforth SOE) DSGE model with recursive (Epstein–Zin) preferences, and calibrate it while attempting to match macroeconomic as well as bond-pricing moments—in particular, nominal and real term premia on government bonds—in the US (the "global economy") and Israel (the SOE). At the same time, we require that the model be able to fit international correlations between major economic variables. We seek to understand the economic forces—global and domestic—that drive bond risk premia in the small open economy. We use the model to explore how the openness of the economy affects the term premium on long-term government bonds.

In the majority of their uses, DSGE models are solved by applying linearization (or log-linearization) of the model; however, the linearized model implies zero risk premia. In a second-order approximation the model may generate constant non-zero risk premia, and an approximation to the third order allows for time-variant risk premia (Andreasen, 2012b). We thus solve the model up to a third-order approximation. We follow the methodology employed by Rudebusch and Swanson (2012), Andreasen (2012a), van Binsbergen et al. (2012), and Swanson (2016), among others, who introduce EZ preferences into a DSGE model and solve the model up to the third-order approximation. The previous literature has demonstrated the importance of EZ preferences with a high risk-aversion parameter for the ability of DSGE models to generate meaningful risk premia.

We find that a standard open economy DSGE model where all shocks in the global and domestic economies are mutually independent may not account for domestic and foreign macroeconomic and bond-pricing moments (particularly realistic nominal term premia) together with realistic correlations between the domestic and global economies. By contrast, a model where global technological changes directly affect domestic technology does succeed in concurrently fitting domestic and foreign macroeconomic and financial moments as well as international correlations. Thus, our preferred specification includes such "technological spillovers."

Similarly to the works on closed economies, which have found that the positive nominal term premium is explained by a highly persistent technological shock, in our open-economy model the positive term premium in the small open economy is attributed to two technological shocks: a global shock and a domestic one. The contribution of the rest of the shocks in the model to the term premium is small. The real term premium (i.e., on CPI-indexed government bonds) in our model is small; thus the nominal term premium is driven mainly by inflation risk. This result is in line with Andreasen (2012a) and Swanson (2016).

We use our model to explore how the openness of the economy affects the term premium on nominal government bonds. We find that in a model without technological spillovers, opening up the economy to trade in goods and financial assets (bonds) results in a reduction of the term premium—thanks to the enhanced ability of the open economy to accommodate domestic shocks. By contrast, in our preferred specification with technological spillovers, opening up the economy hardly changes the term premium. This result stems from the fact that absent technological spillovers, the foreign technological shock generates a negligible term premium in the domestic economy. By contrast, the same shock contributes to a larger term premium in the open economy in the presence of technological spillovers. Thus, when opening up the economy in the presence of technological spillovers, the decrease in the term premium induced by the enhanced ability to accommodate domestic shocks is offset by an increase in the risk premium generated by the foreign technological shock. Our results underscore the importance of taking into account possible correlations between shocks when assessing the implication of these shocks for risk premia.[1]

The remainder of the paper is organized as follows. Section 2 describes the DSGE model consisting of a small open economy and the world economy. Section 3 describes the calibration of the model using a grid search in an attempt to fit empirical moments. Section 4 presents the results in terms of the ability of the model to fit macroeconomic and bond pricing moments as well as comovement between the small open economy and the world. Section 5 studies how the openness of the economy affects the term premium on government bonds. Section 6 concludes and offers directions for further research. In Appendix A we discuss why the foreign technological shock generates a meaningful term premium in the global economy but not in the small open economy in the baseline model. An online Appendix presents the impulse response functions for all the shocks in the model.

---

[1] In our model there is a positive correlation between technological progress in the world economy and in the small open economy, since the world technological shock (innovation in the $AR(1)$ process) also enters the technological process in the small open economy.

2 The Model

We employ a standard New Keynesian DSGE model for a small open economy (SOE), except that we add the following features in order to study the determination of the yield curve and term premia. First, we introduce long-term bonds and their associated term premia into the model, both domestically and abroad. Second, we include both nominal and real (CPI-indexed) bonds. Third, we assume Epstein–Zin preferences instead of the standard preferences typically assumed within these models. Finally, we solve the model using a third-order perturbation rather than the first-order approximation typically employed for solving DSGE models.

The model consists of two blocks: a small open economy and the global economy. In this section we describe the model's fundamentals. We elaborate on the parts of the model that relate to the pricing of bonds and the corresponding risk premia.

2.1 The Small Open Economy

2.1.1 Households

Epstein–Zin preferences: There is a representative agent who has Epstein–Zin (EZ) preferences. Following Rudebusch and Swanson (2012) (henceforth R&S), the EZ preferences are presented by

$$V_t = u(C_t, N_t) + \beta \left[E_t \left(V_{t+1} \right)^{1-\bar{a}} \right]^{1/(1-\bar{a})} \tag{1}$$

if the utility kernel $u_t \equiv u(C_t, N_t)$ satisfies $u_t \geq 0 \ \forall t$, and

$$V_t = u(C_t, N_t) - \beta \left[E_t \left(-V_{t+1} \right)^{1-\bar{a}} \right]^{1/(1-\bar{a})} \tag{2}$$

if $u_t \leq 0 \ \forall t$. The utility kernel of household h is given by[2]

$$u_t(h) = \frac{e^{s_t} (C_t(h) - H^c C_{t-1})^{1-\gamma}}{1-\gamma} - \frac{(N_t(h) - H^N N_{t-1})^{1+\vartheta}}{1+\vartheta} \tag{3}$$

where we denote by $C_t(h)$ household h's consumption, by $H^c C_{t-1}$ external habit

in consumption, by $N_t(h)$ the household's working hours, by $H^N N_{t-1}$ external habit in labor, and by s_t a consumption preference shock following an $AR(1)$ process.

It turns out that for our calibration, the kernel utility is negative in the steady state of the model. Therefore, in the vicinity of the steady state, u_t is negative and the household's EZ preferences are given by equation (2).

---

[2] The specification of the utility kernel and of the production side of the economy below follows De Paoli et al. (2010). However, De Paoli et al. did not try to obtain realistic term premia and, accordingly, they assumed standard expected utility preferences rather than EZ preferences. Also, their model is of a closed economy.

In the EZ preferences given by equation (2), $\bar{a}$ is a risk aversion parameter. Note that for $\bar{a} = 0$, the EZ preferences boil down to the standard expected utility preferences. By varying the value of $\bar{a}$ in the EZ preferences, one can change the household's attitude toward risk without affecting its attitude toward consumption smoothing. This separation between the intertemporal elasticity of substitution and risk aversion has gained EZ preferences popularity in the macro-finance literature. It was proven key to the ability of models to match both macroeconomic moments and risk premia; see, e.g., R&S (2012), Andreasen (2012a, 2012b), and Dew-Becker (2014). Swanson (2012, 2013) analytically shows that the parameter $\bar{a}$ plays a key role in determining the degree of (absolute and relative) risk aversion of households, and thus affects the premium they require for holding risky assets.

The budget constraint: The budget constraint of a representative domestic household (in real terms with respect to the price of the consumption good P_t) is given by

$$\begin{aligned}
&C_t + \frac{T_t}{P_t} + \sum_{j=1}^{J} \frac{V_{j,t}^{bn}}{P_t} B_{j,t}^{d,n} + \sum_{j=1}^{J} V_{j,t}^{br} B_{j,t}^{d,r} + \sum_{j=1}^{J} \frac{S_t \Gamma_{j,t}^{FX,bn} VF_{j,t}^{bn}}{P_t} BF_{j,t}^{d,n} + \sum_{j=1}^{J} \frac{S_t P_t^*}{P_t} \Gamma_{j,t}^{FX,br} VF_{j,t}^{br} BF_{j,t}^{d,r} = \\
&\frac{W_t}{P_t} N_t + \frac{D_t}{P_t} + \frac{D_{X,t}}{P_t} + \Xi_t + \sum_{j=1}^{J} \frac{V_{j-1,t}^{bn}}{P_t} B_{j,t-1}^{d,n} + \sum_{j=1}^{J} V_{j-1,t}^{br} B_{j,t-1}^{d,r} + \\
&\sum_{j=1}^{J} \frac{S_t \Gamma_{j-1,t}^{FX,bn} VF_{j-1,t}^{bn}}{P_t} BF_{j,t-1}^{d,n} + \sum_{j=1}^{J} \frac{S_t P_t^*}{P_t} \Gamma_{j-1,t}^{FX,br} VF_{j-1,t}^{br} BF_{j,t-1}^{d,r} \qquad (4)
\end{aligned}$$

On the LHS of (4) are households' expenditures on consumption (C_t), lump-sum taxes (T_t), and bond holdings. There are four types of zero-coupon bonds, each with time to maturity between 0 (i.e., expiration) and J quarters: nominal and real (CPI-indexed) domestic bonds, and nominal and real foreign bonds. A domestic nominal bond pays one domestic currency at maturity, whereas a domestic real bond pays one unit of consumption (or P_t domestic currencies) at maturity. Similarly, the foreign nominal bond pays one foreign currency at maturity, and the foreign real bond pays one unit of the foreign consumption good (or P_t^* foreign currencies) at maturity. We denote by $B_{j,t}^{d,n}$ the amount of domestic nominal bonds with j periods to maturity held by the household at the end of period t, and by $V_{j,t}^{bn}$ the price (in domestic currency) of a nominal bond with j periods to maturity at time t. We denote by $B_{j,t}^{d,r}$ the amount of domestic real bonds with j periods to maturity held in period t at price $V_{j,t}^{br}$ (in domestic consumption good units). Similarly, $BF_{j,t}^{d,n}$ and $BF_{j,t}^{d,r}$ are the amounts of foreign nominal and real bonds, respectively. Their prices abroad—determined in the world economy—are denoted by $VF_{j,t}^{bn}$ and $VF_{j,t}^{br}$, respectively. Domestic households pay an "intermediation cost" on foreign bonds; thus the effective prices of these bonds for domestic households are $\Gamma_{j,t}^{FX,bn} VF_{j,t}^{bn}$ and $\Gamma_{j,t}^{FX,br} VF_{j,t}^{br}$, respectively. We discuss the motivation and specification of these intermediation costs below. We denote by S_t the nominal exchange rate (domestic currency per foreign currency). On the RHS of (4), the household's resources consist of its earnings from supplying labor ($W_t N_t$), profits (dividends) distributed to it from domestic firms (D_t) and from exporting firms ($D_{X,t}$), and the value of its holdings of bonds carried over from the previous period. Finally, Ξ_t is the profit of the "foreign exchange intermediation firms" that is transferred to households; i.e., it is a (lump-sum) rebate of the intermediation costs paid on foreign bonds.

Households' optimization problem: Households choose their own consumption plans, supply of labor, and holdings of various types of bonds so as to maximize their utility (equation (2)) subject to the budget constraint (equation (4)).

Bond pricing: The pricing equations of the different types of bonds are derived from households' first-order conditions (FOCs) with respect to their holdings of these bonds.

The FOCs with respect to the holdings of domestic nominal bonds with time to maturity $j = 1, ..., J$ yield the following pricing equations for domestic nominal bonds:

$$V_{j,t}^{bn} = E_t \left[\frac{SDF_{t+1}}{\pi_{t+1}} V_{j-1,t+1}^{bn} \right] \tag{5}$$

where

$$SDF_{t+1} \equiv \frac{\beta u_C(C_{t+1}, N_{t+1})}{u_C(C_t, N_t)} [\frac{-V_{t+1}}{\left(E_t(-V_{t+1}^{1-\bar{a}})\right)^{\frac{1}{1-\bar{a}}}}]^{-\bar{a}} = e^{s_{t+1}-s_t} \beta \left\{ \frac{(C_{t+1} - H^c C_t)}{(C_t - H^c C_{t-1})} \right\}^{-\gamma} [\frac{-V_{t+1}}{\left(E_t(-V_{t+1}^{1-\bar{a}})\right)^{\frac{1}{1-\bar{a}}}}]^{-\bar{a}} \tag{6}$$

is the (real) stochastic discount factor, $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is domestic CPI inflation, and $V_{0,t}^{bn} \equiv 1$ (i.e., the "price" of a bond at maturity equals 1).

Thus, the prices of nominal bonds of different maturities are recursively determined by $V_{1,t}^{bn} = E_t \left[\frac{SDF_{t+1}}{\pi_{t+1}} \right]$, $V_{2,t}^{bn} = E_t \left[\frac{SDF_{t+1}}{\pi_{t+1}} V_{1,t+1}^{bn} \right]$, and so on.

Similarly, for domestic real bonds, we obtain the following pricing equations:

$$V_{j,t}^{br} = E_t \left[SDF_{t+1} V_{j-1,t+1}^{br} \right], \tag{7}$$

where $V_{0,t}^{br} \equiv 1$. Thus, real bonds are recursively priced by $V_{1,t}^{br} = E_t \left[SDF_{t+1} \right]$, $V_{2,t}^{br} = E_t \left[SDF_{t+1} V_{1,t+1}^{br} \right]$, and so on.

For all types of bonds, the (continuously compounded) yield on a bond with $j = 1, ..., J$ periods to maturity is defined as $y_{j,t} \equiv -\frac{1}{j} \log V_{j,t}$ (that is, $V_{j,t} \equiv e^{-j y_{j,t}}$), where $V_{j,t}$ is the price of the bond.

The yield on the short (one-period) nominal bond is determined by the central bank interest rate i_t^{CB} (see equation (35) below), namely, $y_{1,t} \equiv -\log V_{1,t}^{bn} = i_t^{CB}$.

The pricing of foreign bonds: For nominal and real foreign bonds held by domestic agents we get from the corresponding FOCs, for $j = 1, ..., J$:

$$\Gamma_{j,t}^{FX,bn} VF_{j,t}^{bn} = E_t \left[\frac{SDF_{t+1}\Delta S_{t+1}}{\pi_{t+1}} \Gamma_{j-1,t+1}^{FX,bn} VF_{j-1,t+1}^{bn} \right] \tag{8}$$

and

$$\Gamma_{j,t}^{FX,br} VF_{j,t}^{br} = E_t \left[SDF_{t+1} \frac{\Delta S_{t+1}\pi_{t+1}^{*}}{\pi_{t+1}} \Gamma_{j-1,t+1}^{FX,br} VF_{j-1,t+1}^{br} \right] \tag{9}$$

where $\Delta S_t \equiv \frac{S_t}{S_{t-1}}$ is the depreciation rate of the nominal exchange rate, and $\pi_{t+1}^{*} \equiv \frac{P_t^{*}}{P_{t-1}^{*}}$ is CPI inflation abroad. Note that $VF_{j,t}^{bn}$ and $VF_{j,t}^{br}$, i.e., the prices of foreign nominal and real bonds abroad, are determined in the world economy by the FOCs for the optimization problem of foreign agents. Thus, similarly to the pricing equations in the domestic economy ((5) and (7)), the FOCs for the optimization problem of foreign households yield the following pricing equations for nominal and real bonds abroad:

$$VF_{j,t}^{bn} = E_t [\frac{SDF_{t+1}^{*}}{\pi_{t+1}^{*}} VF_{j-1,t+1}^{bn}] \tag{10}$$

and

$$VF_{j,t}^{br} = E_t [SDF_{t+1}^{*} VF_{j-1,t+1}^{br}] \tag{11}$$

where SDF_{t+1}^{*} is the real stochastic discount factor of the foreign households and $VF_{0,t}^{bn} = VF_{0,t}^{br} \equiv 1$.

The intermediation costs on foreign bonds: We assume that the prices of foreign bonds traded by domestic investors include intermediation costs denoted by $\Gamma_{j,t}^{FX,bn}$and $\Gamma_{j,t}^{FX,br}$ for foreign nominal and real bonds, respectively. The intermediation costs are introduced in order to "close the small-open-economy model" following Schmitt-Grohe and Uribe (2003). We thus extend Schmitt-Grohe and Uribe (2003) to the case where there are several types of bonds, namely, bonds of different times to maturity, and both nominal and real bonds.

The "price" of every bond at maturity is identically 1; hence in (8) and (9) for the case $j = 1$ we define $\Gamma_{0,t}^{FX,bn} = \Gamma_{0,t}^{FX,br} \equiv 1$.

We specify the intermediation cost on the foreign nominal bond with one period to maturity as follows:

$$\Gamma_{1,t}^{FX,bn} = \exp(\gamma_{NFA}(\frac{NFA_t}{P_t}) + \gamma_{DS}(E_t\Delta S_{t+1}\Delta S_t - 1) - u_t^{FX}) \tag{12}$$

where NFA_t is the economy's net foreign asset position, as defined in Section 2.1.4.[3] The dependence of the intermediation costs on the net foreign asset

---

[3] We set the steady-state value of the net foreign asset position to be zero. Otherwise, the argument in the intermediation premium would be the deviation of the (real) NFA position from its steady-state value.

position is assumed in order to ensure the existence of a unique steady state, following Schmitt-Grohe and Uribe (2003).[4] We also include the expected depreciation of the exchange rate in the intermediation costs, following Adolfson et al. (2008), as this induces more gradualism in the response of the exchange rate to shocks, bringing the behavior of the real exchange rate closer to the empirical evidence.[5] Finally, u_t^{FX} is an exogenous shock (following an $AR(1)$ process), which may be interpreted as an "exchange rate shock."

Recall the pricing equation for the one-period nominal foreign bond held by the domestic households (equation (8) for $j = 1$):

$$\Gamma_{1,t}^{FX,bn} VF_{1,t}^{bn} = E_t[SDF_{t+1} \frac{\Delta S_{t+1}}{\pi_{t+1}}] \tag{13}$$

where the price of the foreign bond abroad $VF_{1,t}^{bn}$ is determined abroad. Note that the yield on the one-period nominal bond abroad is determined by the foreign central bank's interest rate i_t^{CB*}, that is, $VF_{1,t}^{bn} = \exp(-i_t^{CB*})$. We have also seen that $V_{1,t}^{bn} = E_t\left[\frac{SDF_{t+1}}{\pi_{t+1}}\right] = \exp(-i_t^{CB})$. Thus, (13) is a form of an uncovered interest parity relation determining the dynamics of the nominal exchange rate.[6]

Now, once the dynamics of the exchange rate are determined, the domestic pricing equations for all other foreign bonds (namely, foreign nominal bonds with two or more periods to maturity as well as foreign real bonds of all maturities) will determine the intermediation costs for these different bonds. Thus, $\Gamma_{j,t}^{FX,bn}$ for $j = 2, ..., J$ are recursively determined by the pricing equations for the corresponding bonds (equation (8)). Similarly, $\Gamma_{j,t}^{FX,br}$ for $j = 1, ..., J$ are recursively determined by the domestic pricing equations for the foreign real bonds (equation (9)).

Note that, for our purposes, we could assume without loss of generality that domestic households hold only one type of foreign bonds, say, one-period nominal foreign bonds. The FOCs for the domestic households' holdings of all other types of foreign bonds (i.e., longer nominal bonds as well as real bonds with different periods to maturity) merely determine the intermediation costs on these other types of bonds and are inconsequential for the rest of the model.

The term (risk) premium: We now turn to the determination of the term premium on long-term bonds. Following R&S (2012), the term (or risk) premium on a government bond is defined as the difference between the yield on that bond and the yield on a hypothetical "risk-neutral" bond with a similar time to maturity.

---

[4] It also captures a direct effect of the current account on the exchange rate, on top of its effect through the interest rate differential.

[5] A similar specification is employed in Argov et al. (2012).

[6] A linearization of (13) would yield the familiar UIP relationship stating that (abstracting from the intermediation costs) expected depreciation should equal the interest rate differential. Note that since we will use a higher order of approximation, we cannot substitute $e^{-i_t^{CB}}$ for $E_t[\frac{SDF_{t+1}}{\pi_{t+1}}]$.

The pricing of a nominal bond is given by (5). The pricing of the corresponding "risk-neutral" nominal bond $\widetilde{V}^{bn}_{j,t}$ is recursively defined by

$$\widetilde{V}^{bn}_{j,t} = e^{-i^{CB}_t} E_t \widetilde{V}^{bn}_{j-1,t+1}$$

where $\widetilde{V}^{bn}_{0,t} \equiv 1$ and i^{CB}_t is the risk-free interest rate (the yield on the one-period nominal bond, which is the interest rate set by the central bank).

The nominal term premium $\psi^{bn}_{j,t}$ of a bond with j periods to maturity is defined as the difference between the corresponding yields:

$$\psi^{bn}_{j,t} \equiv \frac{1}{j}(\log \widetilde{V}^{bn}_{j,t} - \log V^{bn}_{j,t}) \tag{14}$$

Note that by forward recursion (and the law of iterated expectations), we can write

$$\widetilde{V}^{bn}_{j,t} = E_t e^{-\sum_{k=0}^{j-1} i^{CB}_{t+k}};$$

hence the "risk-neutral" yield corresponds to the sum of expected short-term interest rates,[7] or to the "expectation hypothesis," and the term premium is the difference between the bond yield and this theoretical yield.

To get an economic intuition for the risk premium, R&S (2012) show that the difference between the prices of the two bonds satisfies

$$V^{bn}_{j,t} - \widetilde{V}^{bn}_{j,t} = E_t \sum_{i=0}^{j-1} e^{-i_{t,t+i+1}} cov_{t+i}(m_{t+i+1}, V^{bn}_{j-i-1,t+i+1}), \tag{15}$$

where $m_{t+i+1} \equiv \frac{SDF_{t+i+1}}{\pi_{t+i+1}}$ is the nominal stochastic discount factor and $i_{t,t+i} \equiv \sum\limits_{m=0}^{i-1} i^{CB}_{t+m}$.

Then, using a first-order approximation to the logs in the definition of the term premium ((14)) (i.e., $\log V^{bn}_{j,t} \approx \frac{1}{V^{bn}_j} V^{bn}_{j,t}$, where V^{bn}_j denotes the price of the bond in the nonstochastic steady state), the term premium can be written as

$$\begin{aligned} \psi^{bn}_{j,t} &= \frac{1}{j}(\log \widetilde{V}^{bn}_{j,t} - \log V^{bn}_{j,t}) \\ &\approx \frac{-1}{jV^{bn}_j} E_t \sum_{i=0}^{j-1} e^{-i_{t,t+i+1}} cov_{t+i}(m_{t+i+1}, V^{bn}_{j-i-1,t+i+1}). \end{aligned} \tag{16}$$

Thus, the nominal term premium depends on the expected covariance over the lifetime of the bond between the price of the bond and the nominal SDF. The bond risk premium is larger the more negative this covariance is.

---

[7] Up to Jensen's inequality.

In R&S (2012) there are only nominal bonds. To define the term premium on real bonds we similarly define the pricing equation for the hypothetical "risk-neutral" real bond by[8]

$$\widetilde{V}^{br}_{j,t} = e^{-r_t} E_t \widetilde{V}^{br}_{j-1,t+1},$$

where r_t is the yield on the one-period real bond, namely, $e^{-r_t} = V^{br}_{1,t} = E_t\left[SDF_{t+1}\right].$

Then, the risk premium on a real bond with j periods to maturity is defined as

$$\psi^{br}_{j,t} \equiv \frac{1}{j}(\log \widetilde{V}^{br}_{j,t} - \log V^{br}_{j,t}), \tag{17}$$

and, analogously to equation (16) for the nominal premium, we can write the real term premium as

$$\psi^{br}_{j,t} \approx \frac{-1}{jV^{br}_j} E_t \sum_{i=0}^{j-1} e^{-r_{t,t+i+1}} cov_{t+i}(SDF_{t+i+1}, V^{br}_{j-i-1,t+i+1}).$$

Other conditions for the household sector: Consumption is allocated between domestically produced $(C_{H,t})$ and imported $(C_{F,t})$ goods. Total consumption is given by a CES aggregate:

$$C_t = \left[(1-\varpi)^{\frac{1}{\varsigma}} (C_{H,t})^{\frac{\varsigma-1}{\varsigma}} + \varpi^{\frac{1}{\varsigma}} (C_{F,t})^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}} \tag{18}$$

where the parameter ϖ is the weight of imported consumption in total consumption in the steady state; thus it is a measure of the degree of openness of the economy,[9] and the parameter ς measures the elasticity of substitution between domestic and imported goods.

The CPI P_t is the price of a unit of the consumption good, given by

$$C_t P_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{19}$$

where the determination of $P_{H,t}$ and $P_{F,t}$—the prices of the domestic and imported components of consumption—is discussed below.

Optimal allocation of consumption between domestic and imported goods implies

$$C_{H,t} = (1-\varpi)\left(\frac{P_{H,t}}{P_t}\right)^{-\varsigma} C_t \tag{20}$$

$$C_{F,t} = \varpi\left(\frac{P_{F,t}}{P_t}\right)^{-\varsigma} C_t \tag{21}$$

---

[8] A similar definition is used by Swanson (2016).

[9] We make some simplifying assumptions since the solution method using a third-order approximation is costly in terms of computing time. In particular, we assume that the domestic economy imports consumption goods only. (See equation (24) below.)

$$P_t = \left[(1-\varpi)\left(P_{H,t}\right)^{1-\varsigma} + \varpi \left(P_{F,t}\right)^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}} \tag{22}$$

We assume immediate pass-through from foreign prices to the prices of imported goods (i.e., producer currency pricing)[10]; thus

$$P_{F,t} = S_t P_t^* \tag{23}$$

We further assume that only consumption goods are imported by the domestic economy; thus

$$IMP_t = C_{F,t} \tag{24}$$

Finally, we obtain the standard condition for the household's labor supply from the first order condition with respect to labor:

$$-\frac{u_N(C_t, N_t)}{u_C(C_t, N_t)} = \frac{W_t}{P_t} \tag{25}$$

2.1.2 Domestic firms

There is a continuum of monopolistically competitive firms indexed by $z \in [0,1]$. Each firm produces a differentiated intermediate good using capital $K_{t-1}(z)$ and labor $N_t(z)$ with a Cobb–Douglas production function:

$$Y_t(z) = A_t K_{t-1}^{\alpha}(z) N_t^{1-\alpha}(z), \tag{26}$$

where $A_t \equiv e^{a_t}$ is a (stationary) technological shock (see the specification of the exogenous processes in Section 2.1.7).

The differentiated-good firms sell their products in a monopolistic competition to a composite firm, producing the aggregate domestic good:

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\mu_t - 1}{\mu_t}} dz \right)^{\frac{\mu_t}{\mu_t - 1}} \tag{27}$$

where μ_t is the (time-varying) elasticity of substitution between the domestically produced differentiated goods, thus serving as a "mark-up shock" to domestic inflation $\pi_{H,t}$. The shock $\log(\mu_t)$ follows an $AR(1)$ process.

The aggregate good Y_t is then sold in perfect competition at the price $P_{H,t} = \left(\int_0^1 P_{H,t}(z)^{1-\mu_t} dz \right)^{\frac{1}{1-\mu_t}}$ and is used for private consumption, government consumption, investment, and exports. Minimizing production costs by the composite firm implies the following demand functions for the differentiated goods:

---

[10] This is another assumption made for simplification. Otherwise, we could assume local price rigidity in imports.

$$Y_t(z) = [\frac{P_{H,t}(z)}{P_{H,t}}]^{-\mu_t} Y_t \tag{28}$$

We assume nominal price rigidities à la Rotemberg (1982). Each domestic firm z seeks to maximize its expected profits:

$$\max E_t \sum_{i=0}^{\infty} m_{t,t+i} D_{t+i}(z), \tag{29}$$

where $m_{t,t} \equiv 1$, $m_{t,t+i} \equiv \prod_{l=1}^{i} m_{t+l}$ for $i > 0$, and

$$D_t(z) = P_{H,t}(z)Y_t(z) - W_t N_t(z) - P_{H,t} I_t(z) - \frac{\chi}{2} \left(\frac{P_{H,t}(z)}{P_{H,t-1}(z)} - \left(\pi^H\right)^{1-\hbar} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\hbar} \right)^2 P_{H,t} Y_t.$$

Note that we adjusted the specification of the price adjustment costs (introducing some rigidity with respect to lagged inflation) in order to allow for more inertia in inflation.[11]

The evolution of capital used by firm z is given by

$$K_t(z) = (1 - \delta)K_{t-1}(z) + G(I_t(z)/K_{t-1}(z))K_{t-1}(z) \tag{30}$$

where the adjustment costs associated with the creation of capital are given by[12]

$$G(I_t(z)/K_{t-1}(z)) = \frac{a}{1 - \frac{1}{\chi^k}} \left(\frac{I_t(z)}{K_{t-1}(z)} \right)^{1-\frac{1}{\chi^k}} + b.$$

The FOCs of firms with respect to their production inputs (labor and capital) and prices are standard and are omitted here for the sake of brevity.

2.1.3 Exports

We assume that exporters are price takers in the global economy.[13] Hence the demand for exports is linked to global demand, represented by the world's GDP:

$$EXP_t = \Phi Y_{world,t} \tag{31}$$

where the parameter Φ measures the weight of domestic exports in world output (in per capita terms).

---

[11] Similar to the assumption of indexation to past inflation in the Calvo price rigidity setting. Using the grid search we set the parameter h to be zero.

[12] The specification follows De Paoli et al. (2010). The determination of the parameters a and b is shown in De Paoli et al. (2009).

[13] We make this assumption mainly in the interest of limiting the size of the open economy model.

Exporting firms buy the domestic good at price $P_{H,t}$ and sell it abroad at the world's price $P^*_{EXP,t}$. Their profits (in domestic currency)—which are distributed to the households—are thus given by

$$D_{X,t} = EXP_t(P_{EXP,t} - P_{H,t}) \tag{32}$$

where $P_{EXP,t} = P^*_{EXP,t}S_t$, and we assume $P^*_{EXP,t} = P^*_t$.

2.1.4 The trade balance and net foreign assets

We assume for simplicity that foreign households do not hold domestic bonds. Hence the economy's net foreign asset position is the value of foreign assets held by the domestic economy's agents:

$$\begin{aligned} \frac{NFA_t}{P_t} \quad \equiv \quad & \sum_{j=1}^{J} S_t \frac{VF^{bn}_{j,t}}{P_t} BF^{d,n}_{j,t} + \sum_{j=1}^{J} \frac{S_t P^*_t}{P_t} VF^{br}_{j,t} BF^{d,r}_{j,t} = \\ & \sum_{j=1}^{J} S_t \frac{VF^{bn}_{j-1,t}}{P_t} BF^{d,n}_{j,t-1} + \sum_{j=1}^{J} \frac{S_t P^*_t}{P_t} VF^{br}_{j-1,t} BF^{d,r}_{j,t-1} + \\ & \frac{P_{EXP,t}}{P_t} EXP_t - \frac{P_{F,t}}{P_t} IMP_t \end{aligned} \tag{33}$$

The second equality describes the evolution of net foreign assets as the current value of assets held from the previous period plus net exports (the trade balance).

2.1.5 The government and central bank

Government expenditures follow an $AR(1)$ process:

$$\log(G_t) = (1-\theta)\log(G) + \theta\log(G_{t-1}) + \epsilon^G_t,$$

where ϵ^G_t is a white noise shock. The government budget constraint (in real terms) is given by

$$\frac{T_t}{P_t} + \sum_{j=1}^{J} \frac{V^{bn}_{j,t}}{P_t} B^{G,n}_{j,t} + \sum_{j=1}^{J} V^{br}_{j,t} B^{G,r}_{j,t} = \frac{P_{H,t}}{P_t} G_t + \sum_{j=1}^{J} \frac{B^{G,n}_{j,t-j}}{P_t} + \sum_{j=1}^{J} B^{G,r}_{j,t-j} \tag{34}$$

where $B^{G,n}_{j,t}$ ($B^{G,r}_{j,t}$) denotes the amount of nominal (real) government bonds with j periods to maturity issued at time t. Thus, the government finances its expenditures on G_t plus the redemption of bonds that arrived at their maturity at time t either by taxes or by issuing new bonds. The market-clearing conditions in the domestic bond markets—nominal and real—read:

$$B^{d,n}_{j,t} = \sum_{s=j}^{J} B^{G,n}_{s,t-(s-j)},$$

and

$$B^{d,r}_{j,t} = \sum_{s=j}^{J} B^{G,r}_{s,t-(s-j)}.$$

The central bank sets the one-period nominal interest rate according to the following Taylor rule:

$$\frac{i^{CB}_t}{i^{CB}} = \left(\frac{i^{CB}_{t-1}}{i^{CB}}\right)^{\rho} \left((\frac{\pi_t}{\pi})^{\theta_1}(\frac{Y_t}{Y})^{\theta_2}\right)^{1-\rho} e^{u^i_t} \tag{35}$$

where u^i_t is a monetary policy shock.

2.1.6 Aggregate resource constraint

Combining the household and government budget constraints with the evolution of net foreign assets and the clearing conditions in all markets we can arrive at the economy's aggregate resource constraint:

$$Y_t + \frac{P^*_t S_t}{P_{H,t}} IMP_t = \frac{P_t}{P_{H,t}} C_t + G_t + I_t + EXP_t + \frac{\chi}{2}\left(\pi^H_t - \left(\pi^H\right)^{1-\hbar}\left(\pi^H_{t-1}\right)^{\hbar}\right)^2 Y_t.$$

We can also write the market-clearing condition for the domestic good Y:

$$Y_t = C_{H,t} + G_t + I_t + EXP_t + \frac{\chi}{2}\left(\pi^H_t - \left(\pi^H\right)^{1-\hbar}\left(\pi^H_{t-1}\right)^{\hbar}\right)^2 Y_t.$$

2.1.7 Exogenous processes

There are six domestic "shocks" (i.e., exogenous processes): technology ($a_t \equiv \log(A_t)$), preference (s_t), elasticity of substitution (or markup) ($\log(\mu_t)$), government consumption ($\log(G_t)$), monetary policy (u^i_t), and foreign intermediation costs (u^{FX}_t). The monetary policy shock u^i_t is assumed to be white noise, whereas the other five shocks follow $AR(1)$ processes of the form: $x_t = (1-\rho_x)x + \rho_x x_{t-1} + \epsilon^x_t$, where x is the steady-state value of x_t and ϵ^x_t is an i.i.d. normally distributed innovation with standard deviation σ_x.

2.2 The World Economy

The world economy is essentially a closed-economy version of the NK small-open-economy model outlined in the previous sections.[14] Thus, the world economy consists of households, firms (producing differentiated intermediate

---

[14] Except that in the world economy there is no investment and no government consumption.

goods and a composite good), government (issuing nominal and real government bonds), and a central bank, with specifications for the utility function (EZ preferences and the kernel utility), Taylor rule, etc., similar to the ones in the small open economy.

In our attempt to economize on the size of the aggregate model (consisting of the world block and the small-open-economy block)—due to the computing time needed to solve the third-order approximation of the model—we made the following simplifying assumptions in the world model: (a) there is no investment. Thus, the production function is given by (capital is fixed and normalized to 1):

$$Y_t^* = A_t^* N_t^{*1-\alpha}$$

and (b) there is no government consumption.

As mentioned above, we assume that foreign households hold their local government bonds (nominal and real) only; thus they do not hold the SOE's government bonds. As noted above, the corresponding pricing equations for the foreign bonds held by foreign households are given by (10) and (11).

The FOCs for the households and firms in the world economy as well as the central bank's Taylor rule are analogous to those in the SOE.

There are four exogenous processes ("shocks") in the world economy with corresponding innovations: technology (ϵ_t^{*a}), preference (ε_t^{*s}), elasticity of substitution (markup) ($\epsilon_t^{*\mu}$), and monetary shock (ε_t^{*r}). As in the SOE, the first three shocks follow $AR(1)$ processes whereas the monetary shock is white noise. Compared to the SOE, two innovations are absent in the world economy: the foreign intermediation cost (exchange rate) and the government consumption shock.

3 Solution and Calibration

We solve the model up to a third-order approximation using DYNARE (see Adjemian et al., 2011). In a first-order approximation risk premia are zero, whereas in a second-order approximation risk premia are constant. Thus, in order to allow for time-variable term premia we need to go to a third-order approximation (or higher) (see Andreasen, 2012b).[15] Following Rudebusch and Swanson (2012), we try to fit both macroeconomic and financial moments, including bond term premia.[16] Thus, after calibrating part of the model parameters according to standard values in the literature, we perform a grid search for other parameter values in an attempt to best match model moments with empirical moments. We first parameterized the world economy model (Model W), which is exogenous to the small open economy. Then, given the world economy

---

[15] Alternatively, one could consider other—particularly global—solution methods. However, such methods are generally far more computationally demanding, especially for large models.

[16] Rudebusch and Swanson (2012) examine nominal term premia only. We consider also real (CPI-indexed) bonds.

model, we set the parameters of the small open economy (SOE). In this section we describe this calibration process in more detail.[17]

3.1 The Grid Search

Similarly to R&S (2012), estimating our fairly large model solved to a third-order approximation by maximum likelihood or Bayesian methods was infeasible for us at this point.[18] Rather, we performed a grid search on a limited number of parameters—and a limited number of values for each parameter—in order to choose certain parameter values.

We searched for parameter values for 15 parameters in the world economy and 16 parameters in the SOE—as specified in Tables 1 and 2 below—so as to minimize the distance between several unconditional moments from the model and in the data. Specifically, the loss function is of the form $L = \sum_i (X_i^{Model} - X_i^{Data})^2$, where X^{Model} is a vector of unconditional moments in the model and X^{Data} is the corresponding vector in the data. For the world economy, the vector X consists of the following 8 moments: the mean and standard deviation (SD) of the term premium for 10-year nominal bonds, as well as the SD of the following 6 variables: 10-year nominal bond yields, the FED's interest rate, CPI inflation, consumption, labor, and real wages.[19] For the small open economy, the vector X includes 12 moments: in addition to the aforementioned 8 moments,[20] we also included the mean and SD of the term premium for 5-year real bonds, the SD of 5-year real yields, and the SD of investment.[21]

3.2 Calibration

Table 1 summarizes the calibration of the structural parameters for the world economy (W) and for the small open economy (SOE). Table 2 presents the serial correlation and the standard deviation of shocks in the exogenous $AR(1)$ processes. In the tables, the abbreviation "LIT" denotes that the calibration is based on previous literature, "DATA" stands for a calibration that is based on the data, and "GS" stands for a calibration based on a grid search. Finally,

---

[17] We use the term "calibration" interchangeably with "parameterization" to refer to parameters that were calibrated based on previous literature as well as parameters whose values were determined using the grid search.

[18] Moreover, the open-economy setup consisting of a world block and a SOE block, and including also imports, exports, foreign exchange, etc., makes the model considerably larger than a closed-economy model.

[19] The list of moments we try to fit follows R&S (2012). They also include the slope of the yield curve and excess holding period returns as additional measures for the term premium.

[20] For the small open economy we set the maximum maturity for nominal and real bonds to be 5 years instead of 10. Note that the recursive pricing of bonds implies that increasing the maturity from 20 to 40 quarters amounts to adding the prices of 20 nominal bonds and 20 real bonds to the model's variables. This increase in the size of the model significantly increases the computational burden in solving the third-order approximation of the model and thus further limits the grid search.

[21] Notice that in the grid search we did not include international comovement moments in the loss function, mainly in the interest of avoiding too large a number of moments in the loss function.

for parameters in the SOE, "W" means that the parameter value in the SOE is calibrated according to the corresponding parameter in the world economy.

In the world economy model (W) there are 22 parameters (14 structural in Table 1, and 8 in the exogenous processes in Table 2). Out of these 22 parameters, 6 parameters were calibrated based on common assumptions in the literature (LIT), one was based on the data (DATA), and the remaining 15 parameter values (including the serial correlation and SD of the innovations in the exogenous processes) were calibrated based on a grid search (GS). The combined model (including both the world and the SOE) includes 56 parameters, of which 22 parameters are in the world economy block (W), and 34 in the SOE. For the SOE, 7 parameter values were based on the literature (LIT), 6 parameter values were taken from the world model (W),[22] 4 parameter values were based on the data (DATA), and 1 parameter was determined by a steady-state condition (SS).[23] The remaining 16 parameter values were determined using a grid search (GS), of which 6 are structural parameters and 10 are parameters of the exogenous processes. We applied the grid search mainly for parameters that are important for the term premia and/or are unique to the small open economy and for which we had no good external source for setting their values.

---

[22] Kulish and Rees (2011) also assume symmetry in parameter values between the small open economy and the world economy (e.g., the habit formation parameter, the parameters in the Taylor rule, the households' discount factor, etc.).

[23] The share of domestic exports in world output is determined by a steady-state condition given the calibration of relative prices in the steady state (normalized to be 1) and the calibration of the levels of the domestic and foreign technological shocks in the steady state (both calibrated to be 1).

Table 1: Calibration of Structural Parameters

Parameter	**Value of parameter**		**Description**
	SOE	W	
β	0.99 (LIT)	0.99 (LIT)	discount factor of households
π	3 (W)	3 (DATA)	inflation target (annual)[24]
γ	6 (W)	6 (GS)	curvature of utility w.r.t. consumption
H^c	0.1 (W)	0.1 (GS)	habit in consumption
ϑ	2.5 (W)	2.5 (GS)	curvature of utility w.r.t. labor
H^N	0 (W)	0 (GS)	habit in labor
α	0.36 (LIT)	0.36 (LIT)	share of capital in the production function
ρ	0.75 (LIT)	0.75 (LIT)	degree of smoothing in the CB interest rate rule
θ_1	1.5 (LIT)	1.5 (LIT)	reaction of CB policy rule to inflation
θ_2	0.2 (LIT)	0.2 (LIT)	reaction of CB policy rule to output gap
χ	95 (W)	95 (GS)	parameter of price adjustment in cost function
h	0 (GS)	0 (GS)	weight on past inflation in price adjustment cost function
$\bar{a}$	-90 (GS)	-30 (GS)	parameter of EZ preferences
μ	6 (LIT)	6 (LIT)	elasticity of substitution between intermediate goods
γ_{NFA}	0.1 (GS)	-	effect of NFA on the intermediation costs
γ_{DS}	0.5 (GS)	-	effect of expected change in exchange rate on intermediation costs
ϖ	0.3 (DATA)	-	share of imported goods in private consumption
ς	0.5 (GS)	-	elasticity of substitution between domestic and imported goods
δ	0.025 (LIT)	-	depreciation rate of capital
χ^k	3 (GS)	-	elasticity of investment/capital ratio w.r.t. Tobin's q
Φ	0.42 (SS)	-	share of domestic exports in world output (per capita)
$\bar{G}/\bar{Y}$	0.23 (DATA)	-	share of government spending in GDP
$CRRA$	187.1	63.5	relative risk aversion (result of calibration)[25]

Notes: "GS" – from grid search; "LIT" – from the literature; "DATA" – from the data; "SS" – determined by SS conditions; "W" – parameter in the SOE is taken from the W model.

---

[24] The non-stochastic steady-state inflation of 3% in the world model was set to fit the average inflation rate of 2% in the data for the US. We assume that in the non-stochastic steady state, the inflation rate in the SOE is equal to the one in the world.

[25] The calculation of the relative risk aversion is based on Swanson (2013).

Table 2: Calibration of Exogenous Processes

Type of shock		**Value of parameters**	
		SOE	W
Technology	SE	0.5% (GS)	0.5% (GS)
	ρ	0.97 (GS)	0.97 (GS)
Monetary	SE	0.25% (GS)	0.25% (GS)
	ρ	0 (GS)	0 (GS)
Preference	SE	1% (GS)	1.3% (GS)
	ρ	0.95 (GS)	0.95 (GS)
Elasticity of substitution	SE	5% (GS)	2% (GS)
	ρ	0.7 (GS)	0.7 (GS)
Fiscal	SE	1.2% (DATA)	-
	ρ	0.8 (DATA)	-
Intermediation cost	SE	1% (GS)	-
	ρ	0.6 (GS)	-

Notes: "GS" – from grid search; "DATA" – the AR(1) process for government consumption was separately estimated.

4 Results

In this section we assess the moments generated by the model as compared to the moments in the data.[26] We start with the model for the world economy (the US). This is similar to R&S's (2012) inquiry of the ability of a DSGE model of a closed economy to fit both macro and financial (bond pricing) empirical moments (Table 2 in their paper). We then proceed to examine the ability of the model to fit similar moments in the small open economy (Israel). In the open-economy setup, we also examine the ability of the DSGE model to account for the interrelationships between the SOE and the global economy.

4.1 The World Economy Model

For the global economy we use data for the US.[27] Most of the empirical moments for the US are taken from R&S (2012) (see Table 2 in R&S, 2012, and column (1) in Table 3 below).

Table 3 presents the moments from the world-economy model (Model W), alongside the moments from the data and the moments from R&S's (2012) best-fit specification. As Table 3 shows, the model generates an average 10-year nominal term premium of about 1 pp, similar to the estimate from the affine model,[28] and fits most of the macro moments, including the variability of

---

[26] The unconditional moments from the model were calculated based on simulations of 10,000 periods of the third-order approximation of the model using DYNARE.

[27] This is quite common in the literature. See, e.g., Kulish and Rees (2011), Uribe and Yue (2006), Bunda et al. (2009), Piljak (2013), and Dahlquist and Hasseltoft (2013).

[28] The average premium estimated from the affine model for the US is similar to the average slope of the yield curve. The average slope of the 10-year nominal yield curve reported by R&S (2012) in the 1961–2007 sample is 1.43 pp. Sutton (2000) reported an average slope of

the federal funds rate, inflation, private consumption, and labor. However, the standard deviation of the 10-year nominal premium in the model is smaller than the estimate from the data. As can be seen in Table 3, R&S (2012) managed to obtain a larger SD, similar to the estimate from the data, with their best fit specification. Yet, the smaller SD we obtain is in line with the results reported by Andreasen (2012a,b), Swanson (2016), and Fuerst and Mau (2016).

The model generates a high SD of real wages compared to the data. The relatively high volatility of wages in the model may stem from our simplifying assumption of wage flexibility. Note however that despite the high variability of wages, the model succeeds in matching the variability of inflation. A second apparent weakness of the model is the low varibility of the 10-year real yields obtained in the model compared to the data.[29]

As in the previous literature (e.g., R&S, 2012, Andreasen, 2012a), the major factor that generates a significant positive nominal term premium is a highly persistent technological shock. The technological shock moves the welfare of households and the prices of long-run nominal bonds in the same direction. Therefore, nominal bonds are risky assets and households require positive risk premia on these assets. The real term premium in the model is small (and close to the assessment of Swanson, 2016). A small real premium is obtained due to the moderate reaction of the prices of long-run real bonds to the technological shock, resulting in a rather weak covariance between real bond prices and the welfare of households. We further discuss these results in Appendix A.

Comparing the moments obtained in our model to the moments from R&S's (2012) "best fit" specification (column (3) in Table 3), the main difference lies in the ability of R&S (2012) to generate a highly volatile nominal term premium, which is similar to the estimate from the affine term structure model. As mentioned earlier, to the best of our knowledge, R&S's (2012) model is unique in generating such a high SD of the premium within a standard DSGE model (that is, within a standard rational expectations New Keynesian DSGE model, without assuming a time-varying risk aversion parameter or time-varying stochastic processes of shocks).[30]

1.06 pp in the 1961–1992 sample. Swanson (2016) reported an average slope of 1.29 pp in the 1971–2015 sample.

[29] R&S (2012) have only nominal bonds in their model; thus they do not report the SD of long-term real yields in their model. However, they do report the SD of the short-term ex-ante real rate; see Table 3. Considering that the SD of the short real rate in their model is comparable to our model, as well as the SD of short and long nominal yields, it is plausible that in their model too the SD of the long-term real yield would have been too low compared to the data. Note in particular that both in R&S and in our model (as well as in the data), the SD of the long-term nominal yield is smaller than the SD of the short rate. The same is true for real yields in our model and in the data, and this would plausibly be the case in R&S's model too.

[30] R&S (2012) also report the results on a baseline calibration of their model, made before they applied a grid search ("best fit"). Under the baseline calibration the SD of the nominal term premium in their model is tenfold smaller than under the best-fit specification: 0.047 vs. 0.47 (see Table 2 in their paper).

Table 3: Moments for the World Model

Unconditional moments	**US data, 1961–2007**	**Model W**	**R&S (best fit)**
	(1)	(2)	(3)
$Mean[pr\_n^{10Y}]$	1.06	1.05	1.12
$SD[pr\_n^{10Y}]$	0.54	0.12	0.47
$Mean[pr\_r^{10Y}]$	0*	0.18	NaN
$SD[pr\_r^{10Y}]$	NaN	0.05	NaN
$SD[i^{*10Y}]$	2.41	1.67	2.14
$SD[i^{FED}]$	2.71	2.67	3.09
$SD[\pi]$	2.52	2.49	2.67
$SD[C]$	0.83	1.01	1.10
$SD[N]$	1.71	1.56	2.42
$SD[RW]$	0.82	2.30	1.13
$SD[ri]$	2.30	1.53	1.80
$SD[r^{*10Y}]$	1.37**	0.38	NaN

Notes: Column (1) presents the unconditional moments in US data (taken from R&S, 2012, unless stated otherwise); column (2) presents the moments from our model; column (3) presents the moments from R&S's (2012) "best-fit" specification. pr_n^{10Y} – term premium on 10-year nominal bonds; pr_r^{10Y} – term premium on 10-year real bonds; i^{*10Y} – 10-year nominal yield; i^{FED} – FED interest rate; π – CPI inflation rate; C – private consumption; N – labor; RW – real wages; ri – one-quarter real interest rate; r^{*10Y} – 10-year real yield. * The mean of the 10-year real term premium is based on Swanson (2016), who examined the slopes of real yield curves in the US and in the UK in different samples and concluded that the average real premium is close to zero. ** The SD of the 10-year real yields is calculated from US data for the 1997–2014 sample.

4.2 The Small Open Economy

We now turn to the small open economy. Note that it is not clear to what extent the success of closed-economy DSGE models in concurrently generating both macro and financial (bond pricing) moments carries over to the SOE case. For one thing, the ability of a small open economy to trade with the world may better enable it to accommodate domestic shocks, thereby attenuating consumption and labor risks. This may result in smaller risk premia. On the other hand, the exposure of the economy to foreign shocks—to the extent that these shocks induce a positive covariance between bond prices and household wealth—may work in the opposite direction, namely, to increase risk premia. We will explore these effects of the openness of the economy in more depth in Section 5. Moreover, we impose further requirements on the open-economy model, compared to the closed-economy model: we require the model not only to generate both macro and financial moments within each of the two economies, but also to fit cross-correlations between the small economy and the world economy. Thus, we impose more stringent requirements on the open-economy model compared to the closed-economy one.

4.2.1 Data

For the empirical moments of the small open economy we use data for Israel. The prevalence of CPI-indexed government bonds in Israel enables us to examine the real-term structure along with the nominal one. The empirical moments for Israel are shown in column (1) of Table 4. The moments in the data were computed as follows: consumption (C) is total consumption divided by workers and it is expressed as a logarithmic deviation from a Hodrick–Prescott trend. The same holds for investment (I). Labor (N) is total hours per worker. Real wage (RW) is the hourly nominal wage divided by the CPI and it is expressed as a logarithmic deviation from a Hodrick–Prescott trend. The moments of the inflation rate (π), 5-year zero-coupon nominal and real yields (i^{5Y}, r^{5Y}), the central bank interest rate (i^{CB}), the one-period real rate (ri), the nominal exchange rate depreciation (ΔS), and nominal and real term premia (pr_n^{5Y}, pr_r^{5Y}) were computed from the raw series; all of them, except the exchange rate, are in annual terms.[31]

For the nominal and real term premia in Israel we use estimates from an affine term structure model for Israel by Nathan (2015). Nathan (2015) obtained an average nominal (real) premium for 5 years of 1.38 (0.5) percentage points (pp) and a SD of 1.1 (0.62) pp.[32] We also considered the average slopes of the nominal and real yield curves as indicators of the average term premia: the average nominal slope (5-year yield minus 1-year yield) is 1.1 pp and the real slope is 0.3 pp.[33] Both measures, however, may capture not only the model-equivalent term premia but also the country's default risk premium. An indication of the country's default risk premium may be derived from country default swaps (CDS) traded in the capital market.[34] The average gap between Israeli CDS for 10 years and for 5 years is 0.25 pp. We do not have data on CDS for 1 year, but we presume that a positive gap should also hold for default risk between 1-year and 5-year yields. Therefore, based on the estimates mentioned above, we conjecture that the average nominal term premium for 5 years excluding the country's default risk premium is between 0.8 and 1.1 pp and the real premium is between 0 and 0.2 pp. In the loss function we included the estimated average term premia from the affine model, excluding 0.3 pp for the default risk premium, namely, 1.1 pp for the nominal premium and 0.2 pp for the real premium.

The bottom panel of Table 4 refers to the international comovement of Israel with the US. In examining the comovement between the SOE and the world, we focus on the correlation (and covariance) between yields in Israel and their counterparts in the US (5-year nominal and real yields and the central bank interest rates), as well as the correlations of output and inflation rates between the two

---

[31] The data source, except for the term premia is the Central Bureau of Statistics in Israel and the Bank of Israel.

[32] Based on the 2001.Q1–2014.Q4 sample period.

[33] Based on the 1998.Q1–2014.Q4 sample period.

[34] Bunda et al. (2009) point out that bond yields in emerging markets include country default risk premia.

countries.[35] As the bottom panel of Table 4 shows, the data exhibit significant comovement in these variables. High international comovement of nominal and real variables has been widely documented in the literature. For example, Sutton (2010), Jotikasthira et al. (2015), Dahlquist and Hasseltoft (2013), Byrne et al. (2012), Kulish and Rees (2011), Uribe and Yue (2006), Bunda et al. (2009) and Piljak (2013) underscore the strong comovement between yields in different countries and yields in the US. Moreover, it is often the case that the correlation is higher for long-term interest rates than for short rates (Byrne et al., 2012, and Kulish and Rees, 2011). Henriksen et al. (2013) report high correlations between inflation and output in five major countries and the US. They find that the average correlation for output is 0.27 (in the range of 0.21–0.72) and the average correlation for inflation is 0.52 (in the range of[36] 0.47–0.76). Kollmann (2012) examined the comovement between seven industrial economies and the US. He reports average correlations of output and inflation of 0.61 and 0.64, respectively,[37] and points to the importance of nominal rigidities for the ability of the model to generate such positive correlations.

---

[35] The data for the US was taken from Bloomberg and the Federal Reserve Bank of St. Louis.

[36] In the 1960–2006 sample period.

[37] Based on the 1973–1994 sample period.

Table 4: Moments for the SOE

(A) Domestic Moments for the SOE (Israel)	**Sample, 2001–14**	**SM1**	**SM2**	**SM3**
	(1)	(2)	(3)	(4)
$Mean[pr\_n^{5Y}]$	1.1/0.8*	1.21	0.01	1.07
$SD[pr\_n^{5Y}]$	1.08	0.17	0	0.15
$Mean[pr\_r^{5Y}]$	0.2/0*	-0.23	0	0.0
$SD[pr\_r^{5Y}]$	0.62	0.05	0	0.05
$SD[i^{5Y}]$	2.19	2.11	0.47	1.91
$SD[r^{5Y}]$	1.73	0.39	0.26	0.39
$SD[i^{CB}]$	2.43	2.75	0.99	2.72
$SD[\pi]$	2.98	3.70	1.0	3.58
$SD[C]$	1.33	0.87	0.23	0.86
$SD[I]$	3.82	3.38	0.92	3.17
$SD[N]$	1.36	1.37	0.34	1.30
$SD[RW]$	2.0	3.83	1.89	3.62
$SD[ri]$	2.5**	2.14	0.65	2.11
$SD[\Delta S]$	3.75	2.39	0.87	2.23
(B) International Comovement of SOE (Israel) with W (US)				
Correlations and covariances	**Sample, 2001–14**	**SM1**	**SM2**	**SM3**
$\rho(i^{5Y}, i^{*5Y})$	0.63	0.14	0.79	0.61
$cov(i^{5Y}, i^{*5Y})$	1.84	0.79	0.81	2.46
$\rho(r^{5Y}, r^{*5Y})$	0.78	0.29	0.63	0.36
$cov(r^{5Y}, r^{*5Y})$	1.78	0.07	0.08	0.07
$\rho(i^{CB}, i^{*CB})$	0.51	0.23	0.80	0.54
$cov(i^{CB}, i^{*CB})$	2.13	2.0	2.23	4.04
$\rho(\pi, \pi^{*})$	0.27	0.14	0.58	0.36
$cov(\pi, \pi^{*})$	0.48	0.09	0.09	0.21
$\rho(y, y^{*})$	0.64***	0.08	0.51	0.54
$cov(y, y^{*})$	1.14	0.19	0.14	0.78

Notes: $SM1$ – basic model with domestic and foreign independent shocks; $SM2$ – $SM1$ with only foreign shocks activated; $SM3$ – model with technological spillovers; pr_n^{5Y} – term premium on 5-year nominal bonds; pr_r^{5Y} – term premium on 5-year real bonds; i^{5Y} – yield on 5-year nominal bonds; r^{5Y} – yield on 5-year real bonds; i^{CB} – central bank interest rate; π – CPI inflation rate; C – private consumption; I – investment; N – labor; RW – real wage; ri – one-period real interest rate; ΔS – nominal exchange rate depreciation; ρ – correlation; cov – covariance; y – output gap. * denotes foreign variables (Model W). * Estimates from an affine model and from the average slope of the yield curve as described in the text. ** The ex-ante real interest rate (ri) was calculated as the Bank of Israel interest rate minus inflation expectations for the next four quarters derived from the capital market. *** The comovement between outputs was calculated using detrended output from an HP filter in each country.

4.2.2 A basic model (SM1)

We start with a basic model consisting of a small open economy and a world economy, as described in Section 2. We denote this model SM1. Recall that the model features 10 mutually independent shocks: 6 domestic shocks and 4 foreign shocks (see Table 2). The unconditional domestic and international moments implied by SM1 are presented in column (2) of Table 4. Regarding the domestic moments (Panel (A) in Table 4), SM1 fits fairly well most of the domestic macro and financial moments, including the level of the nominal term premium. Noticeable exceptions are similar to the results obtained for the world model: the SD of the term premia in the model are smaller than the estimates from the affine term structure model for Israel, and also the variability of the 5-year real yield is much smaller in the model than in the data. Note that the last two moments in Panel (A) of Table 4 (namely, the SD of the short-term real rate and of the nominal depreciation rate) were not included in the loss function used in the grid search, yet the corresponding moments in the model are reasonably close to those in the data.

Turning to international comovement, Panel (B) of Table 4 reveals that the basic model SM1 fails to replicate the international moments: the correlations and covariances of yields, inflation rates, and output are noticeably lower in the model compared to the data. This property of the model may indicate that the basic model is missing some important transmission channels from the world economy to the small open economy, and/or that the model attributes too much of the variability of the SOE to domestic shocks rather than to shocks originating in the world. In order to further explore this issue we next examine the effect of the foreign shocks alone on the SOE.

4.2.3 The effect of foreign shocks alone (SM2)

In order to further investigate the effects of the global economy on the small open economy we conduct the following exercise: we shut off all domestic shocks and leave only the four world shocks active. Thus we explore how foreign shocks alone affect the domestic economy. We denote this version of the model by SM2.

The corresponding moments are presented in column (3) of Table 4. We note the following observations: first, as evident in Panel (B) of Table 4, the foreign shocks induce noticeable positive comovement between the SOE and the world economy.[38] Thus, the lack of sufficient correlation in SM1 may be attributed to the effect of domestic shocks on the SOE. Second, the foreign shocks alone generate low variability in the domestic macro and finance variables—despite them generating sufficient variability in global variables, as seen in Table 3. Finally, the foreign shocks generate null level and null volatility of domestic term premia. We further explore this result in Appendix A, where we examine the differing effects of the foreign technological shock on the foreign and domestic economies. In the next section we show that acknowledging the existence of

---

[38] Notice that correlations are high. Low covariances in many cases stem from the low variances of domestic variables.

technological spillovers from the world economy to the SOE greatly improves the fit of the model, thereby helping us match the various moments for the SOE, i.e., domestic macro and finance moments as well as correlations with the world.

4.2.4 A model with technological spillovers (SM3)

In reality, many technological changes are not country-specific but rather are internationally shared. In this section we show that taking such technological spillovers into account may circumvent the difficulty of SM1—a model where domestic and foreign shocks are mutually independent—in concurrently fitting both domestic moments (including term premia) and international comovement.

Thus, we amend the exogenous process for the technological progress in the SOE (see Section 2.1.7) to include also a global component, in addition to the idiosyncratic domestic shock, as follows:

$$\log(A_t) = (1-\rho^A)\log(A) + \rho^A \log(A_{t-1}) + w\epsilon_t^a + (1-w)\epsilon_t^{*a} \tag{36}$$

where ϵ_t^{*a} is the technological innovation in the world economy (W) and ϵ_t^a is a domestic i.i.d. shock in the small open economy. The process in (36) reflects that some of the technological innovations in the domestic economy are common to the SOE and the world economy. We denote the model with technological spillovers by SM3. Other than this modification to the domestic technological process, the model is identical to SM1. Performing a grid search again for SM3, we obtain the calibration of[39] $w = 0.6$.

Our focus on technological spillovers is consistent with the empirical literature that found technology to be a dominant driving force of international business cycles (see Ahmed et al., 1993, and Crucini et al., 2011). Our specification of technological spillovers assumes that the technological innovation abroad has a contemporaneous effect on the technological level in the small open economy. Further lagged effects occur through the $AR(1)$ process. Our specification is somewhat different from Henriksen et al. (2010), Heathcote and Perri (2002), and Rabanal et al. (2011). First, they assume that the spillovers between the two countries go in both directions. Secondly, in addition to the contemporaneous effect of a technological shock abroad on the domestic economy, they also included the effect of a shock with a one-period lag, although the latter effect is set to be small.

The moments from the model with technological spillovers (SM3) are shown in column (4) of Table 4. The assumption of a common technological shock ensures that the world technological shock induces similar supply-side effects in the domestic economy. This helps the model to generate a positive nominal term premium and noticeable comovement with the world concurrently. The nominal and real term premia are close to the measures in Israel. Note that the term premium on 5-year nominal bonds in Israel is similar to the term premium on 10-year nominal bonds in the US. Thus, for a similar period to maturity

---

[39] For the rest of the parameters, the grid search yielded similar calibration in SM3 as in SM1; see the calibration in Tables 1 and 2.

of, say, 5 years, the premium in Israel is larger than in the US.[40] The model attributes the higher risk premium in Israel to a higher risk aversion parameter: the EZ parameter was found to be -90 in Israel compared to -30 in the US. The volatility of the nominal and real premia are low also in SM3 compared to the estimates from the affine model but are similar to the results of Andreasen (2012a) for the UK and Swanson (2016) for the US and the UK.[41] Similarly to the results on SM1, SM3 is able to fit most of the macro moments.

The proposed modification in the exogenous process for the technological shock (36) greatly improves the international moments of the model, as evident in Panel (B) of Table 4. Both correlations and covariances are fairly close to the data. A noticeable exception is the low covariance between 5-year real yields generated by the model, which mainly reflects the low variability of long-term real yields in the model, as mentioned before.[42]

The model also generates a (slight) upward correlation pattern with respect to time to maturity in nominal yields, consistent with the data. Notice that such a pattern is generated in the model with technological spillovers (SM3) but not in SM1—the model with no technological spillovers—or SM2—the model with no spillovers and in which only foreign shocks are activated. Such a pattern was also found for other countries (Kulish and Rees, 2011). Kulish and Rees (2011) explained the upward pattern by a lower persistence of domestic shocks relative to foreign shocks.[43] This is not the case in our model, as in our model shocks of the same type in both countries are similarly persistent (with similar SD). In our model, two main factors contribute to an upward pattern in correlations between yields: (1) a highly persistent spillover effect magnifies the correlation between domestic and foreign long-run yields by generating a comovement in term premia of long-term bonds;[44] (2) domestic shocks affect only domestic yields, thereby disconnecting domestic yields from their counterparts abroad. In this regard, the exchange rate shock is found to be important. This shock has a greater effect on short-term yields due to its relatively moderate persistence, thereby reducing the correlation between short-term yields in the SOE and abroad.[45] We note that although the model generates a fairly high correlation (of 0.6)

---

[40] The nominal term premium on 5-year bonds in the world economy (W) in the model is 0.59 pp.

[41] Andreasen (2012) obtained a quarterly SD of the nominal term premium for 5 years of 0.1 pp and a much lower SD for the real premium. Swanson (2016) obtained a quarterly SD of nominal premia in the model of about 0.08 pp.

[42] Also the correlation between 5-year real yields in the model is a little low relative to the data. In this respect SM3 does not improve upon SM1 by much.

[43] In their model, the expectation hypothesis holds; thus they exclude the possibility that correlation in yields may also reflect correlation in term premia.

[44] The model generates a correlation of 0.6 between the nominal term premia in the two economies. When we derive a second-order approximation of the model—thereby imposing constant term premia in both economies—the correlation between short-run yields turns out to be similar to the correlation between long-run yields. Thus, without time-varying term premia the upward pattern of correlation between yields vanishes.

[45] Without the foreign exchange shock, the correlations generated by the model (SM3) between short- term yields and between long-term yields in the global and domestic economies are similar.

between risk premia in the domestic economy and the world economy, the role of risk premia in the comovement (covariance) between long-term yields in the two countries in the model is small. This is due to the low variances of domestic and foreign premia in the model. By contrast, statistical and affine models have found a larger role for risk premia in explaining the covariation in yields, consistent with the higher variance of the term premia found in these models (Sutton, 2000; Dahlquist and Hasseltoft, 2013; Jotikasthira et al., 2015).

We conclude that the model with technological spillovers (SM3) succeeds in matching most domestic and international moments fairly well. We thus use this model to explore how the openness of the economy affects the term premia on government bonds.

5 The Effect of the Openness of the Economy on the Term Premium

What is the contribution of global factors to the risk premium on government bonds in the SOE? How does the openness of the economy affect the risk premium? What are the channels through which the openness of the economy affects the premium? In this section we explore these questions. To this end, Table 5 presents the term premium on 5-year nominal bonds under alternative assumptions with regard to the small open economy.

Table 5: Average Term Premium on 5-Year Nominal Bonds

	Open?[46]	**Tech. spillovers?**	**Active shocks**	**Term premium (pp)**
(a)	Yes	Yes	All	1.07
(b)	No	No	All (/Domestic only)[47]	0.76
(c)	No	Yes	All (/Domestic+Foreign tech.)[48]	1.08
(d)	Yes	No	Domestic only	0.46
(e)	Yes	Yes	Domestic+Foreign tech.	1.03
(f)	Yes	Yes	Foreign tech. only	0.57
(g)	No	Yes	Foreign tech. only	0.32

In Table 5, row (a) corresponds to our benchmark model of a small open economy with technological spillovers (SM3). Then, in row (b) of Table 5, we examine what the term premium would be if the economy were completely closed, that is, if the domestic economy had no connection with the world whatsoever. This includes trade in goods and services, trade in financial assets (foreign

---

[46] Open with respect to trade in goods and financial assets (bonds).

[47] Note that a completely closed economy (row (b) of Table 5) where all shocks are active is effectively exposed to domestic shocks only, since we shut off all transmission channels from the global economy to the domestic economy.

[48] An economy which is closed to trade is effectively not exposed to foreign shocks except for the tech. shock–if the latter has a direct technological effect on the SOE.

bonds), and technological spillovers from the world.[49] We see that the term premium in the closed economy is 0.76 pp, compared to 1.07 in the open economy. The difference of 0.3 pp in the premium reflects the contribution of the openness of the economy, including openness with respect to technological innovations.

Next, we turn to examine the effects of "classical" openness of the economy—that is, openness with respect to trade in goods and financial assets—on the term premium. Thus, we separate out the direct effect of the foreign technological shock. Row (c) of Table 5 represents an economy that is closed to trade in goods and foreign bonds, but with access to foreign technology. Thus, starting from our benchmark model (SM3), we close the economy with respect to trade in goods and financial assets, but not with respect to direct technological spillovers. We can see that the term premium in row (c) is similar to the premium in row (a) for the original open-economy model. Thus, comparing rows (a) and (c), it seems that closing the economy in the "classical" sense does not affect the term premium. We next seek to better understand why this is the case. Does openness to trade in goods and financial assets not affect the risk premium on bonds?

When opening the economy to trade in goods and financial assets, there are two effects that are likely to affect risk premia. First, we would expect that the improved ability of domestic households to smooth consumption in response to domestic shocks through imports and/or exports should lower the term premium in the open economy. On the other hand, the exposure of the economy to global shocks—and thus to additional sources of risk—may increase the term premium, though the direction of the latter effect on the premium is not clear since it depends on the covariance between the SDF and the bond price that is induced by the foreign shocks.

In order to examine the first effect—that is, the effect of the enhanced ability to smooth consumption in response to domestic shocks in the open economy compared to the closed one—we compare the term premium obtained in the completely closed economy (row (b)) to the term premium in an open economy where only the domestic shocks are activated (the latter specification is presented in row (d) of Table 5). Thus, in rows (b) and (d) we compare two (otherwise identical) economies where only domestic shocks are active: one economy is open to trade in goods and bonds and the other is closed. As we can see, the resulting term premium is 0.46 in the open economy, compared to 0.76 in the closed one (rows (b) and (d) in Table 5). Thus, the enhanced ability to smooth consumption enjoyed by the open economy indeed reduces the term premium—by 0.30 pp. We refer to this effect as the "consumption smoothing channel."

Given that the consumption smoothing channel works to reduce the term premium in the open economy relative to the closed economy, why is the premium in our open economy model SM3 (row (a)) similar to the premium in the

---

[49] Recall that the exogenous process for the technological shock in the SOE that is exposed to direct technological spillovers is given by (36). For the case where the economy is not exposed to technological spillovers, we assume that the process is similar except that the term $(1-w)\epsilon_t^{*a}$ is omitted.

closed economy model (row (c))? It could be that the exposure of the open economy to other foreign shocks (in addition to the technological shock) acts to increase the premium in the open economy relative to the closed one. However, this is not the case here: looking at row (e) in Table 5 we can see that the contribution of the rest of the foreign shocks (other than the technological shock) to the term premium is negligible: the term premium is 1.07 pp when all the shocks are active (row (a)) and it is 1.03 pp when we shut off the foreign shocks other than the technological shock (row (e)). Put differently, comparing rows (c) and (e) we can see that the term premia in the open and closed economies are similar also when the two economies are exposed to the same set of shocks, namely, the domestic shocks and the foreign technological shock.

What is it then that works to increase the term premium in the open economy, thereby offsetting the effect of the consumption smoothing channel? The answer lies in the different effect of the foreign technological shock on the two economies. Note that in the case of the closed economy, the effect of the foreign technological shock on the domestic economy is through the direct technological channel only. By contrast, in the open economy, the shock affects the SOE both directly (through the technological exogenous process) and indirectly (through trade). In rows (f) and (g) of Table 5 we evaluate the effect of the foreign technological shock alone on the open economy and on an (otherwise similar) closed economy. We can see that the foreign technological shock generates a larger term premium in the open economy than in the closed one: 0.57 vs. 0.32. It turns out that, quantitatively, in our model this latter effect roughly offsets the opposite effect of the consumption smoothing channel so that on net the term premia in the open and closed economies are similar, provided that both enjoy technological spillovers from abroad.

Note that when we open up the economy, the increase in the required risk premium associated with the foreign technological shock stems from the "indirect effect" of that shock on the SOE. However, there is a subtle point here: this indirect effect alone does not generate a significant term premium in the SOE. We saw this in model SM2 where we activated only the foreign shocks and obtained a negligible term premium in the SOE.[50] Thus, it is the fact that the indirect effect occurs on top of the direct technological effect that makes it a significant contribution to the term premium in the SOE. Put differently, the fact that the foreign technological shock occurs concurrently with a domestic technological shock is important for the effect it has on the term premium. We elaborate on this point in what follows.

Recall that the term premium depends on the expected covariance between the (nominal) SDF and the bond price throughout the lifetime of the bond. The indirect effect of the technological shock through the standard open-economy channels increases this covariance (in absolute values), provided that the direct effect is also at work. Figure 1 may help us better understand why the indirect effect increases the term premium in this case but not when working alone. Fig-

---

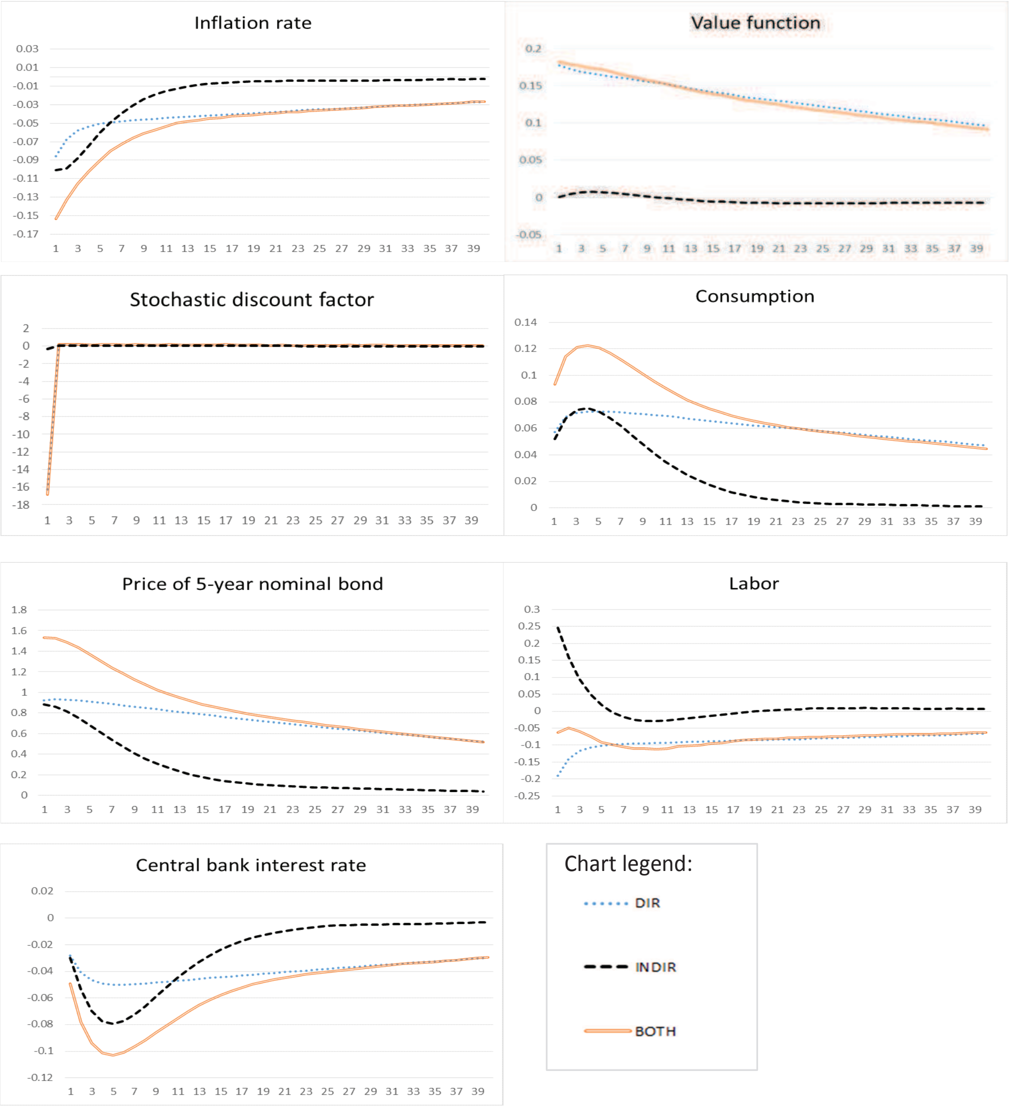
[50] The role of the rest of the foreign shocks in generating term premia is negligible; thus the result we obtained in SM2 with regard to the term premium holds when we activate the foreign technological shock only.

ure 1 presents the effect of a one-SD foreign technological shock on the domestic economy in three cases: (a) when the economy is closed to trade and is affected by the global economy through direct technological spillovers only (denoted by the suffix DIR in Figure 1) (this case represents the direct technological effect alone); (b) in an open economy where there are no direct technological spillovers (denoted by the suffix $INDIR$) (reflecting the indirect "open economy" effect of the foreign technological shock alone); and (c) in the benchmark open economy with technological spillovers where both effects co-occur (denoted by the suffix $BOTH$). Looking at case (b) (the indirect effect alone), we can see that the negligible term premium in this case may largely be explained by the very small effect this shock has on the SDF (and on the value function).[51] Consequently, the covariance of the SDF with the bond price is very small; hence the negligible term premium. Note that unlike the effect on the SDF, the indirect effect on the bond price is more pronounced: this is reflected in Figure 1 in the reaction of the bond's price in case (b) ($INDIR$) as well as in the larger reaction in case (c) ($BOTH$) compared to case (a) (DIR). This extra reaction of the bond price in case (c) compared to case (a)—due to the indirect effect—increases the covariance of the bond price with the SDF—which varies due to the direct effect. Thus, it is the combination of the indirect effect on the bond price and the direct effect on the SDF that increases the term premium in the open economy that is exposed to direct technological spillovers. The enhanced reaction of the bond price in the open economy compared to the closed one reflects the fact that both the direct and indirect effects of the positive foreign technological shock work to lower domestic inflation (thereby increasing nominal bond prices): directly due to the domestic technological upturn and indirectly through lower inflation abroad due to the technological improvement there (see the upper-left panel of Figure 1). To summarize, the foreign technological shock generates a higher term premium in the open economy compared to the closed one due to the larger effect the shock has on inflation and bond prices. However, absent technological spillovers, the enhanced response of bond prices is not sufficient for generating a higher term premium, because of the muted effect on the household's SDF.

---

[51] The small increase in the household's value function (and accordingly the minor decrease in the SDF) in case (b) is partly a result of the increase in labor, which works to offset the effect of the increased consumption on the household's utility. In addition, the effect on consumption is less persistent than the direct effect. See Appendix A for more details.

Figure 1: Impulse Responses to a Foreign Technological Shock


Inflation rate

Value function

Stochastic discount factor

Consumption

Price of 5-year nominal bond

Labor

Central bank interest rate

Chart legend:

Notes: The figure shows the responses of selected variables in the domestic economy to a one SD positive technological shock abroad for the following specifications: DIR – a closed economy with technological spillovers from the world (direct effect only); INDIR – an open economy with no direct technological spillovers (indirect effect only); and BOTH – our benchmark open-economy model with technological spillovers (both effects concurrently). The value function is in absolute value (so that an increase in the value function reflects an improvement). All variables are expressed in quarterly terms (percentage deviations from steady-state values).

6 Conclusions

In this paper we study the determination of the term premium on nominal and real government bonds in a small open economy. Previous work on closed economy models underscored the prominent role of a technological shock combined with EZ preferences in generating meaningful positive nominal term premia. For the open economy, we find that a global technological shock that spills over to the domestic economy alongside a domestic technological shock is essential to the ability of the model to generate noticeable nominal term premia and at the same time match domestic and international moments, including cross-correlations between the domestic economy and the world.

We used the model to examine how the openness of the economy affects the nominal term premium. We identified two opposing forces with regard to the effect of openness on the term premium on nominal bonds. On the one hand, the openness of the economy may help domestic households better accommodate domestic shocks through trade in goods and financial assets, thereby serving to reduce the risk premium. On the other hand, we found that the effect of the foreign technological shock on the open economy is more pronounced than its effect on a closed economy that is subject to technological spillovers from the world. The latter channel works to increase the term premium in the open economy. By contrast, without the technological spillovers, the foreign technological shock does not generate a meaningful term premium in the small open economy. Thus, the openness of the economy works to increase the term premium induced by a foreign technological shock, but only in the presence of technological spillovers. Quantitatively, in our preferred specification of the model—including technological spillovers—the two effects roughly offset each other, so that the nominal term premium in the open economy is similar to the premium in an otherwise identical closed economy. Without technological spillovers, opening up the economy reduces the nominal term premium.

The volatility of the term premium generated in the model is small relative to the estimates from affine term structure models. The standard deviations of the term premia obtained in our model are in line with the results obtained by Andreasen (2012a,b), Swanson (2016), and Fuerst and Mau (2016). Fuerst and Mau (2016) point out that the variability of the nominal term premium obtained in a standard DSGE model is low. They offer instead a model with segmented markets between long- and short-term bonds. Alternatively, within the framework of a standard DSGE model (like ours), a larger variability of risk premia over time may be generated by (a) changes in the distribution of shocks over time and (b) changes in the "price of risk" due to changing attitudes toward risk. It is possible to add these two sources for variability in the term premium to our model by introducing the following respective extensions: (a) stochastic volatility of shocks. In particular, we have seen that highly persistent technological (supply side) shocks are the main force behind positive nominal term premia in this class of models. If the relative importance of such shocks were to decrease over time (e.g., because the standard deviation of the technological shock were to decrease), this would result in smaller term premia.

Moreover, if persistent demand shocks become dominant, nominal term premia may turn negative, since such shocks generate a negative covariance between households' well-being and the price of nominal bonds. (b) Time-varying parameter of risk aversion in the EZ preferences. Andreasen (2012b) has examined the effect of stochastic volatility of the technological shock on the nominal term premium, and found that it may considerably increase the volatility of the premium. Dew-Becker (2014) incorporated time-varying risk aversion in a DSGE model with EZ preferences, and obtained considerable variability of the nominal term premium.[52]

The rapid progress of computing capabilities may enable the use of a model like ours (possibly with the extensions just mentioned) for estimating the unobservable time series of the term premia, i.e., for decomposing observed yields into the expected path of future short rates and risk premia by employing a non-linear filter. This may be an interesting and useful application of the model in future research.

It may also be desirable to incorporate default risk into the pricing of government or corporate bonds (Swanson, 2016).

Finally, in future research it may be interesting to explore alternative explanations of risk premia. The extremely high values of the risk aversion parameter required to obtain bond risk premia within current DSGE models suggest that this may be a direction worth pursuing.[53]

References

Adjemian, Stephane, Houtan Bastani, Michel Juillard, Frederic Karame, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto, and Sebastien Villemot (2011), Dynare: Reference Manual, Version 4, Dynare Working Papers, 1, CEPREMAP, URL: http://www.dynare.org.

Adolfson, Malin, Stefan Laseen, Jesper Linde, and Mattias Villani (2008), Evaluating an estimated New Keynesian small open economy model, *Journal of Economic Dynamics and Control* 32, 2690–2721.

Ahmed Shaghil, Barry W. Ickes, Ping Wang, and Byung Sam Yoo (1993), International business cycles, *American Economic Review* 83, 335–359.

Andreasen, Martin M. (2012a), An estimated DSGE model: Explaining variation in nominal term premia, real term premia, and inflation risk premia, *European Economic Review*, 56, 1656–1674.

---

[52] Dew-Becker (2014) also assumes that the technological shock directly enters the exogenous process of the central bank's inflation target—a somewhat nonstandard assumption that is apparently needed to obtain realistic moments for the term premia.

[53] Several justifications have been offered in the literature for why the risk-aversion parameter required in the model may be high. However, it is not clear whether such explanations are quantitatively sufficient for accounting for the extremely high values typically obtained in this literature.

Andreasen, Martin M. (2012b), On the effects of rare disasters and uncertainty shocks for risk premia in non-linear DSGE models, *Review of Economic Dynamics* 15, 295–316.

Argov, Eyal, Emanuel Barnea, Alon Binyamini, Eliezer Borenstein, David Elkayam, and Irit Rozenshtrom (2012), MOISE: A DSGE model for the Israeli economy, Bank of Israel Discussion Paper No. 2012.06.

van Binsbergen, Jules H., Jesus Fernandez-Villaverde, Ralph S.J. Koijen, and Juan Rubio-Ramırez (2012), The term structure of interest rates in a DSGE model with recursive preferences, *Journal of Monetary Economics* 59, 634–648.

Byrne, Joseph P., Giorgio Fazio, and Norbert Fiess (2012), Interest rate comovements, global factors and the long end of the term spread, *Journal of Banking and Finance* 36, 183–192.

Bunda, Irina, Javier Hamann, and Subir Lall (2009), Correlations in emerging market bonds: The role of local and global factors, *Emerging Markets Review* 10, 67–96.

Crucini, Mario, Ayhan Kose, and Christopher Otrok (2011), What are the driving forces of international business cycles, *Review of Economic Dynamics* 14, 156–175.

Dahlquist, Magnus and Henrik Hasseltoft (2013), International bond risk premia, *Journal of International Economics* 90, 17–32.

Dew-Becker, Ian (2014), Bond pricing with a time-varying price of risk in an estimated medium-scale Bayesian DSGE model, *Journal of Money, Credit and Banking* 46, 838–888.

De Paoli, Bianca, Alasdair Scott, and Olaf Weeken (2010), Asset pricing implications of a New Keynesian model, *Journal of Economic Dynamics and Control* 34, 2056–2073.

De Paoli, Bianca, Alasdair Scott, and Olaf Weeken (2009), Asset pricing implications of a New Keynesian model, Bank of England Working Paper no. 326.

Fuerst, Timothy S. and Ronald Mau (2016), Term premium variability and monetary policy, Federal Reserve Bank of Cleveland Working Paper no. 16-11.

Gali, Gordi (1999), Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations?, *American Economic Review* 89, 249–271.

Heathcote, Jonathan and Fabrizio Perri (2002), Financial autarky and international business cycles, *Journal of Monetary Economics* 49, 601–627.

Henriksen, Espen, Finn E. Kydland, and Roman Sustek (2010), The high cross-country correlations of prices and interest rates, National Bureau of Economic Research Working Paper 15123.

Jotikasthira, Chotibnak, Anh Le, and Christian Lundblad (2015), Why do term structures in different currencies comove?, *Journal of Financial Economics* 115, 58–83.

Kollmann, Robert (2001), Explaining international comovements of output and asset returns: The role of money and nominal rigidities, *Journal of Economic Dynamics and Control* 25, 1547–1583.

Kulish, Mariano and Daniel Rees (2011), The yield curve in a small open economy, *Journal of International Economics* 85, 268–279.

Nathan, Daniel (2015), Might inflation-indexed bonds be informative after all? Evidence from a liquid market, Mimeo, Bank of Israel.

Piljak, Vanja (2013), Bond markets co-movement dynamics and macroeconomic factors: Evidence from emerging and frontier markets, *Emerging Markets Review* 17, 29–43.

Rabanal, Pau, Juan F. Rubio-Ramirez, and Vicente Tuesta (2011), Cointegrated TFP processes and international business cycles. *Journal of Monetary Economics* 58, 156–171.

Rudebusch, Glenn D. and Eric T. Swanson (2012), The bond premium in a DSGE model with long-run real and nominal risks, *American Economic Journal: Macroeconomics* 4, 105–143.

Rotemberg, Julio J. (1982), Monopolistic price adjustment and aggregate output, *The Review of Economic Studies* 49, 517–531.

Schmitt-Grohe, Stephanie and Martın Uribe (2003), Closing small open economy models, *Journal of International Economics* 61,163–185.

Sutton, Gregory D. (2000), Is there excess comovement of bond yields between countries?, *Journal of International Money and Finance* 19, 363–376.

Swanson, Eric T. (2012), Risk aversion and the labor margin in dynamic equilibrium models, *American Economic Review* 102, 1663–1691.

Swanson, Eric T. (2013), Risk aversion, risk premia, and the labor margin with generalized recursive preferences, Federal Reserve Bank of San Francisco Working Paper 2012-7.

Swanson, Eric T. (2016), A macroeconomic model of equities and real, nominal and defaultable debt, Presentation at NBER Summer Institute.

Uribe, Martın and Vivian Z. Yue (2006), Country spreads and emerging countries: Who drives whom?, *Journal of International Economics* 69, 6–36.

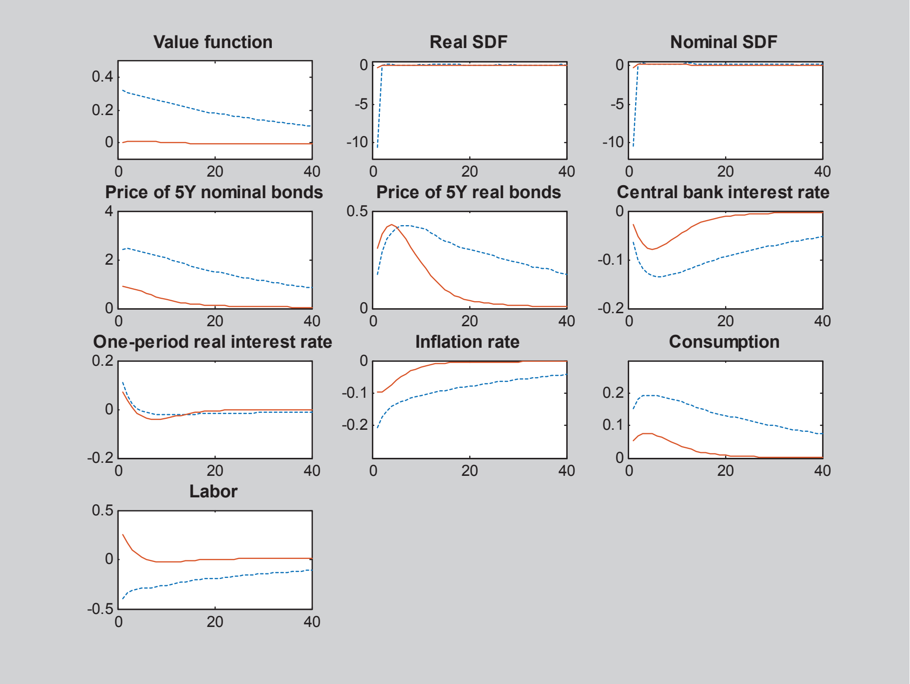
Appendix A: Why does a technological shock in the world not generate a meaningful nominal term premium in the small open economy in SM1?

In the "standard" SOE model (namely, absent technological spillovers, Model SM1 in the text), a foreign technological shock generates a noticeable nominal term premium in the world economy, but only a negligible premium in the small open economy. In this appendix we further examine why this is the case.

Figure 2 displays the responses of several variables in the world economy (Model W; dashed line) and in the small open economy (Model SM1; continuous line) to a 1 SD positive technological shock abroad. The figure reveals two sources for the small term premium generated in the SOE compared to the world economy. (1) In the SOE the increase in consumption is accompanied by an increase in labor, thereby offsetting the increase in the value function (and the corresponding decrease in the SDF). By contrast, in the world economy (Model W), consumption increases but labor decreases, each contributing to an increase in the household's value function (and correspondingly a decrease in its SDF). As recognized in the literature on closed economies, this pronounced negative effect on the SDF alongside the positive effect on long-term bond prices is the reason for the positive risk premium demanded by investors in long-term bonds in Model W. (2) The responses of variables in the SOE are smaller in magnitude and less persistent than those in the world economy. Persistence is important both for the effect on the value function (hence on the SDF)—since this effect depends on the expected path of consumption and labor throughout the future—and for long-term bonds. (1) and (2) result in a tiny effect on the SDF in the SOE compared to the effect on the SDF of households in the world economy (as well as a relatively small effect on long-term bond prices in the SOE). Consequently, there is no sizable covariance between the SDF and bond prices in the SOE, and this leads to a negligible nominal term premium.

The opposing effects of the foreign technological shock on labor in the world and in the SOE reflect the different nature of the shock in the two economies. The decrease in labor in response to a technological improvement in the world is a well-known property of New Keynesian DSGE models, resulting from demand-determined production in the presence of price rigidities (Gali, 1999). In contrast to the world economy, and absent a parallel change in technology in the SOE, such a "supply side" effect on labor is not present. Instead, the increase in world output (and wealth) leads to an increased demand for the SOE's exports, thereby increasing the demand for labor in the SOE. The described transmission mechanism from the foreign technological shock to the domestic economy is not unique to our model but rather is common to many small-open-economy models. For example, Kollmann (2001) describes a similar transmission of the foreign technological shock to consumption, labor, and other variables in the domestic economy.

Figure 2: Responses in the world economy and in the SOE to a foreign technological shock in SM1



Notes: The figure depicts the responses of variables in the world economy (dashed lines) and in the SOE (continuous lines) to a 1 SD positive technological shock abroad in SM1 (the model without technological spillovers). All variables are expressed in quarterly terms (in %). The value function is in absolute value (so that an increase in the value function reflects an improvement).